Thinking deeply about putting numbers in boxes

A brief survey of some of the research that my colleagues and I have done in the past 30 years

Jeff Dinitz

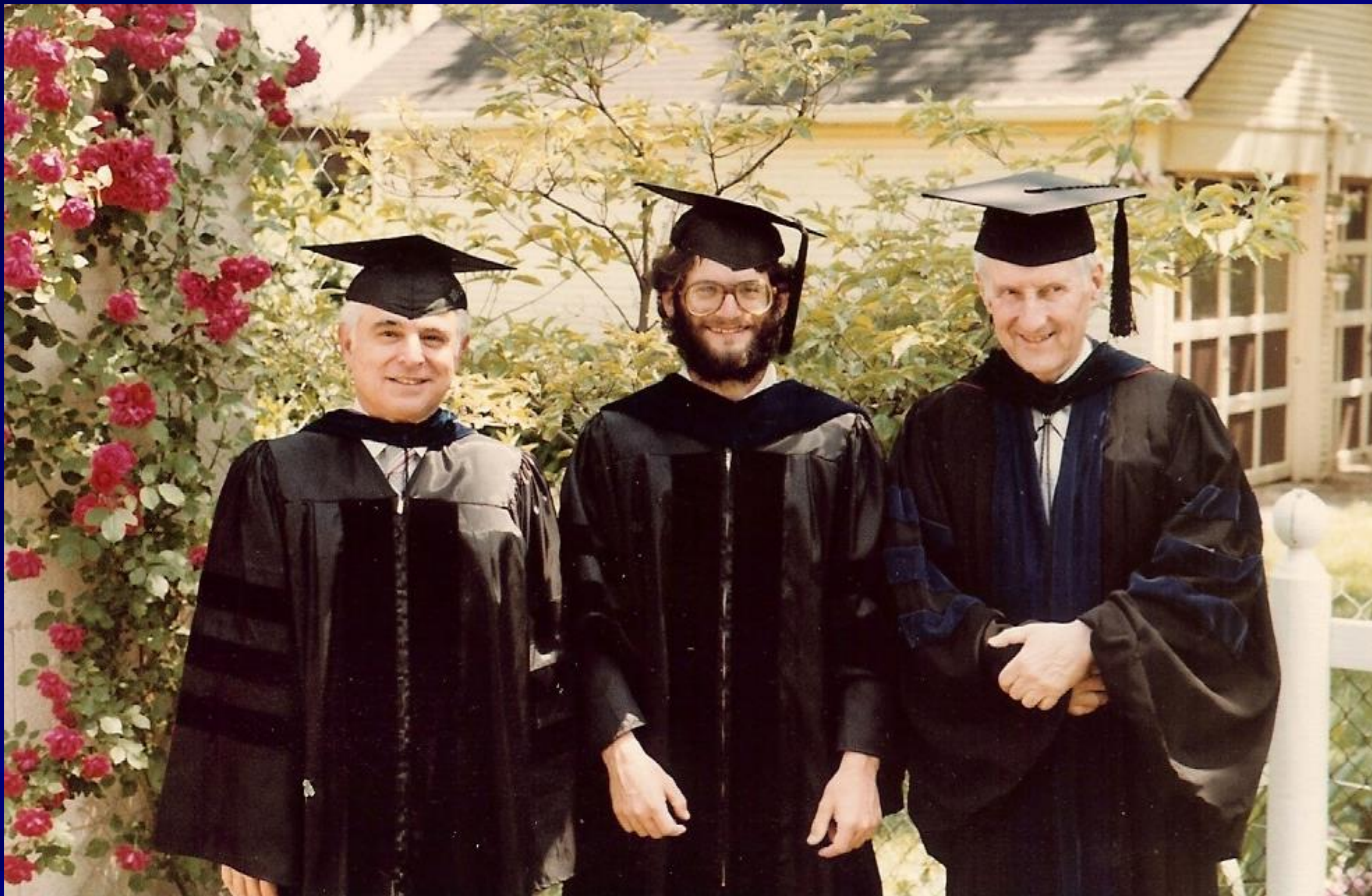
University of Vermont

Dedicate this talk to my father
Simon Dinitz

(1926-2007)

A University Scholar and teaching award winner at The Ohio State University

“Think deeply about simple things”
Arnold Ross
(1906-2002)



Sudoku

Good example of putting numbers in boxes

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Questions about Sudoku

- What is the fewest number of filled cells that are possible in a Sudoku square?

  17 (but not proven yet)

- What is the fewest number of filled cells that can not be completed to a Sudoku square?

1								
			1					
						2	3	4

can you beat 5?

- What about pairs of Sudoku squares?

Is it possible to find two Sudoku squares so that when they are superimposed, all 81 ordered pairs occur in the 81 cells?

1	2	3	7	8	9	4	5	6
4	5	6	1	2	3	7	8	9
7	8	9	4	5	6	1	2	3
3	1	2	9	7	8	6	4	5
6	4	5	3	1	2	9	7	8
9	7	8	6	4	5	3	1	2
2	3	1	8	9	7	5	6	4
5	6	4	2	3	1	8	9	7
8	9	7	5	6	4	2	3	1

8	9	7	4	5	6	3	1	2
2	3	1	7	8	9	6	4	5
5	6	4	1	2	3	9	7	8
1	2	3	9	7	8	5	6	4
4	5	6	3	1	2	8	9	7
7	8	9	6	4	5	2	3	1
6	4	5	2	3	1	7	8	9
9	7	8	5	6	4	1	2	3
3	1	2	8	9	7	4	5	6

Yes!

called *orthogonal* squares

The example above is from
Keely’s Kraus’ 2006 Honors Thesis

A fundamental object in combinatorial design theory is a Latin Square

A Latin Square of side n is an $n \times n$ array where each cell contains a number from an n-set and such that in each row and each column each number occurs exactly once.

An example with $n = 4$

1	2	3	4
3	1	4	2
2	4	1	3
4	3	2	1

Note a finished Sudoku is a 9×9 latin square

First introduced by Leonard Euler in 1784 to solve the “36 officer problem”

Do there exist a pair of orthogonal $6 \leq 6$ latin squares?

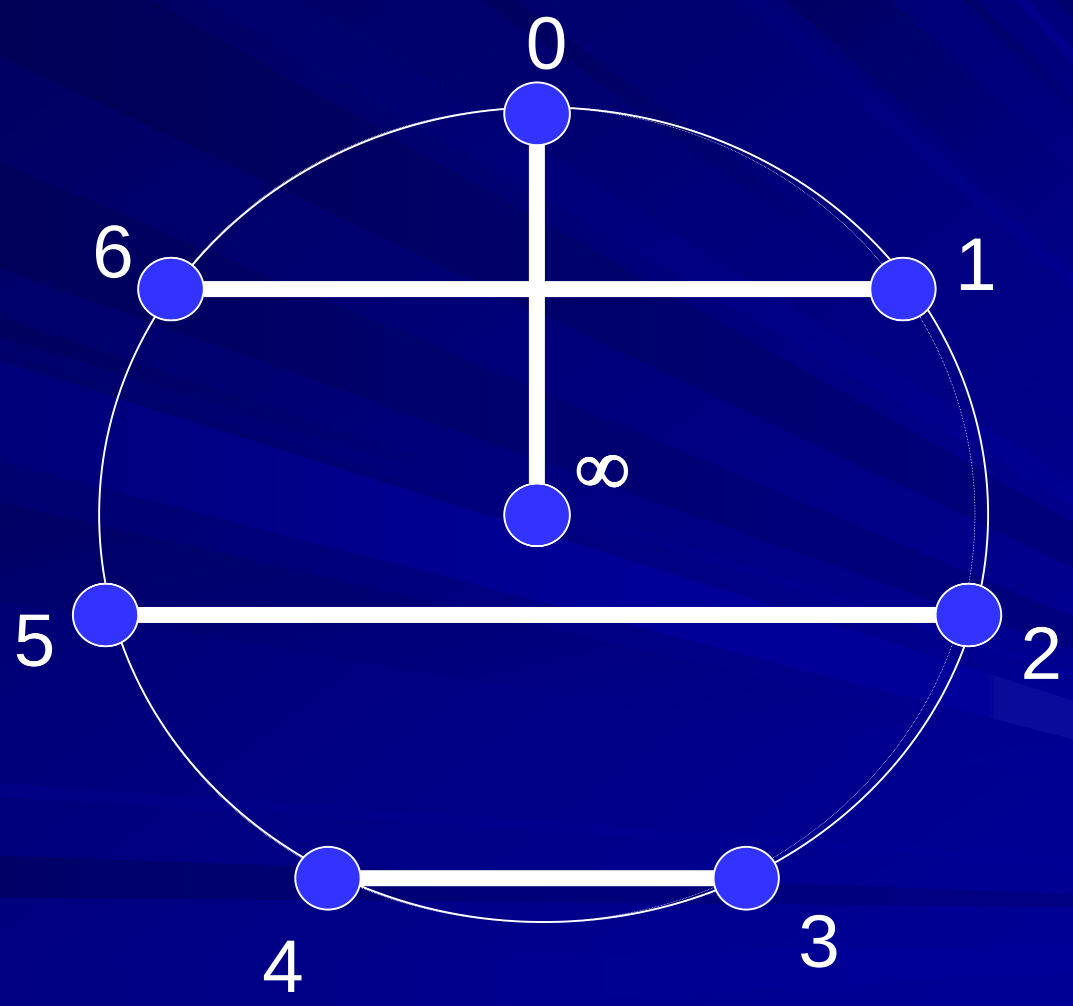
Euler couldn’t find such a pair and conjectured they don’t exist. He further conjectured that no pair of orthogonal latin squares exist for any order n that is “oddly even”, i,e, n= 2,6,10,14,18 …

In 1960 Bose, Parker and Shrikhande proved Euler’s conjecture false, they were called *Euler Spoilers.*

0	4	1	7	2	9	8	3	6	5
8	1	5	2	7	3	9	4	0	6
9	8	2	6	3	7	4	5	1	0
5	9	8	3	0	4	7	6	2	1
7	6	9	8	4	1	5	0	3	2
6	7	0	9	8	5	2	1	4	3
3	0	7	1	9	8	6	2	5	4
1	2	3	4	5	6	0	7	8	9
2	3	4	5	6	0	1	8	9	7
4	5	6	0	1	2	3	9	7	8

0	7	8	6	9	3	5	4	1	2
6	1	7	8	0	9	4	5	2	3
5	0	2	7	8	1	9	6	3	4
9	6	1	3	7	8	2	0	4	5
3	9	0	2	4	7	8	1	5	6
8	4	9	1	3	5	7	2	6	0
7	8	5	9	2	4	6	3	0	1
4	5	6	0	1	2	3	7	8	9
1	2	3	4	5	6	0	9	7	8
2	3	4	5	6	0	1	8	9	7

For me, it all started with the figure below -- which I saw in my first combinatorics course in graduate school in 1974.



It looks cool. But what's it good for?

Constructing Round Robin Tournaments

Every player plays every other player exactly once and the games are played in rounds where each player plays once in each round.

Example: 4 players

Round 1	1-2	3-4
Round 2	1-3	2-4
Round 3	1-4	2-3

With *n* players (n even) there are n -1 rounds and each round contains n/2 games. So total number of games is n(n-1)/2

Example: 8 players (7 rounds, 4 games per round)

Round 1	0-1	2-3	4-5	6-7
Round 2	0-2			
Round 3	0-3			
Round 4	0-4			
Round 5	0-5			
Round 6	0-6			
Round 7	0-7			

Example: 8 players (7 rounds, 4 games per round)

Round 1	0-1	2-3	4-5	6-7
Round 2	0-2	1-3	4-6	5-7
Round 3	0-3	1-2	4-7	5-6
Round 4	0-4			
Round 5	0-5			
Round 6	0-6			
Round 7	0-7			

Example: 8 players (7 rounds, 4 games per round)

Round 1	0-1	2-3	4-5	6-7
Round 2	0-2	1-3	4-6	5-7
Round 3	0-3	1-2	4-7	5-6
Round 4	0-4	1-5	2-3	
Round 5	0-5			
Round 6	0-6			
Round 7	0-7			

Example: 8 players (7 rounds, 4 games per round)

Round 1	0-1	2-3	4-5	6-7
Round 2	0-2	1-3	4-6	5-7
Round 3	0-3	1-2	4-7	5-6
Round 4	0-4	1-5	2-3	oops (need 6-7)
Round 5	0-5			
Round 6	0-6			
Round 7	0-7			

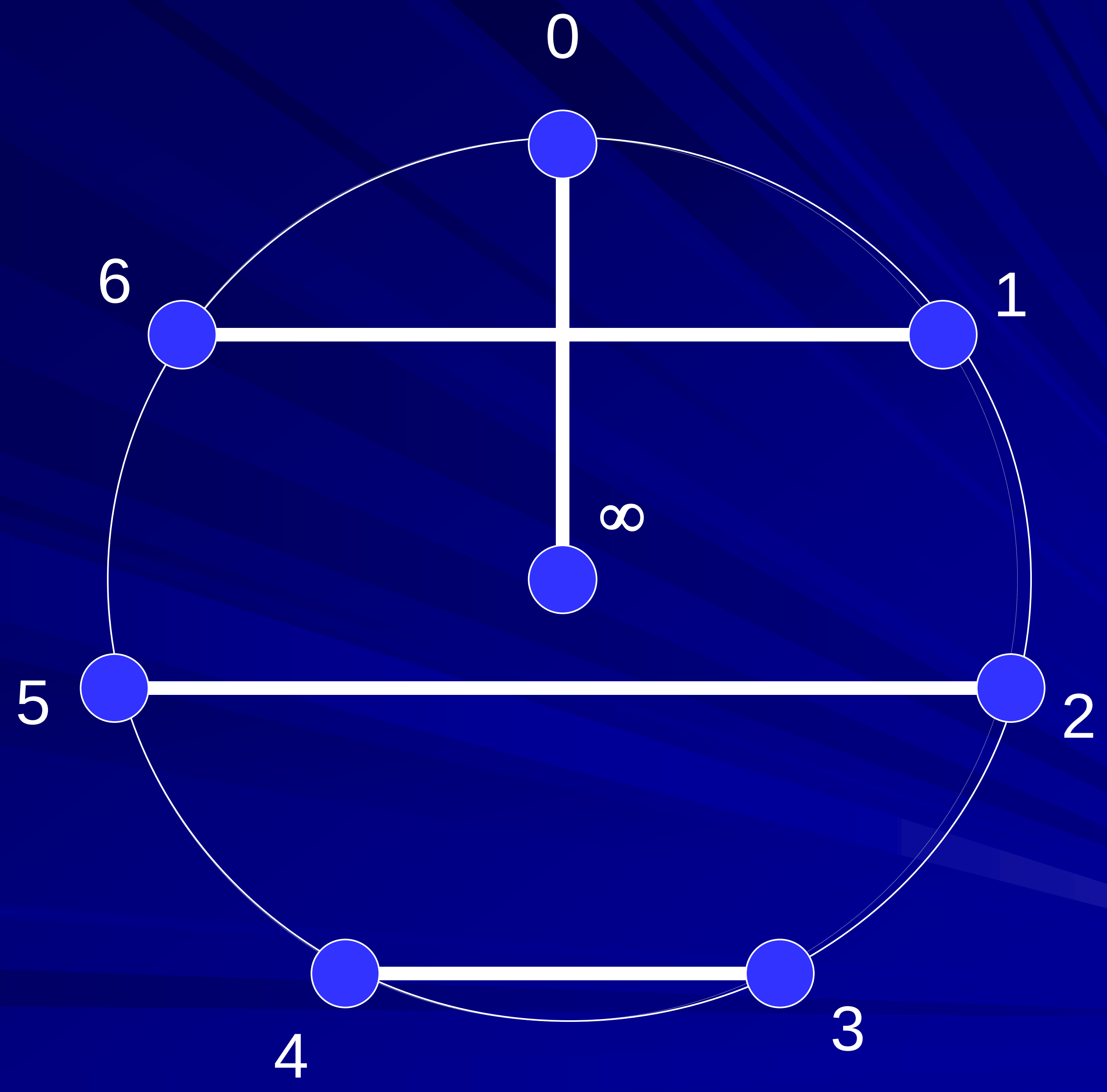
As the number of players gets bigger, the problem gets much harder.

Can we construct a round robin tournament for any number of players?

Starting at 8 players it is tough, so mathematicians try to impose structure in order to reduce the size of the problem and make it more tractable.

There are 7 rounds so might assume a cyclic structure on the weeks (we'll try Z_7). In other words, if we can find a "good" first week we can use it to generate all the other weeks.

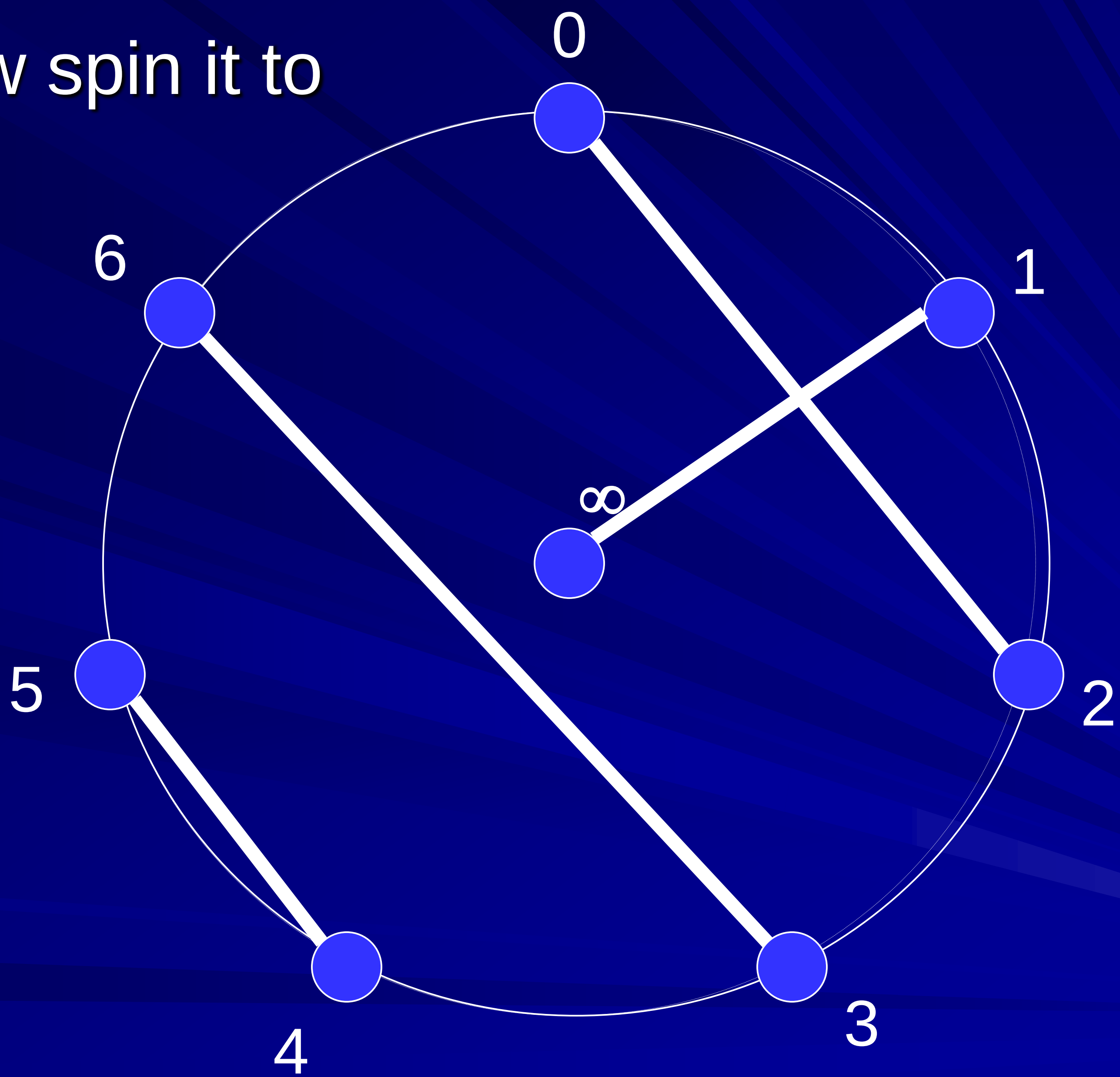
Something like …



0
6
1
∞
5
2
4
3

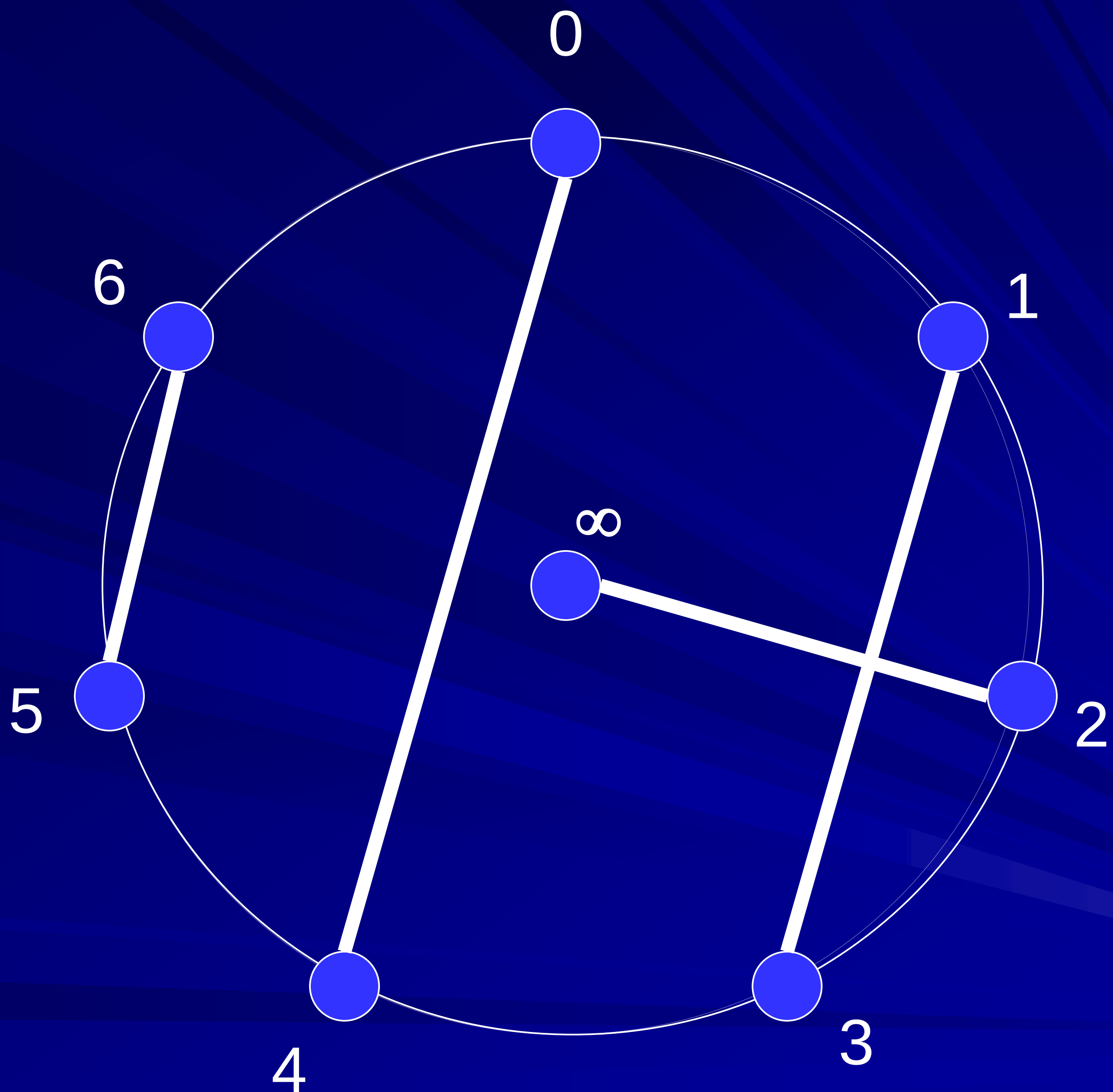
Rounds	0					
games	0, ∞					
	1,6					
	2,5					
	3,4					

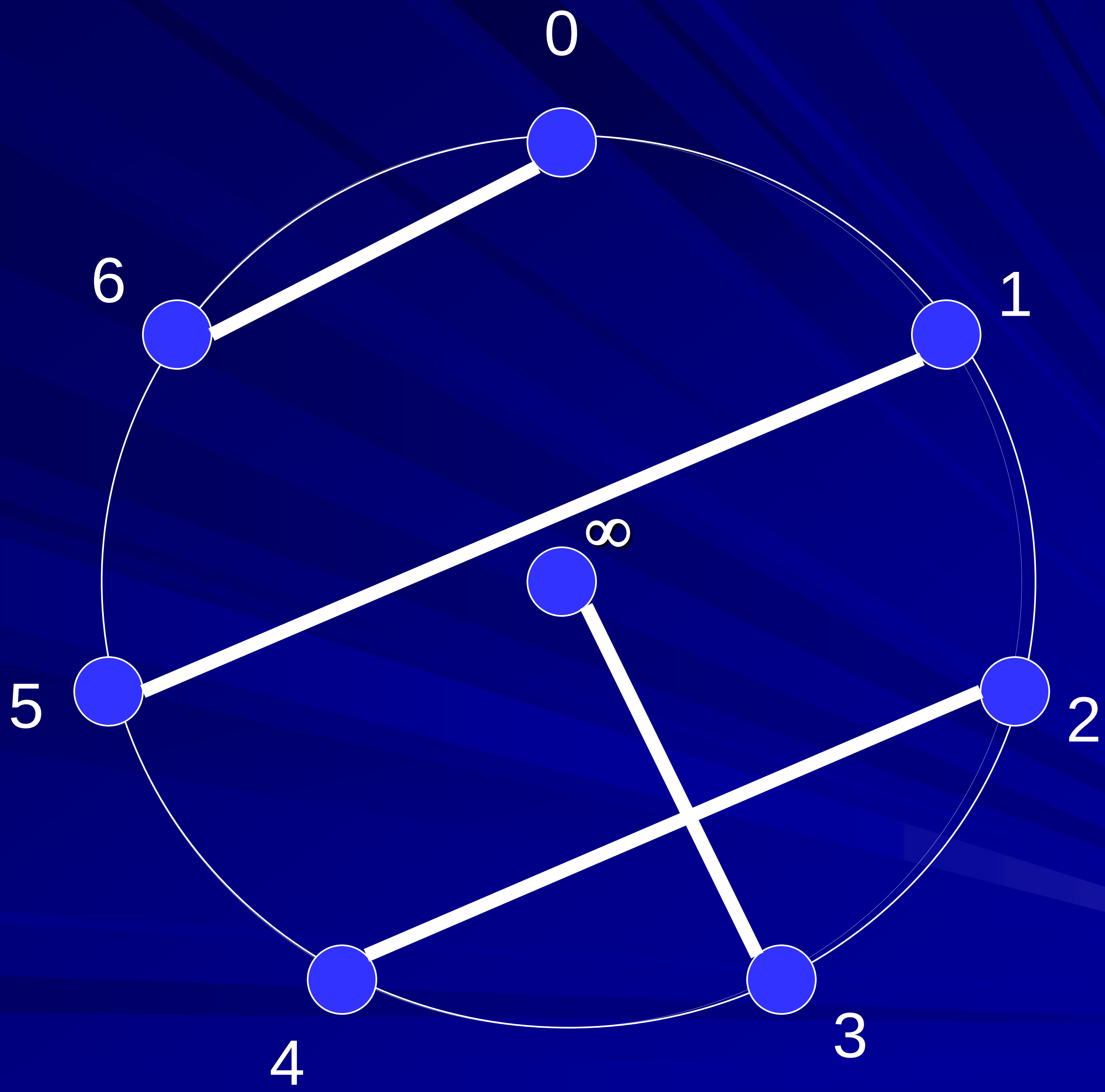
- now spin it to get

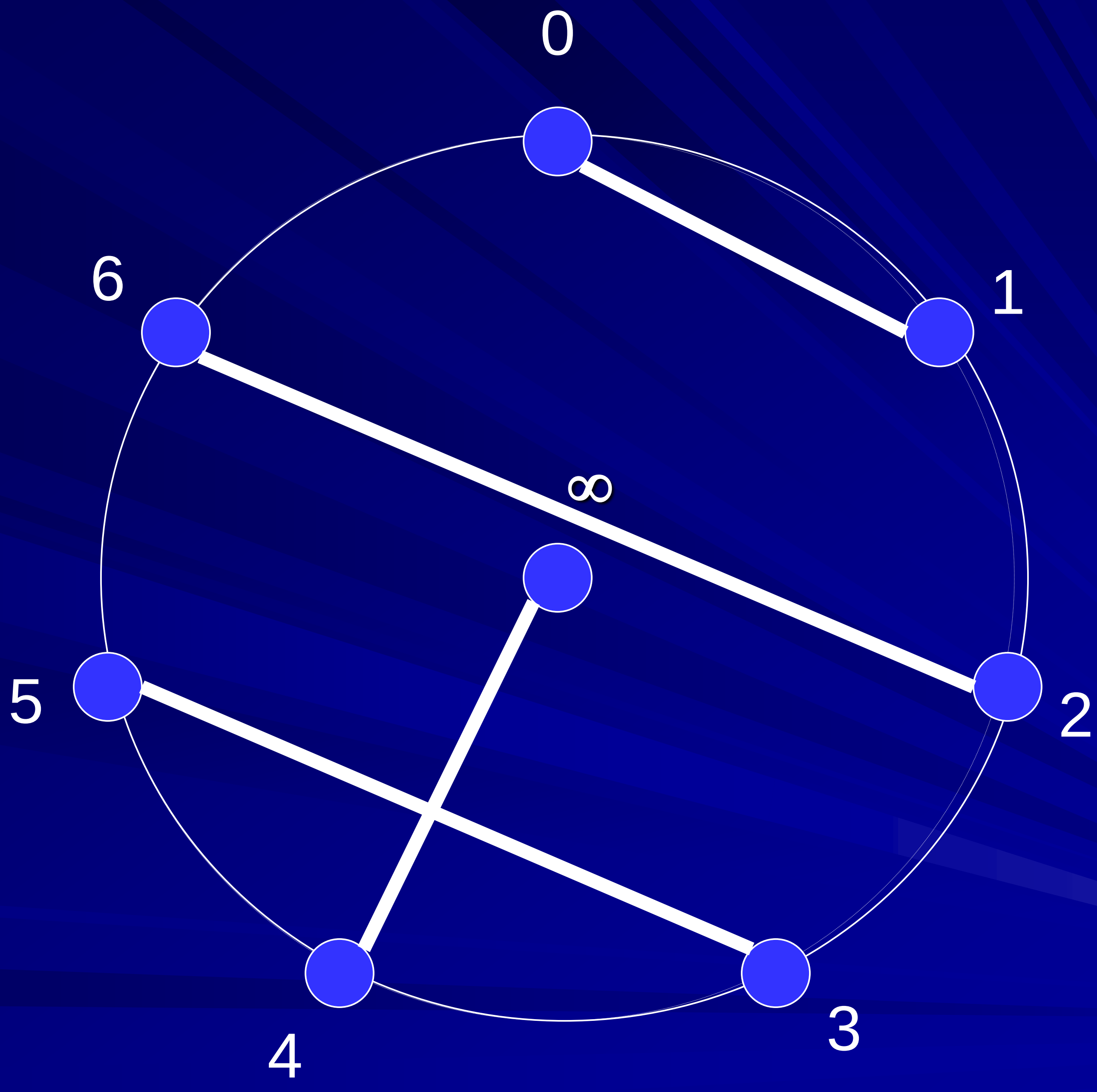


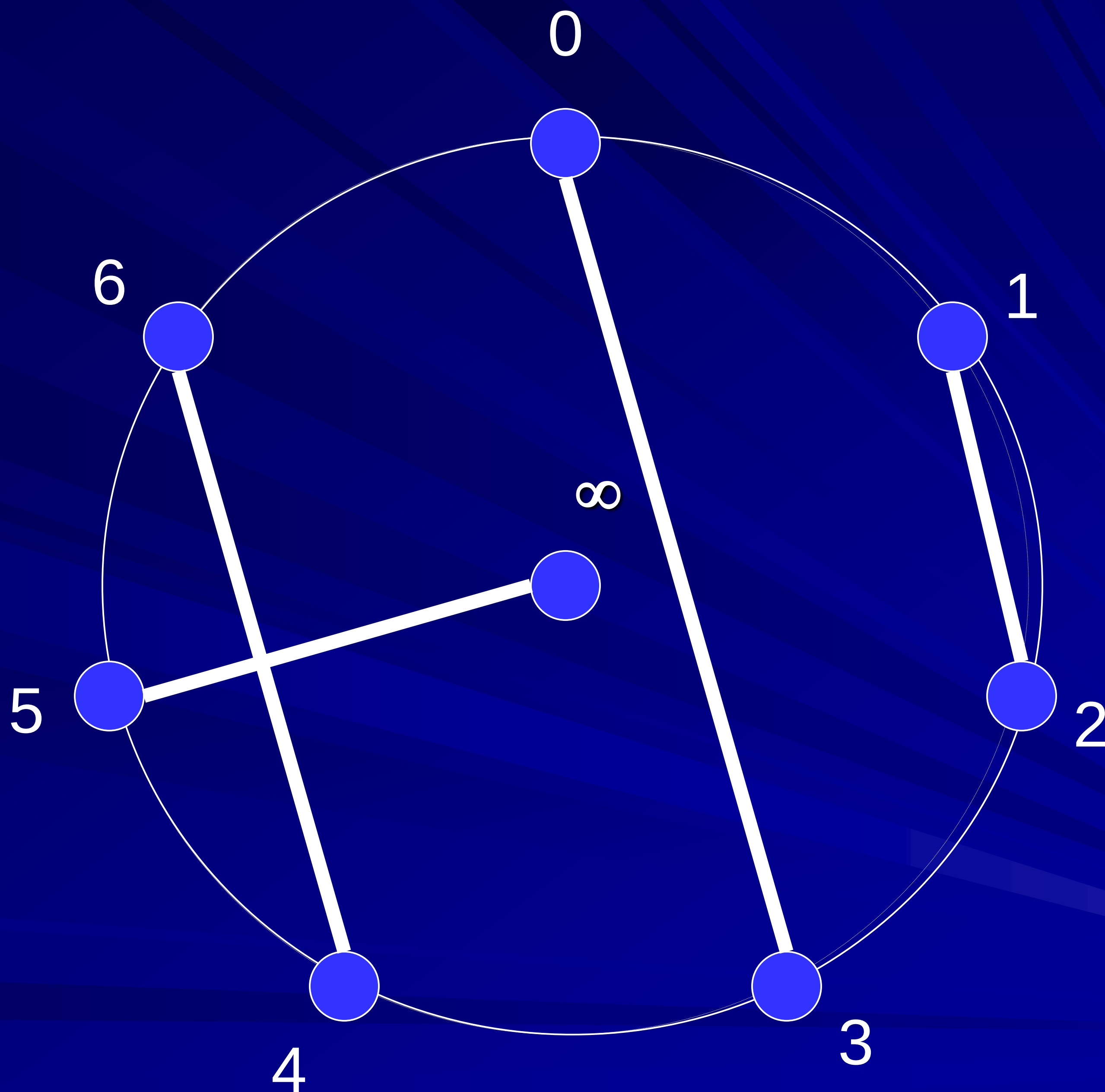
Rounds	0	1					
games	0, ∞	1, ∞					
	1,6	2,0					
	2,5	3,6					
	3,4	4,5					

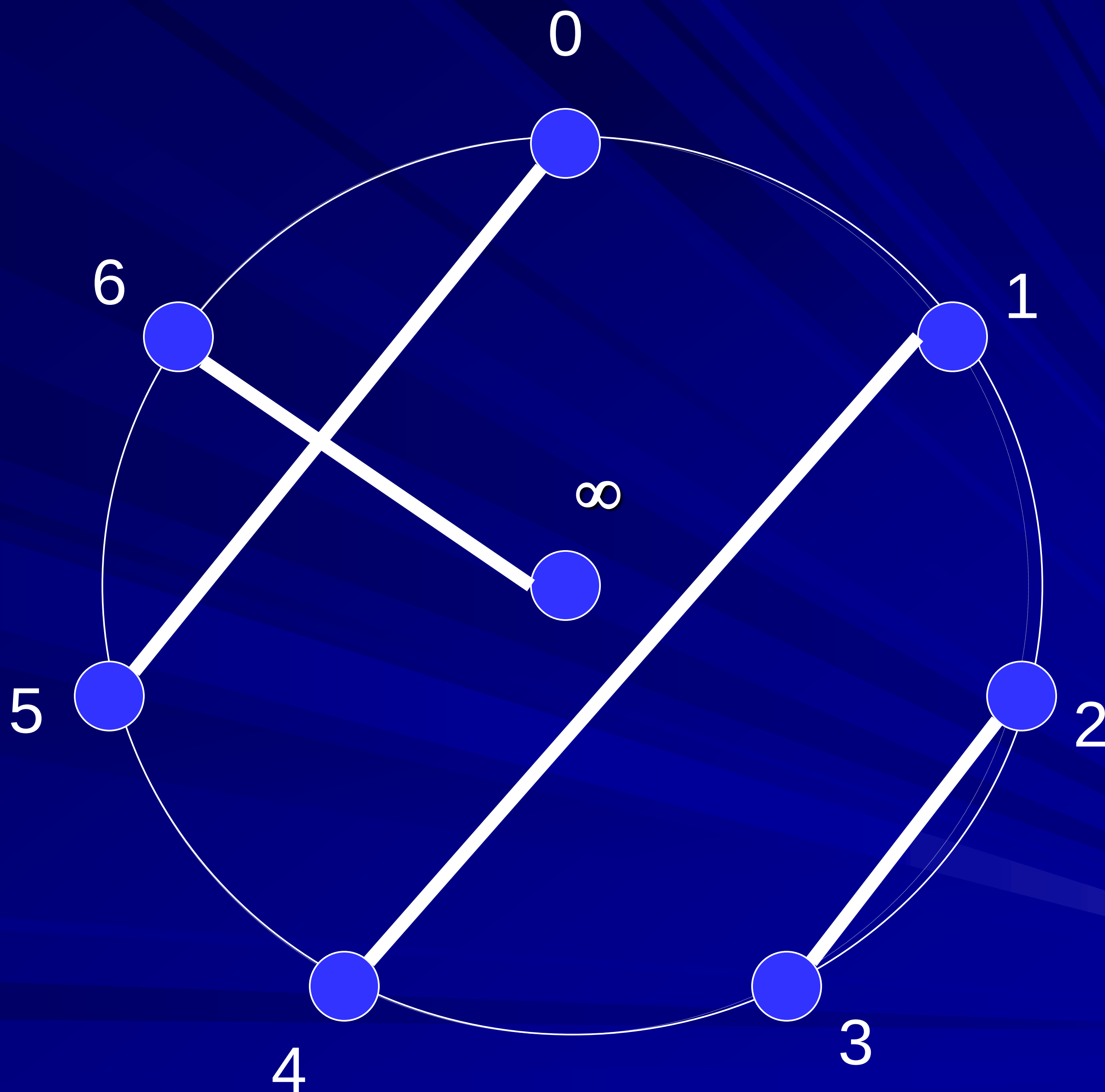
Round 2, 3, …


0
1
2
3
4
5
6
∞


0
6
1
∞
5
2
4
3


0
1
2
3
4
5
6
∞


0
1
2
3
4
5
6
∞

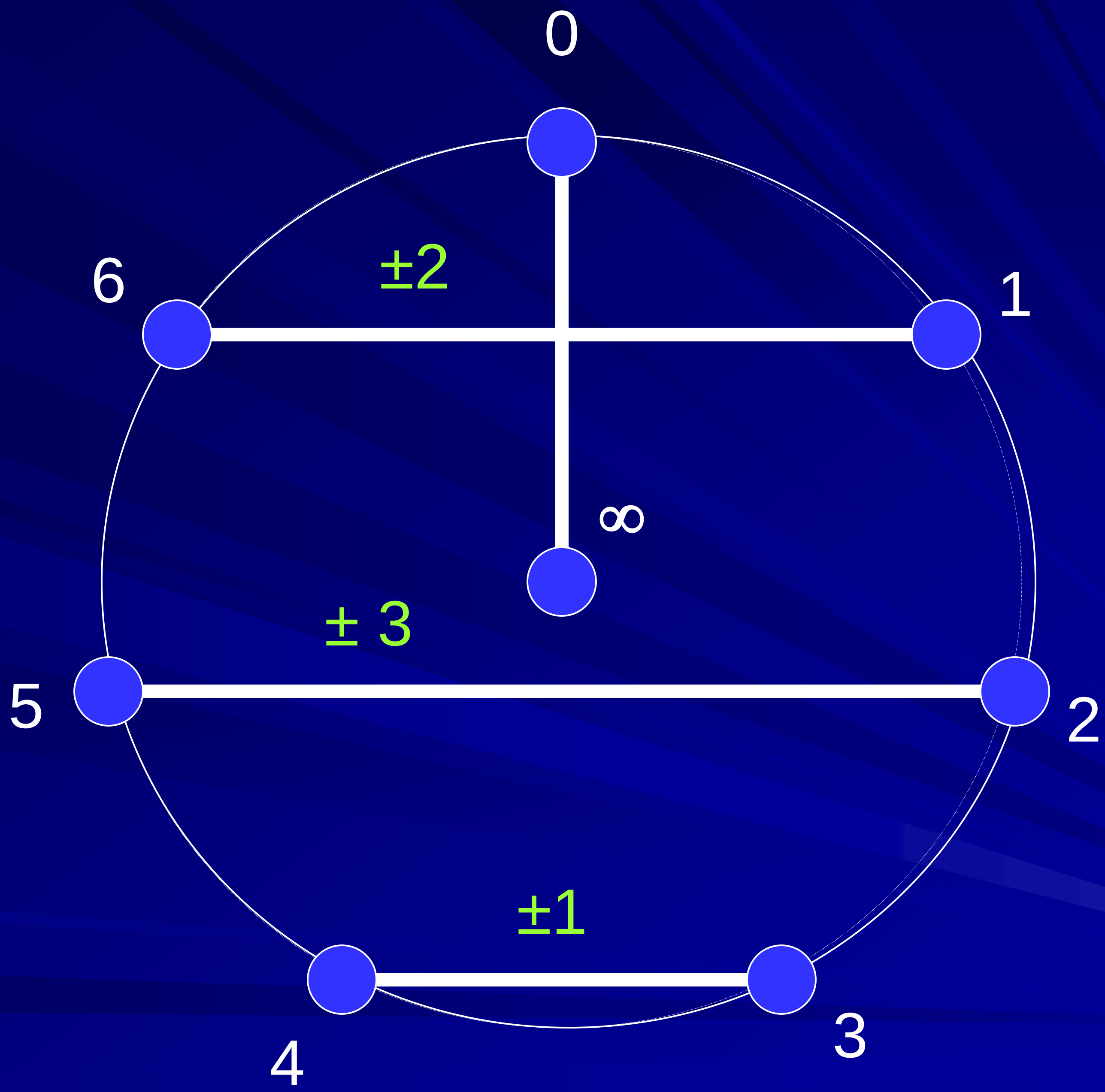

0
6
1
∞
5
2
4
3

Rounds	0	1	2	3	4	5	6
games	0, ∞	1, ∞	2, ∞	3, ∞	4, ∞	5, ∞	6, ∞
	1,6	2,0	3,1	4,2	5,3	6,4	0,5
	2,5	3,6	4,0	5,1	6,2	0,3	1,4
	3,4	4,5	5,6	6,0	0,1	1,2	2,3

This round-robin tournament is called GK(8).

This (cyclic) construction works for 8 players.

Why?


0
6
±2
1
∞
± 3
5
2
±1
4
3

Rounds	0	1	2	3	4	5	6
$d = \infty$	0, ∞	1, ∞	2, ∞	3, ∞	4, ∞	5, ∞	6, ∞
± 2	1,6	2,0	3,1	4,2	5,3	6,4	0,5
± 3	2,5	3,6	4,0	5,1	6,2	0,3	1,4
± 1	3,4	4,5	5,6	6,0	0,1	1,2	2,3

This round-robin tournament is called GK(8).

0,∞	1,∞	2,∞	3,∞	4,∞	5,∞	6,∞
1,6	2,0	3,1	4,2	5,3	6,4	0,5
2,5	3,6	4,0	5,1	6,2	0,3	1,4
3,4	4,5	5,6	6,0	0,1	1,2	2,3

The schedule for a round robin tournament can be encoded in a Latin square.

If teams *a* and *b* play in round *r*, then place an *r* in cells (*a,b*) and (*b,a*) of the latin square.

0	4	1	5	2	6	3
4	1	5	2	6	3	0
1	5	2	6	3	0	4
5	2	6	3	0	4	1
2	6	3	0	4	1	5
6	3	0	4	1	5	2
3	0	4	1	5	2	6

0,∞	1,∞	2,∞	3,∞	4,∞	5,∞	6,∞
1,6	2,0	**3,1**	4,2	5,3	6,4	0,5
2,5	3,6	4,0	5,1	6,2	0,3	1,4
3,4	4,5	5,6	6,0	0,1	1,2	2,3

The schedule for a round robin tournament can be encoded in a Latin square. If teams *a* and *b* play in round *r*, then place an *r* in cells (*a,b*) and (*b,a*) of the latin square.

Example: 3 plays 1 in round 2

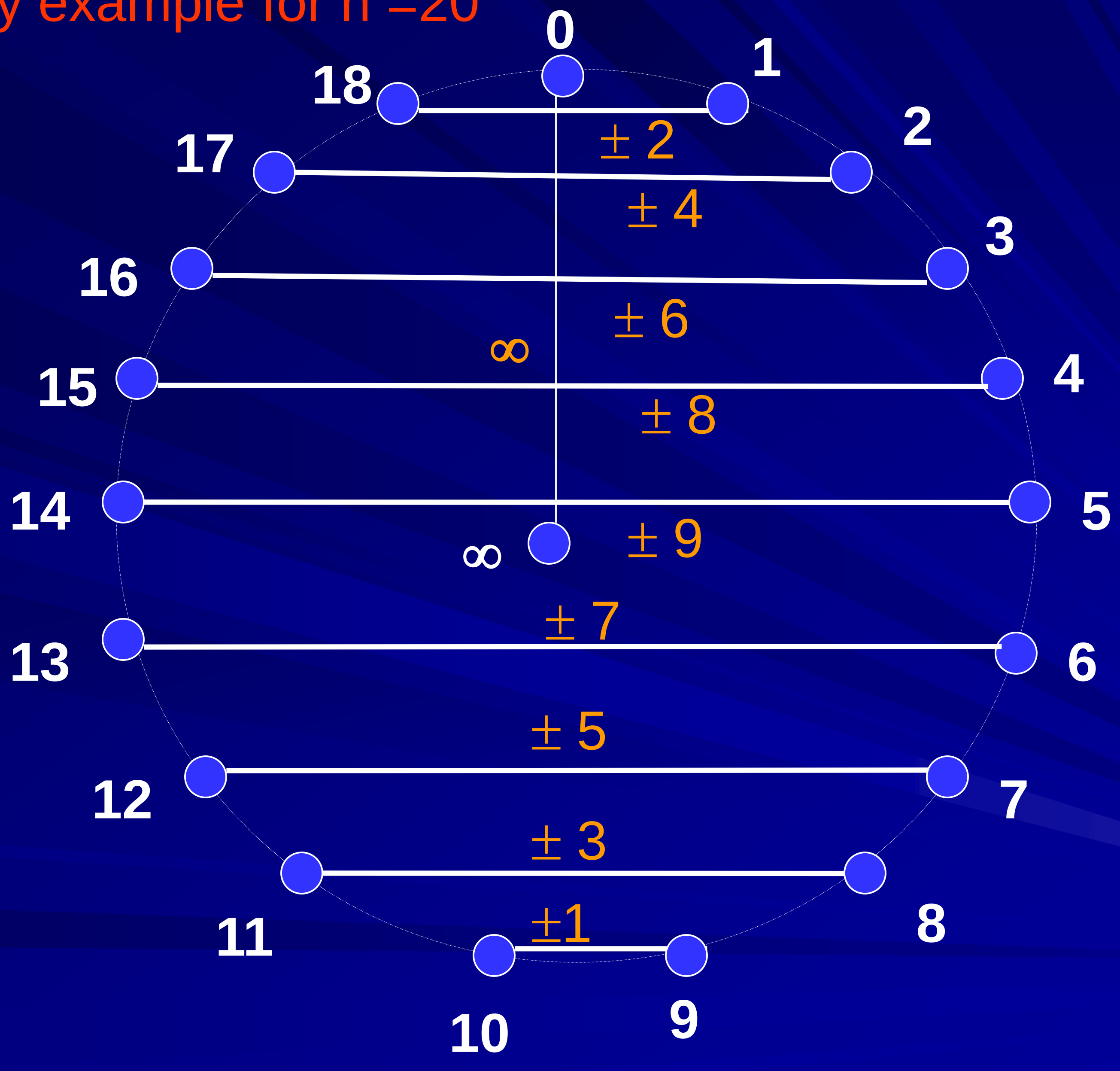
		1		3			
	0	4	1	5	2	6	3
1	4	1	5	**2**	6	3	0
	1	5	2	6	3	0	4
3	5	**2**	6	3	0	4	1
	2	6	3	0	4	1	5
	6	3	0	4	1	5	2
	3	0	4	1	5	2	6

creates a symmetric idempotent Latin square

This (cyclic) construction works for any even number of players.

Why?

Proof: by example for n =20



So answer is yes – we can do this for any even number of players.

What about an odd number of players?

Rounds	0	1	2	3	4	5	6
$d = \infty$	**0, ∞**	**1, ∞**	**2, ∞**	**3, ∞**	**4, ∞**	**5, ∞**	**6, ∞**
± 2	**1,6**	**2,0**	**3,1**	**4,2**	**5,3**	**6,4**	**0,5**
± 3	**2,5**	**3,6**	**4,0**	**5,1**	**6,2**	**0,3**	**1,4**
± 1	**3,4**	**4,5**	**5,6**	**6,0**	**0,1**	**1,2**	**2,3**

Just kick out player ∞.

Her opponent in each round gets a bye.

Rounds	0	1	2	3	4	5	6
$d = \infty$							
± 2	1,6	2,0	3,1	4,2	5,3	6,4	0,5
± 3	2,5	3,6	4,0	5,1	6,2	0,3	1,4
± 1	3,4	4,5	5,6	6,0	0,1	1,2	2,3

Just kick out player ∞.

Her opponent in each round gets a bye.

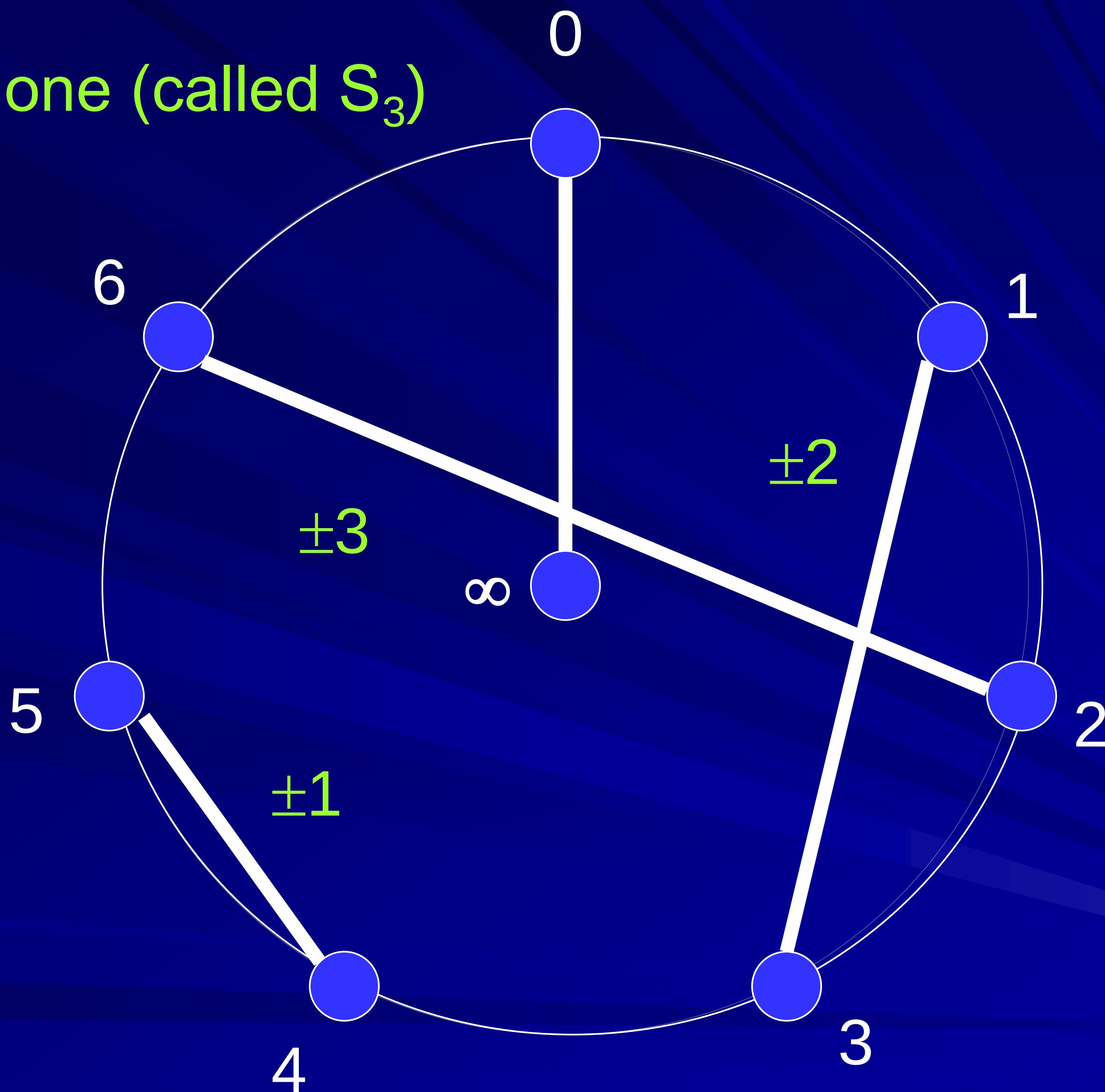
So answer is yes – we can do this for any even number of players.

What about an odd number of players?

Can do!

Are there other constructions for round robin tournaments using symmetries?

Here's one (called S_3)

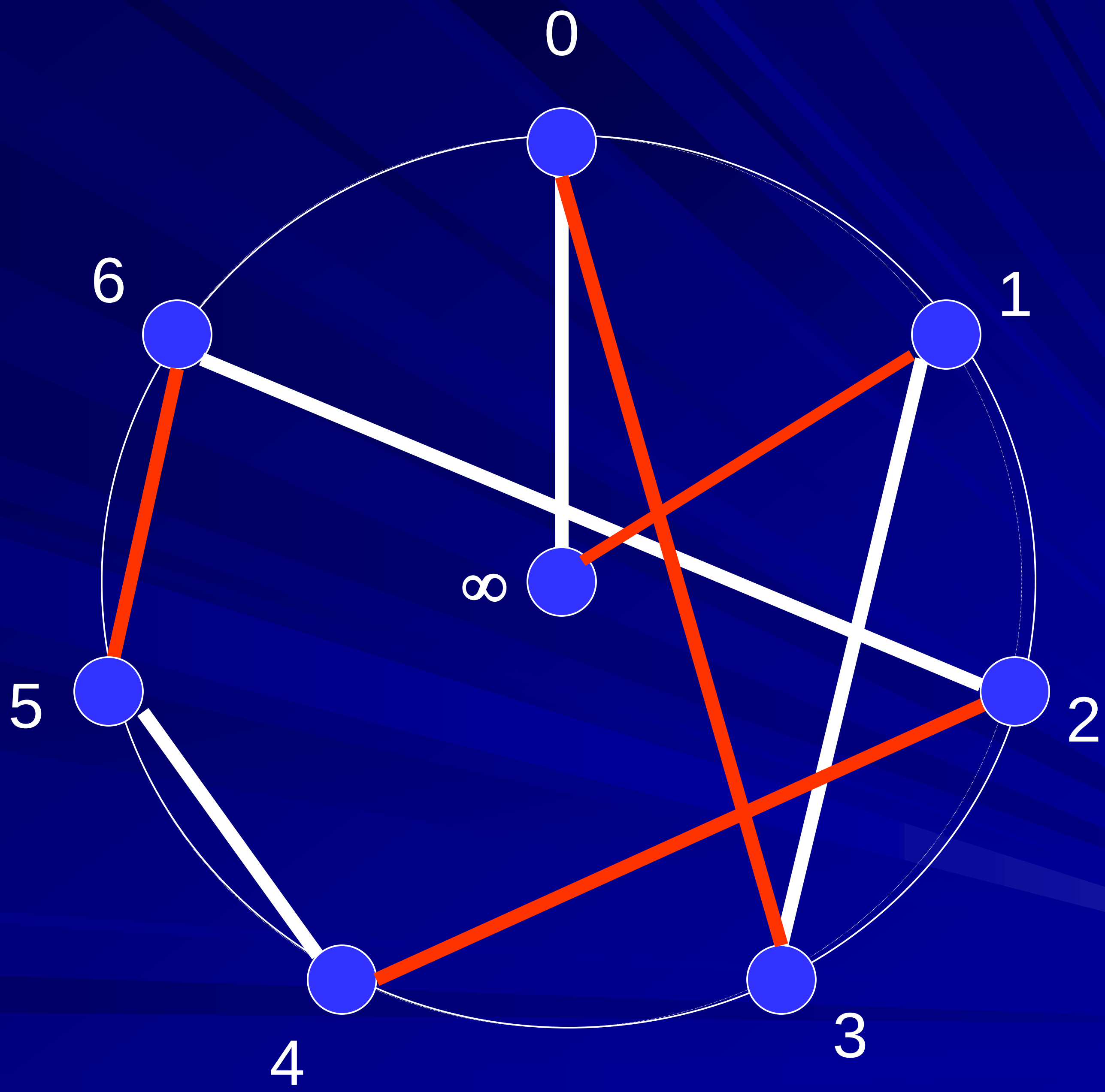


In general these are termed Starters.

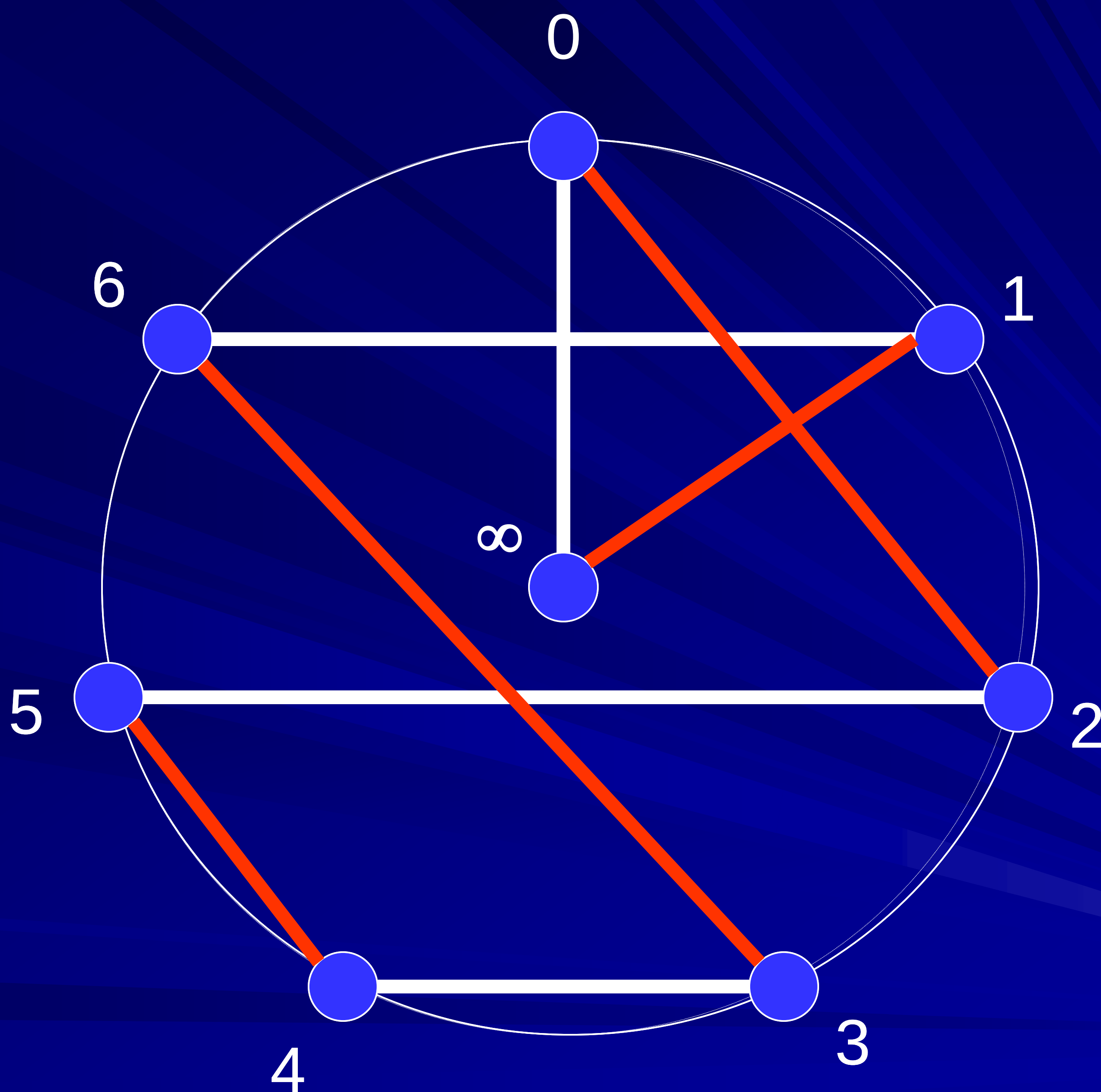
The earlier one was called P, the patterned starter.

The tournament that is obtained from this example is not isomorphic to GK(8).

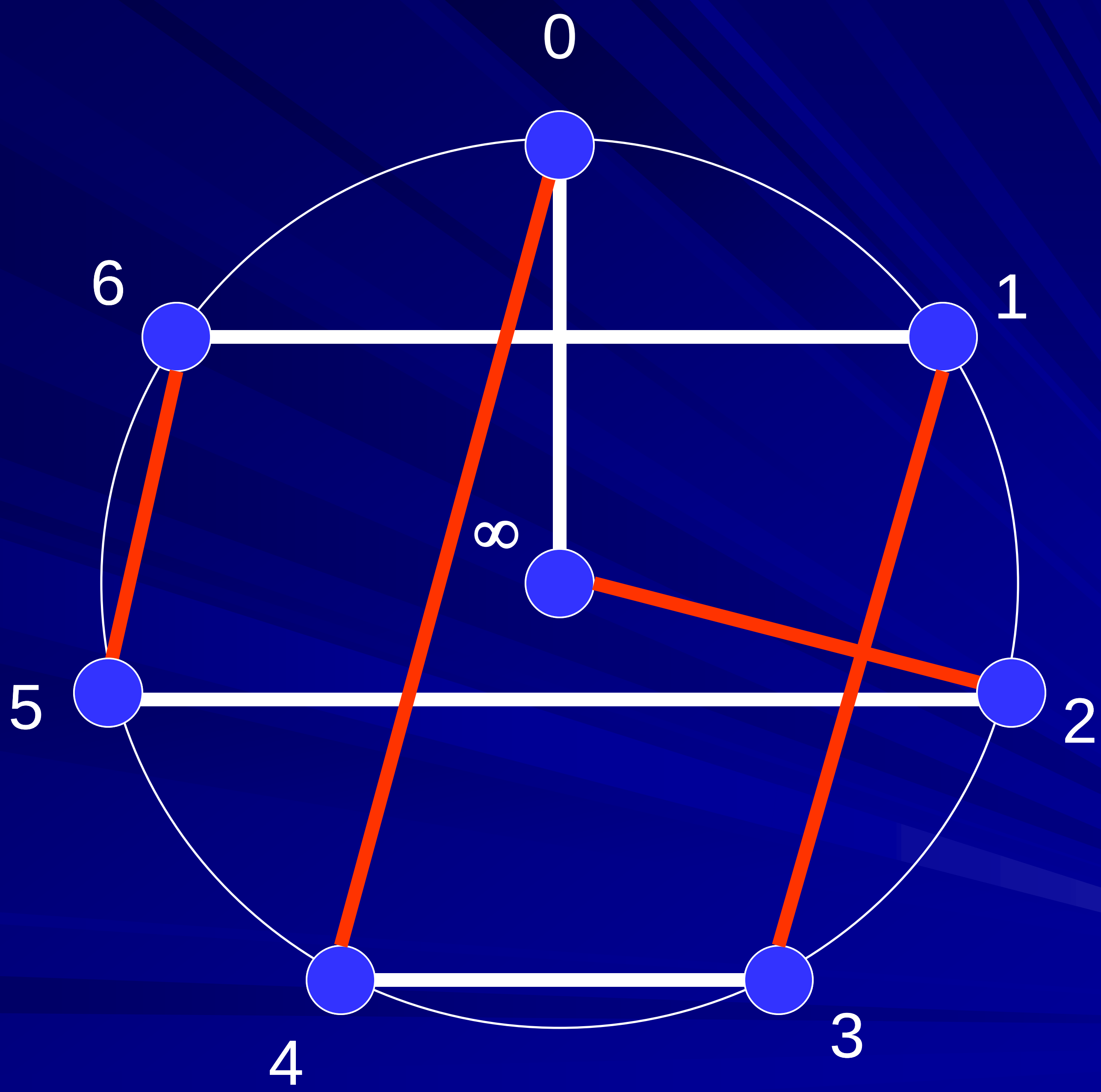
This means that there is no way to relabel the players or reorder the weeks to make the tournaments look alike.



$S_3 \cup (S_3 + 1)$ is composed of two 4-cycles



P ∪ (P+1) is an 8-cycle.



P ∪ (P+2) is an 8-cycle. Same for every pair of weeks!!.

How many nonisomorphic round robin tournaments are there on n players?

At least two for n=8

This is a good example of the so called combinatorial explosion

Here is a table of small orders up to n = 10

n	# nonisomorphic	
2	1	
4	1	
6	1	
8	6	Safford, 1906
10	396	Gelling, 1973

In 1994, we considered the case of n =12. David Garnick of Bowdoin College designed and programmed our orderly algorithm.

Build up the tournament in weeks. Define a lexicographic order on a partial tournament and discard all but the smallest partial tournament in each isomorphism class.

The program took 8.2 CPU years to run, but was parallelized and ran in about 8 months (on and off).

Brendan McKay (ANU) found way to verify answer was correct (counted in two ways).

The current table (as of 1994)

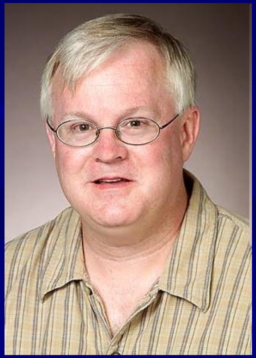
n	# nonisomorphic	
2	1	
4	1	
6	1	
8	6	Safford 1906
10	396	Gelling 1973
12	526,915,620	D,G,M 1994
14	1.132×10^{18} est.	D,G,M 1994
16	7.07×10^{30} est.	D,G,M 1994

The current table

n	# nonisomorphic	
2	1	
4	1	
6	1	
8	6	Safford, 1906
10	396	Gelling, 1973
12	526,915,620	D,G,M, 1994
14	1,132,835,421,602,062,347	Keri, Östergård, 2009
16	7.07×10^{30} est.	D,G,M 1994

Through the years I've researched many questions related to starters and 1-factorizations

- A fast algorithm for finding strong starters (with D.R. Stinson), *SIAM J. on Algebraic and Discrete Methods*, **2** (1981), pp. 50-56.
- Some new perfect 1-factorizations from starters in finite fields (with D.R. Stinson), *Journal of Graph Theory* **13** (1989), pp. 405-415.
- Trains: an invariant for one-factorizations (with W.D. Wallis), *Ars Combinatoria* **32** (1991), pp. 161-180.
- Constructing indecomposable 1-factorizations of the complete multigraph (with D.S. Archdeacon), *Discrete Math* **92** (1991), pp. 9-19.
- Uniform Room frames with five holes (with E.R. Lamken), *Journal of Combinatorial Designs* **1** (1993), pp. 323-328.
- On the structure of uniform one-factorizations from starters in finite fields (with P. Dukes), *Finite Fields and Applications*, **12** (2006), 283 - 300.



Doug Stinson

Wal Wallis

Dan Archdeacon

Esther Lamken

Peter Dukes

To Boxes now

Remember GK(8)

Rounds / games	0	1	2	3	4	5	6
	0, ∞	1, ∞	2, ∞	3, ∞	4, ∞	5, ∞	6, ∞
	1,6	2,0	3,1	4,2	5,3	6,4	0,5
	2,5	3,6	4,0	5,1	6,2	0,3	1,4
	3,4	4,5	5,6	6,0	0,1	1,2	2,3

Assume there are 4 different sites and we want to balance number of times each player plays at each site.

rounds

sites

	0, ∞	**1, ∞**	**2, ∞**	**3, ∞**	**4, ∞**	**5, ∞**	**6, ∞**
±2	**1,6**	**2,0**	**3,1**	**4,2**	**5,3**	**6,4**	**0,5**
±3	**2,5**	**3,6**	**4,0**	**5,1**	**6,2**	**0,3**	**1,4**
±1	**3,4**	**4,5**	**5,6**	**6,0**	**0,1**	**1,2**	**2,3**

Not very balanced (for player ∞)

Balanced tournament designs

An arrangement of the games of a round robin tournament so that each player plays at each site at most twice.

α 4	∞ 2	1 3	5 7	0 6	2 3	4 5	∞ 7	α 1
∞ 3	α 5	4 6	0 2	1 7	∞ 4	α 2	0 5	6 3
5 6	0 3	α 7	∞ 1	4 2	6 7	0 1	α 3	∞ 5
1 2	4 7	∞ 0	α 6	5 3	α 0	∞ 6	1 4	7 2
0 7	1 6	2 5	4 3	α ∞	1 5	3 7	2 6	0 4

A balanced tournament design on 10 players (5 sites)

Can you make one for any (even) number of players

In 1977, Hasselgrove and Leach gave a nice construction of a balanced tournament design on 2n players that started with GK(2n) and just shifted the cells a bit. However this only worked when $2n \equiv 0$ or 2 (mod 3).

Using techniques from combinatorial design theory Schellenberg, van Rees and Vanstone proved (1978) that there is a balanced tournament design for any even number of players (except when $2n = 4$)

so, yes you can.

How many *different* ones are there?

2n	number	Ref.
2,6	1	
8	47	Corriveau (1988)
10	30,220,557	Dinitz, Dinitz (2005)

In general, there are about 90,000 nonisomorphic balanced tournament designs for each of the 396 nonisomorphic tournaments on 10 players. The actual number ranges from a low of 293 to a high of 103,912.

A different site each week?

- Can we make a round robin tournament with 8 teams over 7 weeks at 7 sites where each team:

plays each other team exactly once,

plays one game per week, and

plays at each site exactly once?

Yes – here it is

weeks \ sites	0	1	2	3	4	5	6
0	0,∞	3,4	1,6		5,2		
1		1,∞	4,5	2,0		6,3	
2			2,∞	5,6	3,1		0,4
3	1,5			3,∞	6,0	4,2	
4		2,6			4,∞	0,1	5,3
5	6,4		3,0			5,∞	1,2
6	2,3	0,5		4,1			6,∞

Patterned starter

S_5

Note that the rows and columns both form round robin tournaments (called orthogonal one-factorizations)

Room Squares

A Room square of side *n*, RS(*n*), defined on an *n*+1 set *S* is an $n \le n$ array, *R*, satisfying

1) every cell of *R* either is empty or contains an unordered pair of symbols from *S*,
2) each symbol of *S* occurs once in each row and column
3) every unordered pair of symbols occurs in precisely one cell of R.

0,∞	3,4	1,6		5,2		
	1,∞	4,5	2,0		6,3	
		2,∞	5,6	3,1		0,4
1,5			3,∞	6,0	4,2	
	2,6			4,∞	0,1	5,3
6,4		3,0			5,∞	1,2
2,3	0,5		4,1			6,∞

A RS(7) solved our question.

Questions about Room squares

- Can you make them for all odd orders?
- How many nonisomorphic ones are here?
- Why are they called Room squares?
- What would you do with three mutually orthogonal one-factorizations? More balance?
- Any connection to balanced tournament designs?

Questions about Room squares

- Can you make them for all odd orders?

$n=1$

1,2

$n=3$

1,2	3,4	

No Way!

Theorem (Mullin, Wallis 1975) There exists a Room square of side n for $n=1$ and for all odd $n \geq 7$.

Questions about Room squares

- Can you make them for all odd orders? YES.
- How many nonisomorphic ones are here?
- Why are they called Room squares?
- What would you do with three mutually orthogonal one-factorizations of K_n? More balance?
- Any connection to balanced tournament designs?

How many nonisomorphic ones are here?

Let NR(n) denote number of nonisomorphic Room squares of side n.

n	5	7	9
NR(n)	0	6	257,630

In 1983, Doug Stinson and I derived a general lower bound using recursive constructions.

Theorem (D., Stinson 1983) For all odd $n \geq 153$,

$$NR(n) \geq .19\, e^{.04 n^2}$$

Example: $NR(153) \geq 8.6 \times 10^{405}$

On nonisomorphic Room squares (with D.R. Stinson), *Proc. of American Math Soc.*, 89. 1983, pp. 175-181.

Questions about Room squares

- Can you make them for all odd orders? YES.
- How many nonisomorphic ones are here? LOTS!
- Why are they called Room squares?
- What would you do with three mutually orthogonal one-factorizations? More balance?
- Any connection to balanced tournament designs?

Why are they called Room squares?

Named after the Australian statistician T.G. Room who published a paper in 1955 in which he proved that Room squares of side 3 and 5 do not exist and constructed one of order 7.

But…

VII. *On a Tactical Theorem relating to the Triads of Fifteen Things.* By A. CAYLEY, *Esq.**

THE school-girl problem may be stated as follows:—"With 15 things to form 35 triads, involving all the 105 duads, and such that they can be divided into 7 systems, each of 5 triads containing all the 15 things." A more simple problem is, "With 15 things, to form 35 triads involving all the 105 duads."

In the solution which I formerly gave of the school-girl problem (Phil. Mag. vol. xxxvii. 1850), and which may be presented in the form

	a	*b*	*c*	*d*	*e*	*f*	*g*
abc				35	17	82	64
ade		62	84			15	37
afg		13	57	86	42		
bdf	47		16		38		25
bge	58		23	14		67	
cdg	12	78			56	34	
cef	36	45		27			18

(viz. the things being *a*, *b*, *c*, *d*, *e*, *f*, *g*, 1, 2, 3, 4, 5, 6, 7, 8, the first pentad of triads is *abc*, *d* 35, *e* 17, *f* 82, *g* 64, and so for all the seven pentads of triads), there is obviously a division of the 15 things into (7+8) things, viz. the 35 triads are composed 7 of them each of 3 out of the 7 things, and the remaining 28 each of 1 out of the 7 things, and 2 out of the 8 things: or attending only to the 8 things, there are 28 triads each of them containing a duad of the 8 things, but there is no triad consisting of 3 of the 8 things. More briefly, we may say that in the system there is an 8 without 3, that is, there are 8 things, such that no triad of them occurs in the system.

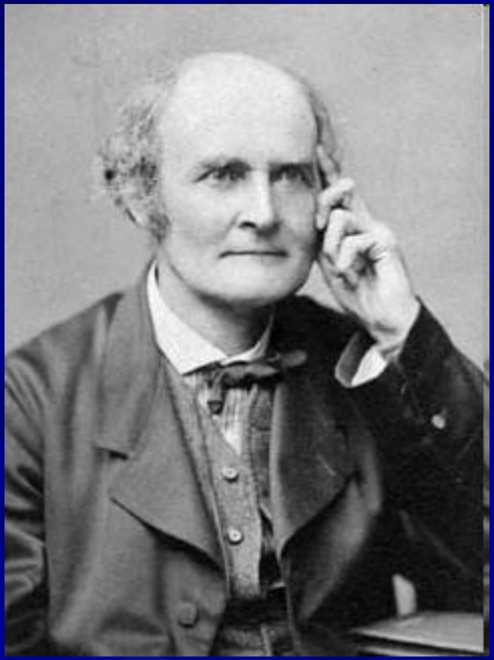
I believe, but am not sure, that in all the solutions which have been given of the school-girl problem there is an 8 without 3.

Now, considering the more simple problem, there are of course solutions which have an 8 without 3 (since every solution of the school-girl problem is a solution of the more simple problem):

* Communicated by the Author.

Phil Mag. 1863 59–61

Arthur Cayley
1821-1895



But…

THE CAMBRIDGE AND DUBLIN

MATHEMATICAL JOURNAL.

EDITED BY W. THOMSON, M.A.

FELLOW OF ST. PETER'S COLLEGE, CAMBRIDGE,
AND PROFESSOR OF NATURAL PHILOSOPHY IN THE UNIVERSITY OF GLASGOW.

VOL. V.

(BEING VOL. IX. OF THE CAMBRIDGE MATHEMATICAL JOURNAL.)

Δυοῖν ὀνομάτων μορφὴ μία.

CAMBRIDGE:
MACMILLAN AND Co.;
GEORGE BELL, LONDON;
HODGES AND SMITH, DUBLIN.
1850

Mag., June 1850), "To make the school walk out every day in the quarter, so that every three shall walk together."

The three final pairs of columns have three systems of subindices; permute cyclically these *systems*, and you get two more sets each of six columns. In each of the three sets of six columns permute cyclically the five letters, and you obtain the solution of Mr. Sylvester's puzzle.

Is it known that a similar feat can be achieved by nine ladies, to walk in threes till every three have walked together?

From the figure

$$1_1 1_2 1_3,\quad 2_1 2_2 2_3,\quad 3_1 3_2 3_3,$$

write out the triplets

$$\begin{matrix} 1_1 1_2 1_3, & 1_1 2_1 3_1, & 1_1 2_2 3_3, & 1_1 3_2 2_3, \\ 2_1 2_2 2_3, & 1_2 2_2 3_2, & 2_1 3_2 1_3, & 2_1 1_2 3_3, \\ 3_1 3_2 3_3, & 1_3 2_3 3_3, & 3_1 1_2 2_3, & 3_1 2_2 1_3; \end{matrix}$$

then, from the six figures

$$\begin{matrix} 1_1 1_2 2_1 & 2_1 2_2 3_1 & 3_1 3_2 1_1 & 3_1 1_1 1_2 & 1_1 2_1 2_2 & 2_1 3_1 3_2 \\ 2_2 2_3 3_1 & 3_2 3_3 1_1 & 1_2 1_3 2_1 & 1_2 2_1 2_2 & 2_3 3_1 3_2 & 3_2 1_1 1_2 \\ 3_2 3_3 1_3 & 1_2 1_3 2_3 & 2_2 2_3 3_3 & 2_3 3_2 3_3 & 3_2 1_2 1_3 & 1_3 2_2 2_3, \end{matrix}$$

arrangements may be made in exactly the same manner, so as to complete the solution. Can 25 be made to walk out in fives, till every five have walked together?

The *school-girl problem* may be shewn to depend on the combination of the triplets made with seven things, with the following curious arrangement of the duads made with eight things:

—	—	—	*hi*	*kl*	*mn*	*op*
—	*il*	*mo*	—	*np*	*hk*	—
—	*no*	*hl*	*mp*	—	—	*ik*
lp	—	*in*	*ko*	*hm*	—	—
im	—	*kp*	—	—	*lo*	*hn*
ho	*km*	—	*ln*	—	*ip*	—
kn	*hp*	—	—	*io*	—	*lm*.

It will be found difficult to imitate this arrangement with more than eight things.

I must not omit to add, that I had no solution of the problem 1), except the cases of $nr = 9$ and $nr = 15$, until I saw the following arrangement of 27 things by the Rev. James Mease, A.M., of Freshford, Kilkenny, who sent it

Room square of side 7 found in 1850 by Rev. Thomas P. Kirkman (1806-1895)

Questions about Room squares

- Can you make them for all odd orders? YES.
- How many nonisomorphic ones are here? LOTS!
- Why are they called Room squares? BY MISTAKE!
- What would you do with three mutually orthogonal one-factorizations? More balance?
- Any connection to balanced tournament designs?

What would you do with three mutually orthogonal one-factorizations? More balance?

In the context of Round Robin Tournaments we can balance for one more thing. For example: referees.

Lets make a round robin tournament with:

8 teams over 7 weeks at 7 sites with 7 referees where each team:

plays every other team exactly once,

plays one game per week

plays at each site exactly once

gets each of the 7 referees exactly once.

Also each referee works at each site at most once.

games per week	games per site	games per referee
0) ∞,0 3,4 1,6 2,5	0) ∞,0 2,3 4,6 5,1	0) ∞,0 4,5 1,3 6,2
1) ∞,1 4,5 2,0 3,6	1) ∞,1 3,4 5,0 6,2	1) ∞,1 5,6 2,4 0,3
2) ∞,2 5,6 3,1 4,0	2) ∞,2 4,5 6,1 0,3	2) ∞,2 6,0 3,5 1,4
3) ∞,3 6,0 4,2 5,1	3) ∞,3 5,6 0,2 1,4	3) ∞,3 0,1 4,6 2,5
4) ∞,4 0,1 5,3 6,2	4) ∞,4 6,0 1,3 2,5	4) ∞,4 1,2 5,0 3,6
5) ∞,5 1,2 6,4 0,3	5) ∞,5 0,1 2,4 3,6	5) ∞,5 2,3 6,1 4,0
6) ∞,6 2,3 0,5 1,4	6) ∞,6 1,2 3,5 4,0	6) ∞,6 3,4 0,2 5,1

Each of these three schedules are round robin tournaments on 8 players but each represents a different balance condition. They are orthogonal since no games which are together in one tournament are together in another.

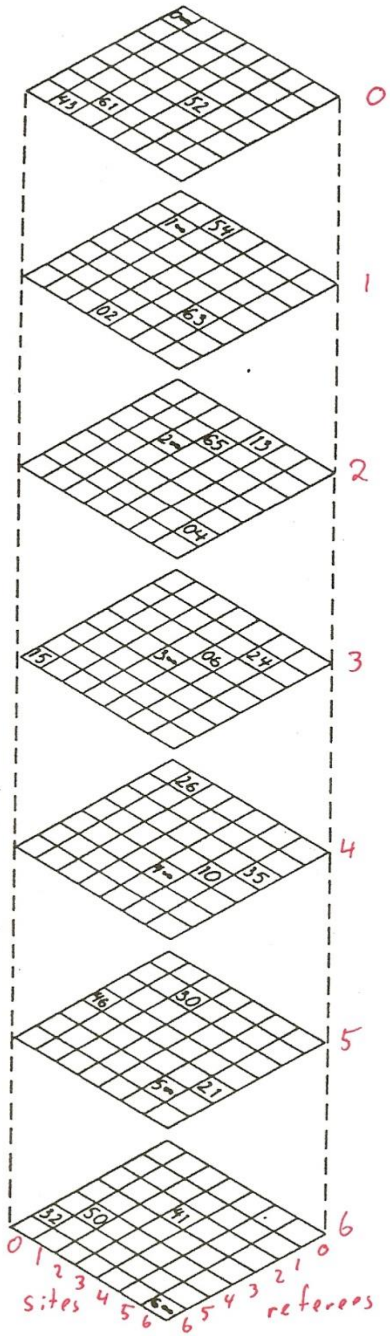
games per week					games per site					games per referee				
0)	∞,0	3,4	1,6	2,5	0)	∞,0	2,3	4,6	5,1	0)	∞,0	4,5	1,3	6,2
1)	∞,1	4,5	2,0	3,6	1)	∞,1	3,4	5,0	6,2	1)	∞,1	5,6	2,4	0,3
2)	∞,2	5,6	3,1	4,0	2)	∞,2	4,5	6,1	0,3	2)	∞,2	6,0	3,5	1,4
3)	∞,3	6,0	4,2	5,1	3)	∞,3	5,6	0,2	1,4	3)	∞,3	0,1	4,6	2,5
4)	∞,4	0,1	5,3	6,2	4)	∞,4	6,0	1,3	2,5	4)	∞,4	1,2	5,0	3,6
5)	∞,5	1,2	6,4	0,3	5)	∞,5	0,1	2,4	3,6	5)	∞,5	2,3	6,1	4,0
6)	∞,6	2,3	0,5	1,4	6)	∞,6	1,2	3,5	4,0	6)	∞,6	3,4	0,2	5,1

Each of these three schedules are round robin tournaments on 8 players but each represents a different balance condition. They are orthogonal since no games which are together in one tournament are together in another.

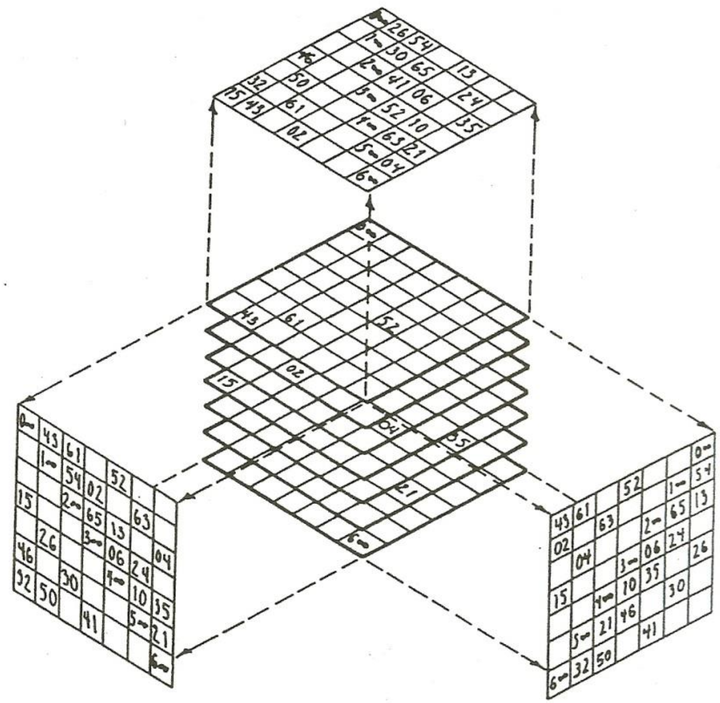
Example: Team 1 plays team 4 in week 6, at site 3 and with referee 2.

Can construct a $7 \times 7 \times 7$ cube where i, j is in cell (w,s,r) if team i plays team j in week w at site s and with referee r.

So in the example 1,4 will be in cell (6,3,2).



Exploded View



Projected Views

Room Cube
of side 7

For 10 players we can add a 4th balance condition

weeks

01	23	45	67	89
02	13	46	58	79
03	12	47	59	68
04	16	25	39	78
05	18	24	37	69
06	19	27	35	48
07	15	28	36	49
08	17	29	34	56
09	14	26	38	57

sites

01	29	36	48	57
02	15	34	69	78
03	16	28	45	79
04	17	26	35	89
05	14	27	39	68
06	12	37	39	58
07	19	25	38	46
08	13	24	59	67
09	18	23	47	56

referees

01	26	39	47	58
02	14	37	56	89
03	17	25	48	69
04	18	27	36	59
05	19	28	34	67
06	15	24	38	79
07	13	29	45	68
08	16	23	49	67
09	12	35	46	78

time of day

01	25	34	68	79
02	18	35	49	67
03	15	27	46	89
04	13	28	57	69
05	16	29	38	47
06	14	23	59	78
07	12	39	48	56
08	19	26	37	45
09	17	24	36	58

Four orthogonal round robin tournaments on 10 players

Balance properties:

Each player plays every other player exactly once.
Each player plays exactly one game per week, one game at each site, one game with each referee and one game at each time of day. Each referee refs at most once at each site and in each week and at each time of day.

How many can we make?

Let $\upsilon(n)$ denote the maximum number of orthogonal round robin tournaments on n players.

Theorem (D. 1984) For every even $n \geq 12$, except possibly for $n=16$, $\upsilon(n) \geq 5$.

n	$\nu(n) \geq$	n	$\nu(n) \geq$	n	$\nu(n) \geq$	n	$\nu(n) \geq$
1	$=\infty$	27	13	53	17	79	39
3	$=1$	29	13	55	5	81	5
5	$=1$	31	15	57	5	83	41
7	$=3$	33	5	59	29	85	5
9	$=4$	35	5	61	21	87	5
11	5	37	15	63	5	89	11
13	5	39	5	65	5	91	5
15	4	41	9	67	33	93	5
17	5	43	21	69	5	95	5
19	9	45	5	71	35	97	5
21	5	47	23	73	9	99	5
23	11	49	5	75	5	101	31
25	7	51	5	77	5	103	51

Questions about Room squares

- Can you make them for all odd orders? YES.
- How many nonisomorphic ones are here? LOTS!
- Why are they called Room squares? BY MISTAKE!
- What would you do with three mutually orthogonal one-factorizations? More balance? DEFINITELY!
- Any connection to balanced tournament designs?

Connection to balanced tournament designs

A MESRS(9) (maximum empty square Room square)

3,7					2,8	5,9	0,4	1,6
	5,6				0,1	4,7	2,9	3,8
		0,2			6,7	1,8	3,5	4,9
			4,8		3,9	2,6	1,7	0,5
				1,9	4,5	3,0	6,8	2,7
1,2	8,0	5,7	6,9	3,4				
4,6	1,3	8,9	0,7	2,5				
5,8	7,9	1,4	2,3	6,0				
9,0	2,4	3,6	1,5	7,8				

Exist for all odd sides $n \geq 9$ except possibly $n=17,21,29$

D., Stinson 2004

A gravity-transformed balanced tournament design ☺

Questions about Room squares

- Can you make them for all odd orders? YES.
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- What would you do with three mutually orthogonal one-factorizations? More balance? DEFINITELY!
- Any connection to balanced tournament designs?

A MESRS(n) gives a GRAVITY-TRANSFORMED BALANCED TOURNAMENT DESIGN !

More numbers in boxes

The Dinitz Conjecture

article | discussion | edit this page | history

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Dinitz conjecture

From Wikipedia, the free encyclopedia

In combinatorics, the **Dinitz conjecture** is a statement about the extension of arrays to partial Latin squares, proposed in 1979 by Jeff Dinitz, and proved in 1994 by Fred Galvin.

The Dinitz conjecture, now a theorem, is that given an $n \times n$ square array, a set of m symbols with $m \geq n$, and for each cell of the array an n-element set drawn from the pool of m symbols, it is possible to choose a way of labeling each cell with one of those elements in such a way that no row or column repeats a symbol.

References [edit]

- F. Galvin (1995). "The list chromatic index of a bipartite multigraph". *Journal of Combinatorial Theory*. Series B **63** (1): 153–158. doi:10.1006/jctb.1995.1011.

External links [edit]

Eric W. Weisstein, *Dinitz Problem* at MathWorld.

This combinatorics-related article is a stub. You can help Wikipedia by expanding it.

Categories: Combinatorics | Latin squares | Mathematical theorems | Combinatorics stubs

A quick example

$n = 4$

1,2,3,4	1,3,5,6	2,3,5,9	1,2,3,5
2,4,5,9	1,6,7,8	4,7,8,9	2,3,4,7
1,2,3,4	1,2,4,7	3,4,8,9	3,4,7,8
2,3,6,7	2,3,4,5	1,2,3,4	4,5,6,7

Find a partial Latin square

A quick example

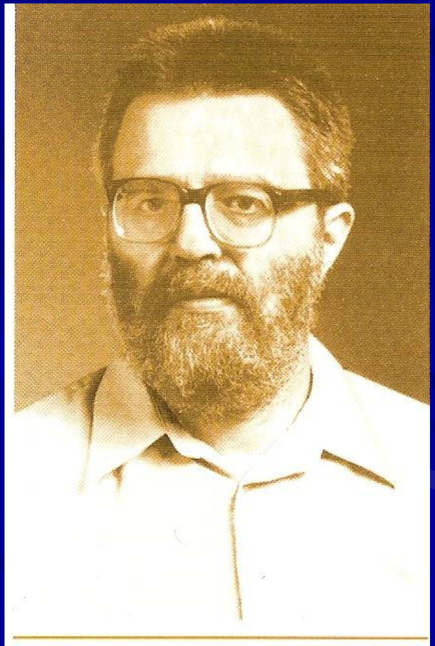
$n = 4$

1,2,3,4	1,3,5,6	2,3,5,9	1,2,3,5
2,4,5,9	1,6,7,8	4,7,8,9	2,3,4,7
1,2,3,4	1,2,4,7	3,4,8,9	3,4,7,8
2,3,6,7	2,3,4,5	1,2,3,4	4,5,6,7

Got it!

In 1979, I conjectured that this was possible for every possible placement of sets and for every size *n.*

In 1994, Fred Galvin (University of Kansas) proved my conjecture to be true.


Fred Galvin

Some Applications

- Statistical design of experiments (pioneered by R.A. Fisher)
- Group testing
- Cryptography (Threshold schemes, Authentication codes, resilient functions)
- Coding theory (error correcting codes)
- Signal processing (fault tolerant optical networks)
- Computers (hash functions, random number generators …)
- Molecular Biology (Oligo DNA microarrays)
- Numerical Integrations ((t,m,s)-nets)
- Construction of tournaments (Howell rotations in duplicate bridge)

XFL

3/09/00

Divisions

East
New York
Washington D.C.
Miami
Orlando

West
LA
San Francisco
Las Vegas
Dallas

Guidelines

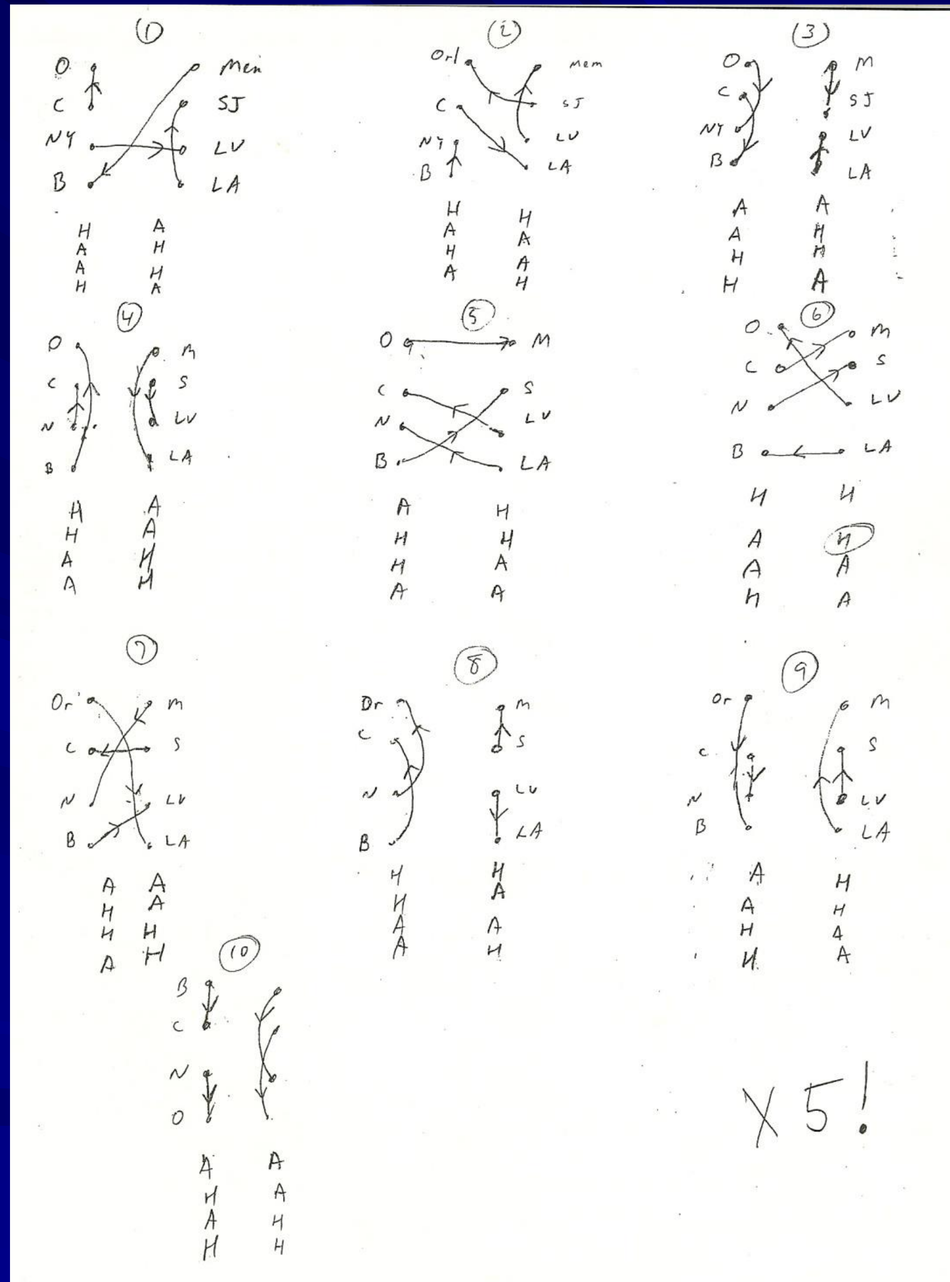
- Ten games
- Each team will play every other team in its own division twice
- Each team will play every other team in the other division once
- There will be five home games and five away games
- Each team will have one divisional home game and one divisional away game against every team in its division
- Each team will have two home games and two away games against the four teams from the other division
- There must be one Eastern standard time game and one Pacific standard time game each weekend

Preferences

- Teams that have non-divisional games along opposite coast will play both non-divisional games in consecutive weeks to avoid scheduling competitive advantages
- No three-week-long road trips
- Every team with at least 1 home game by the third week of season

Other

- Substitute Dallas for Houston on attached schedules



X 5!

XFL™

2001 SEASON SCHEDULE

Saturday, February 3
New York/New Jersey at Las Vegas 8:00 PM
Chicago at Orlando 8:00 PM

Sunday, February 4
Memphis at Birmingham 4:00 PM
Los Angeles at San Francisco 4:00 PM

Saturday, February 10
Chicago at LA 8:00 PM
San Francisco at Orlando 8:00 PM

Sunday, Feb 11
Birmingham at New York/New Jersey 4:00 PM
Las Vegas at Memphis 7:00 PM

Saturday, February 17
San Francisco at Memphis 8:00 PM
Los Angeles at Las Vegas 8:00 PM

Sunday, February 18
Chicago at Birmingham 4:00 PM
Orlando at New York/New Jersey 7:00 PM

Saturday, February 24
New York/New Jersey at Chicago 8:00 PM
Birmingham at Orlando 8:00 PM

Sunday, February 25
Las Vegas at San Francisco 4:00 PM
Memphis at Los Angeles 7:00 PM

Saturday, March 3
Los Angeles at New York/New Jersey 8:00 PM
Birmingham at San Francisco 8:00 PM

Sunday, March 4
Las Vegas at Chicago 4:00 PM
Orlando at Memphis 7:00 PM

Saturday, March 10
Las Vegas at Orlando 8:00 PM
Chicago at Memphis 8:00 PM

Sunday, March 11
New York/New Jersey at San Francisco 4:00 PM
Los Angeles at Birmingham 7:00 PM

Saturday, March 17
Memphis at New York/New Jersey 8:00 PM
Birmingham at Las Vegas 8:00 PM

Sunday, March 18
Orlando at Los Angeles 4:00 PM
San Francisco at Chicago 7:00 PM

Saturday, March 24
Las Vegas at Los Angeles 8:00 PM
Memphis at San Francisco 8:00 PM

Sunday, March 25
New York/New Jersey at Orlando 4:00 PM
Birmingham at Chicago 7:00 PM

Saturday, March 31
Chicago at New York/New Jersey 8:00 PM
Orlando at Birmingham 8:00 PM

Sunday, April 1
Los Angeles at Memphis 4:00 PM
San Francisco at Las Vegas 7:00 PM

Saturday, April 7
Orlando at Chicago 8:00 PM
Memphis at Las Vegas 8:00 PM

Sunday, April 8
New York/New Jersey at Birmingham 4:00 PM
San Francisco at Los Angeles 7:00 PM

Saturday, April 14
PLAYOFF GAME 8:00 PM

Sunday, April 15
PLAYOFF GAME 7:00 PM

Saturday, April 21
"THE BIG GAME AT THE END"℠
XFL CHAMPIONSHIP GAME 8:00 PM

All Times Eastern Standard Time

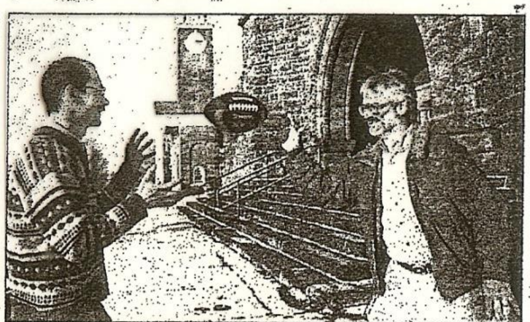
TNN — SUNDAYS AT 4PM EST
NBC — SATURDAYS AT 8PM EST
UPN — SUNDAYS AT 7PM EST

"All the News
That's Fit to Print"

The New York Times

New England Edition
Boston: Becoming sunny, windy an
colder, high 29. Tonight, clear and col
low 21. Tomorrow, increasing clouc
and not as cold, high 35. Weather ma
and details appear on Page B1

VOL. CL . . . . No. 51,653 SATURDAY, FEBRUARY 3, 2001 ONE DOLLA



Paul O. Boisvert for The New York Times

Dalibor Froncek, left, and Jeff Dinitz, experts in combinatorics.

What Good Is Math? An Answer for Jocks

By PATRICIA COHEN

Here's the scheduling problem that faced the XFL, the new smack-in-the-mouth, kick-in-the-groin professional football league that has its premiere tonight.

Two divisions with four teams each; each team plays every other team in its own division twice and the teams in the other division once. So far so good. Then the headaches began. Marquee name games — like the Las Vegas Outlaws against the New York/New Jersey Hitmen — had to take place on Saturday nights to mesh with NBC's schedule, and the openers had to be in warm-weather locations (no blizzards) and smaller stadiums (so the stands would looked packed). Teams were not supposed to have too many away games in a row, but the Chicago Enforcers could not play at Soldier Field in February because the Auto Show needed the parking lots; the Outlaws could not play at home in Week 8 because of the Moto Cross show; and Orlando Rage was probably going to be displaced in Week 4 for a concert.

The organizers could have spent weeks or months matching and rematching teams. But two mathematicians at the University of Vermont — a college that doesn't even have a do better.

So Jeff Dinitz and Dalibor Froncek, experts in an arcane field of math called combinatorics, offered to be the brains behind the brawns' schedule.

"I just called them out of the blue," Mr. Dinitz, the chairman of the mathematics and statistics department, said. "I was surprised they were pretty receptive."

Indeed, the XFL, which was created by the World Wrestling Federation, adopted Mr. Dinitz's and Mr. Froncek's schedule virtually unchanged.

"We were very, very happy," said Rich Rose, a senior consultant to the league who worked with the mathematicians.

Mr. Dinitz has been a fervent football fan ever since graduate school at Ohio State in the late 1970's when the legendary Woody Hayes coached. He invited Mr. Froncek, a visiting professor from the Czech Republic — a nation without the finer art of football — to his house to watch the games. One day last February he mentioned to Mr. Froncek that a new league was forming.

"I said, 'If it's new then they have no schedule,'" Mr. Froncek explained. "'Don't you think we should offer that we would make a schedule

What Good Is Math? Jocks Find Out

Continued From Page A15

for them?' I was pretty familiar with these kinds of schedules. They're quite similar to what I have done for the Czech guys." Mr. Froncek creates the schedules for the 12-team Czech basketball league and the 14-team Czech hockey league. Mr. Dinitz, meanwhile, discusses the N.F.L.'s cumbersome scheduling procedure in a handbook of combinatorial designs for which he was a co-editor.

So Mr. Dinitz got on his computer and went to the XFL's home page and called the number listed. A public relations person answered and, to Mr. Dinitz's amazement, he was quickly put in touch with Mr. Rose. He began to schmooze football with him.

As it turned out, their timing was perfect. "We knew the basics" of what we wanted, Mr. Rose said, "but we hadn't gotten down to developing a schedule."

Mr. Dinitz said that the XFL had taken a crack at it and was in trouble. "They sent us a sample schedule and it wasn't very good," he said, "so we knew that we could help them."

For hundreds of years mathematicians have fiddled with combination problems. Seventeenth-century mathematicians were obsessed with figuring out gambling odds, like what the chances are of getting a pair of aces in five-card poker. Galileo figured out the odds for dice.

Las Vegas isn't the only place where combinatorics mavens gravitate. It can also be used to devise a college course schedule that accommodates all students' class preferences, postal routes that begin and end in the same place but ensure that no block is walked twice, and pharmaceutical experiments that require trying out different combinations of drugs. In the last few decades the field has become critically important because it is used in computers.

"The main problem is to combine constraints," Mr. Froncek said. "You can do one thing relatively well and the other relatively well, but when you want to put these things together, you can't do that without a deep knowledge of some mathematical methods."

For the XFL schedule, Mr. Rose sent Mr. Dinitz and Mr. Froncek a list of must-haves (like one Eastern time game and one Pacific time game each weekend) and a list of it-would-be-nice-to-haves (like minimizing the number of times teams had to travel coast to coast).

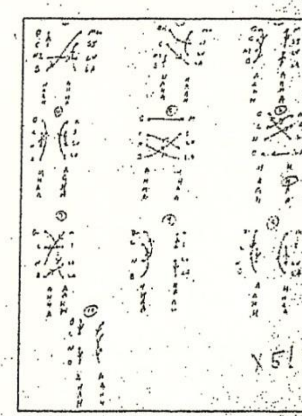
Mr. Dinitz and Mr. Froncek used graphs to visualize the schedule. "Once you picture something, it's a lot easier to work with," Mr. Dinitz explained. "I represent each of the eight teams by just a dot on my page. There's a dot for New York and a dot for Chicago, and if New York is at Chicago, I'll draw a line between them with an arrow from New York to Chicago."

He and Mr. Froncek kept adding dots and flipping arrows, trying out various combinations. "The cool thing is it's mathematics, but there isn't an equation in it," Mr. Dinitz said.

By the end of March, the two professors sent the XFL three possible schedules that satisfied various soups of requirements and preferences. They asked for $1,000 and an expense-paid trip to the league's Super Bowl. Excuse me, Mr. Rose scolded over the telephone, the N.F.L. has a Super Bowl; the XFL has a Championship Game.

They received a check for $1,250, with no mention of championship tickets. But then in July, Mr. Rose called back with the first in a series of new requirements (for example, the San Francisco Demons had to be at home the first week), and wish lists (that Los Angeles be the host to either New York or Chicago). The two mathematicians also made some suggestions of their own, like avoiding three home games in a row, and an intradivisional round robin for the last three weeks.

They asked for $4,000 more, a lot less than the $100 an hour that Mr. Dinitz usually charges for consulting. "I didn't know how much to ask," he said, and "I didn't want to scare them away." Still, he and Mr. Froncek began suggesting other types of remuneration at the end of their letters: "We also remember that the XFL is into performance bonuses — does that include us? How about one of those cool-looking footballs? Stock options?" (So far, only the footballs have shown up.)



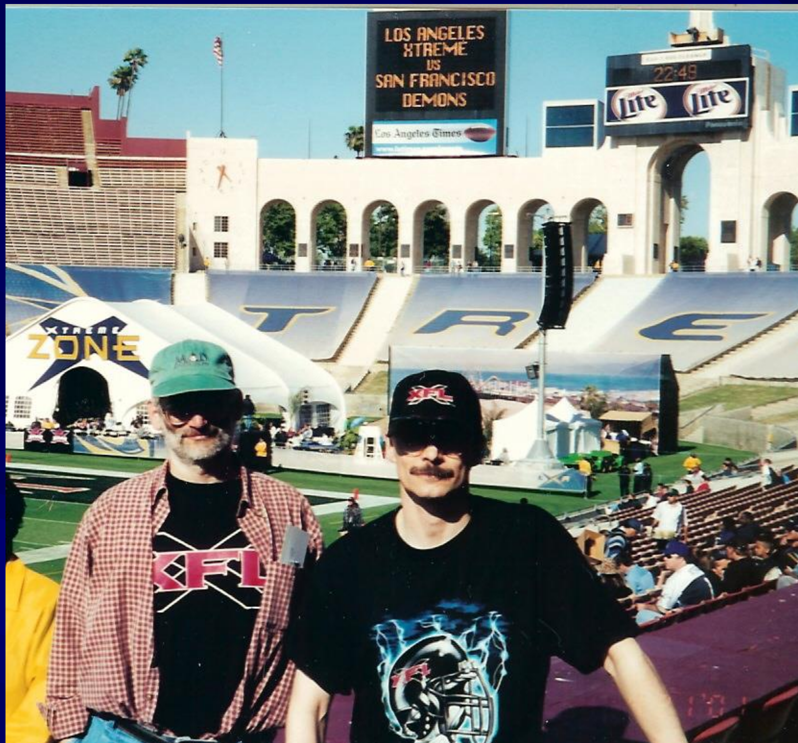
The 14th version of the chart for the XFL schedule, which its creators called X5 and declared perfect. H stands for home and A stands for away.

After the 14th version of the schedule, which they labeled X5 and called perfect, the league was satisfied.

"Jeff and Dalibor were great," Mr. Rose said, adding that there were a couple of last-minute adjustments to X5. "We fully plan on using them in 2002."

Mr. Dinitz is having an XFL party at his house tonight for the math department. And while the XFL has unveiled its schedule to tens of millions of potential fans, he and Mr. Froncek plan to unveil the work behind that schedule to tens of combinatorics fans in the March-April issue of Congressus Numerantium, a journal published in Winnipeg.

So what about the N.F.L.? Mr. Dinitz said he believed they could improve its schedule, too, "but we haven't gotten around to calling them yet."


LOS ANGELES
XTREME
VS
SAN FRANCISCO
DEMONS
Los Angeles Times
22:49
Lite
Lite
XTREME
ZONE
XFL
XFL


2001 INAUGURAL SEASON
SAN FRANCISCO
DEMONS
XFL
XFL
CHAMPIONSHIP
THE MILLION DOLLAR GAME
XTREME
LOS ANGELES
APRIL 21, 2001
LOS ANGELES MEMORIAL COLISEUM
SEAT 101
ROW 30
TUNNEL 22 NORTH

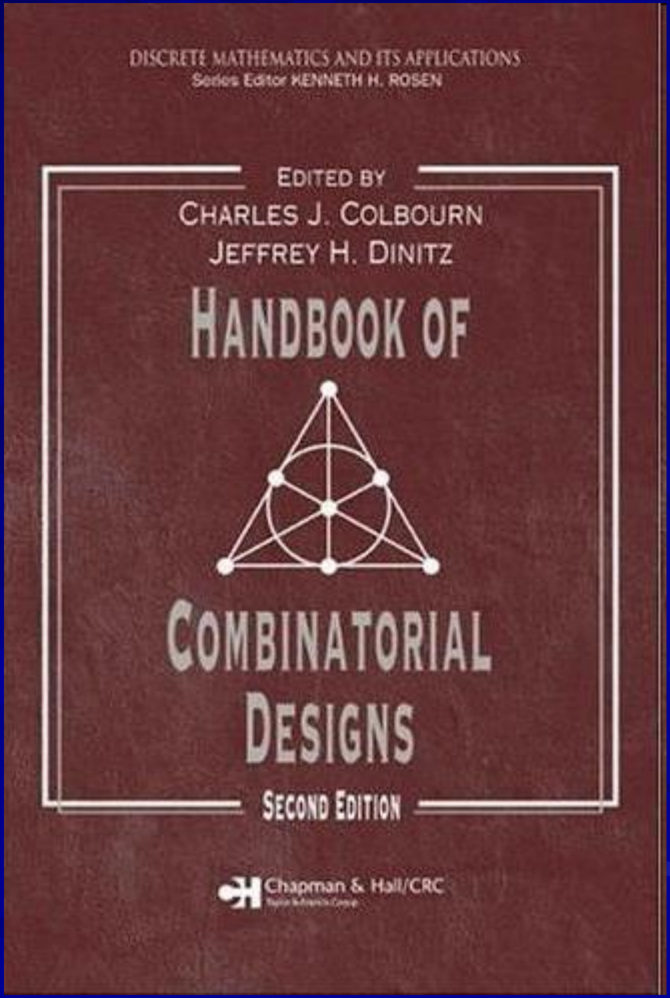
Handbook of Combinatorial Designs

Standard reference in design theory.

982 pages long, 110 sections, over 100 contributors

Because of it I get lots of inquiries.

coauthor:
Charlie Colbourn



Raymond Brownell

WWW.RAYMONDBROWNELL.COM

PROFILE

I was born in Hobart, Tasmania and studied architecture there, but since qualifying in 1959, I have spent most of my life in Europe, engaged in an interesting international career. My experience in architecture has had a definite influence on my painting approach -- particularly the years working for Jorn Utzon on the Sydney Opera House, when I gained valuable insights into the operation of organic integrity in the design process.

However it was while working in Paris in 1969, that I had the really seminal experience, which was an exhibition of the work of Max Bill, leading theoretician of the Swiss concrete art movement. These were artists who based their works on mathematical ideas and logical procedure. This inspired my own ambition to make paintings that would reveal the fascinating structures and patterns inherent in mathematics, particularly permutations and combinations.

Each work has its genesis in a mathematical idea. The first priority is to seek out its essential characteristics, then to develop a framework which will best reflect these. The process should be consistent, so that the finished painting is solely concentrated on providing a clear, natural and comprehensible expression of the original idea . I use acrylic paint on linen canvas in a precise technique, and try to eliminate traces of brushwork (the personal) by applying the paint thinly in layers, leaving the linen texture still visible beneath the paint, expressing the canvas for what it is.

Although these works are driven by a strict mathematical logic, expressed through colour, the colours often generate their own subtle optical effects, so that a gentle opposition can be set up, between the rational idea, and its realisation in that unreliable medium, colour. Nevertheless in overall effect, these paintings do communicate a sense of order and harmony.

A rather more detailed exposition of my approach can be found in the essay "OF MIND AND EYE" on this site.

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r.brownell: 'Centred Latin Squares' (24 of 24)

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Date: Tue, 30 May 2006 21:24:48 +0100
From: Raymond Brownell <r.brownell@ukonline.co.uk>
To: jeff.dinitz@uvm.edu
Subject: 'Centred Latin Squares'
Part(s): 2.1 pntgs5.jpg application/applefile 59.92 KB

Dear Professor Dinitz,

With this email I am hoping that you will kindly offer me some advice and/or information concerning certain types of latin square that I that I use in my work.

My professional career was in architecture, but for many years now I have been absorbed in painting, much influenced by the Swiss 'concrete art' school, which adopted a rational approach, making 'concrete' in their paintings and sculptures abstract, ideas derived from mathematics, (albeit these were often very basic).

Much of my work develops possibilities inherent in combinatorics, especially paired combinations, where colours take the place of numbers. Of course I came to orthogonal latin squares, and then to your and Professor Colbourn's CRC Handbook. Although my mathematical abilities are pretty limited, I've gained enough understanding for my purposes at present. I soon realised the standard or reduced latin square was for me rather unpromising. A preference for configurations involving rotation, equivalent
distribution of each number/colour, and symmetry then led to the types of latin square the subject of this email.

What I would like to know, is set out below, first the questions,then the diagrams:

(a) If, as I expect, any of these types of latin square are already known to mathematicians, do they have names, and or any particular applications?
(b) If any of them haven't yet been identified, do you see them as being of any interest, mathematically speaking?

```
                 1  4  3  2                 1  3  4  2                11  43
34  22                                            1  x  3  4  2
1  3  4  x  2                    11  x3  34  4x  22
                 2  3  4  1    +           3  1  2  4     =        23  31  42
14                                             3  4  2  1  x                   x
2  1  3  4                   3x  42  21 13   x4
                 3  2  1  4                 2  4  3  1                32  24
13  41                                            2  1  x  3  4        +
3  4  x  2  1          =        23  14  xx  32  41
                 4  1  2  3                 4  2  1  3                44  12
21  33                                            x  3  4  2  1
2  1  3  4  x                    x2  31  43  24  1x

                                                                        4  2
1  x  3             4  x  2   1  3                  44  2x  12  x1  33
               Equal distribution of each number within square
               and each symmetric about diagonal axis.
                                Equal distribution of each of four corner
numbers within square.


                                                                          5
6  7  1  2  3  4                                    4  3  2  1  7  6  5
                                      54  63  72  11  27  36  45
                                                                          6
7  1  2  3  4  5                                    5  4  3  2  1  7  6
                                      65  74  13  22  31  47  56
                                                                          7
1  2  3  4  5  6                                    6  5  4  3  2  1  7
                                      76  15  24  33  42  51  67
```

He is interested in
self-orthogonal latin squares.

These are latin squares that are orthogonal to their transpose

1	3	4	2
4	2	1	3
2	4	3	1
3	1	2	4

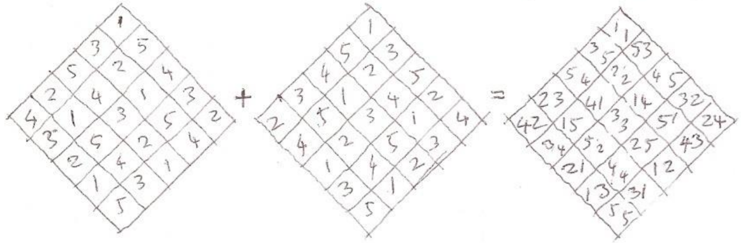
L

1	4	2	3
3	2	4	1
4	1	3	2
2	3	1	4

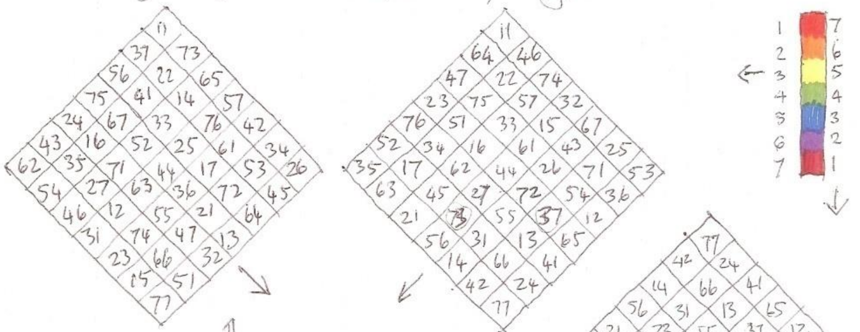
L^T

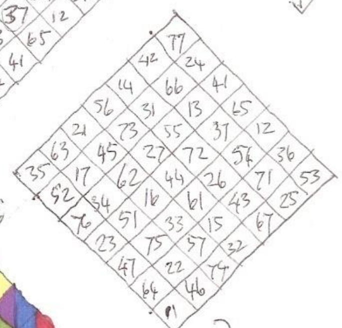
1 1	3 4	4 2	2 3
4 3	2 2	1 4	3 1
2 4	4 1	3 3	1 2
3 2	1 3	2 1	4 4

SELFORTHOGONAL LATIN SQUARES (see Henry's pages) 6/06



n = 5 self orthogonal latin squares. ±1/±2 type

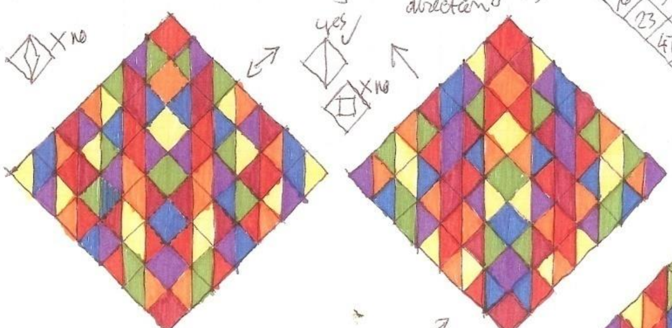




n = 7 ±1/±2 types

±2/±3 type

both have consecutive numbering in vertical (diagonal) direction



n = 7 ±2/±3 type

don't like yellow corners east + west – looks weak

try reversing vertically the numbering →

n = 7 ±1/±2 type

n = 7 ±2/±3 type

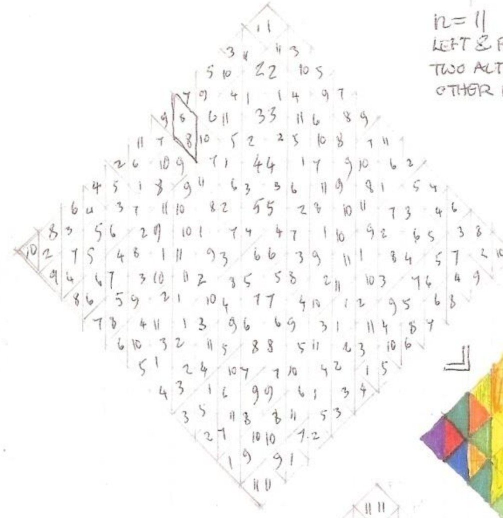
better!

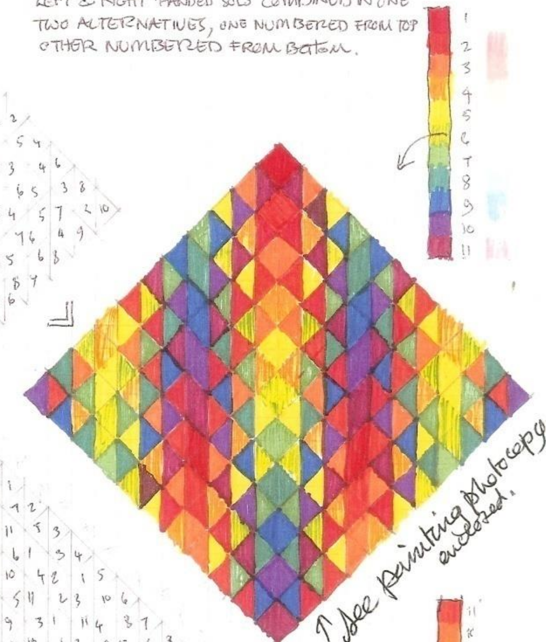
n = 11 SOLS – 6/06

LEFT & RIGHT HANDED SOLS COMBINED IN ONE

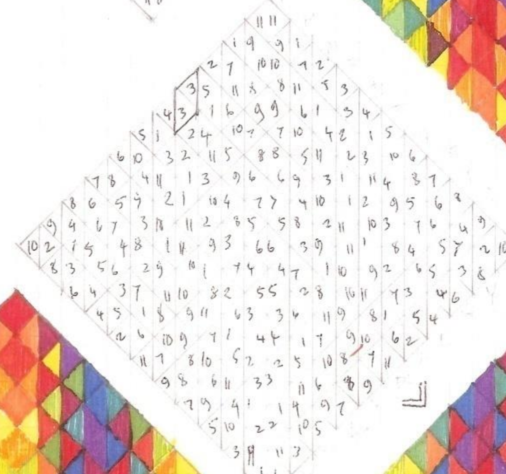
TWO ALTERNATIVES, ONE NUMBERED FROM TOP

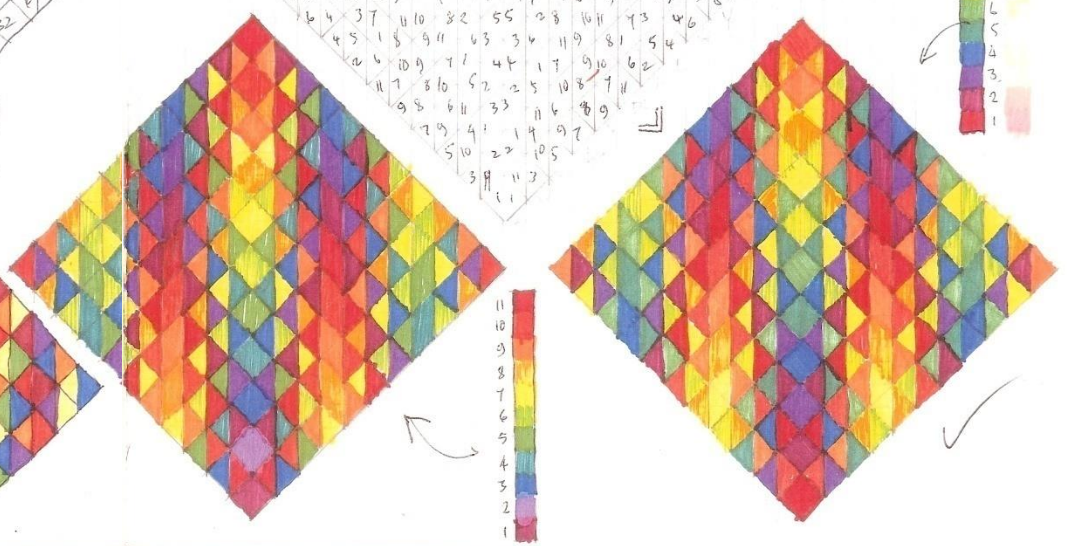
OTHER NUMBERED FROM BOTTOM.

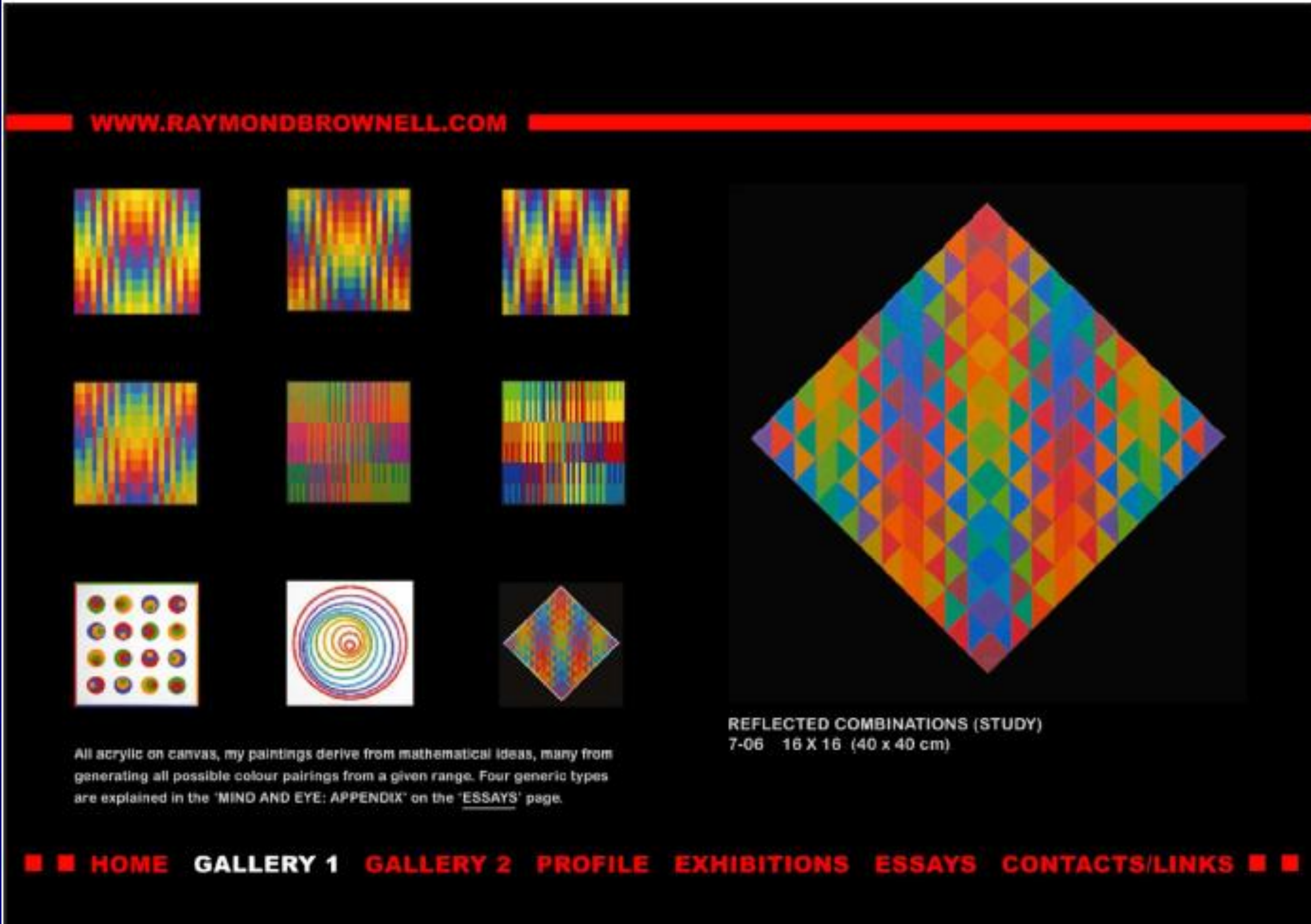




↑ See painting photocopy enclosed.







WWW.RAYMONDBROWNELL.COM
REFLECTED COMBINATIONS (STUDY)
7-06 16 X 16 (40 x 40 cm)
All acrylic on canvas, my paintings derive from mathematical ideas, many from generating all possible colour pairings from a given range. Four generic types are explained in the 'MIND AND EYE: APPENDIX' on the 'ESSAYS' page.
HOME GALLERY 1 GALLERY 2 PROFILE EXHIBITIONS ESSAYS CONTACTS/LINKS

Tom Johnson

An American composer living in Paris

Biography (November 2004)

Tom Johnson, born in Colorado in 1939, received B.A. and M.Mus. degrees from Yale University, and studied composition privately with Morton Feldman. After 15 years in New York, he moved to Paris, where he has lived since 1983. He is considered a minimalist, since he works with simple forms, limited scales, and generally reduced materials, but he proceeds in a more logical way than most minimalists, often using formulas, permutations, and predictable sequences.

Johnson is well known for his operas: The Four Note Opera (1972) continues to be presented in many countries. Riemannoper has been staged more than 20 times in German-speaking countries since its premier in Bremen in 1988. Often played non-operatic works include Bedtime Stories, Rational Melodies, Music and Questions, Counting Duets, Tango, Narayana's Cows, and Failing: a very difficult piece for solo string bass.

His largest composition, the Bonhoeffer Oratorium, a two-hour work in German for orchestra, chorus, and soloists, with text by the German theologian Dietrich Bonhoeffer, was premiered in Maastricht in 1996, and has since been presented in Berlin and New York.

Johnson has also written numerous radio pieces, such as J'entends un choeur (commissioned by Radio France for the Prix Italia, 1993), Music and Questions (also available on an Australian Broadcasting Company CD) and Die Melodiemaschinen, premiered by WDR Radio in Cologne in January 1996. The most recent radio piece is A Time to Listen, premiered by the Irish national radio in 2004.

The principal recordings currently available on CD are the Musique pour 88 (1988) (XI), An Hour for Piano (1971) (Lovely Music), The Chord Catalogue (1986) (XI), Organ and Silence (2000) (Ants), and Kientzy Plays Johnson (2004) (Pogus).

The Voice of New Music, a collection of articles written 1971-1982 for the Village Voice, published by Apollohuis in 1989, is now in the public domain and can be downloaded at www.tom.johnson.org. Self-Similar Melodies, a theoretical book in English, was published by Editions 75 in 1996.

Tom Johnson 75, rue de la Roquette, 75011 Paris TEL 01 43 48 90 57 FAX 01 43 48 85 74
email: tom@johnson.org web site: www.tom.johnson.org

Jeffrey Dinitz
Math Dept.
Univ of Vermont

FAX 802 656 2552

July 4 (not a holiday here), 2005

Dear Jeffrey Dinitz,

I am a composer, not a mathematician, but I have been studying block design for some months, as I find many fascinating musical possibilities here. A piece called *Kirkman's Ladies* was completed earlier this year.

I am writing to you because I am now working with Room squares. I figured out how to calculate 7 x 7 Room squares via two mutually disjoint STS(7,3,2) block designs and found six solutions before stumbling onto your article in the CRC Handbook and realized that you had already done my work for me. Many thanks. I have a few questions still, however, most of which you can no doubt answer quickly and easily, and which will help me define the music I am trying to write, and to set its limits.

For each of the three non-equivalent skew designs you show on page 438 of the *CRC Handblook*, the columns can be arranged in 7 ! = 5040 ways and the lines in another 5040 ways. Does this mean there are 5040 * 5040 = 25,401,600 isomorphisms of each?

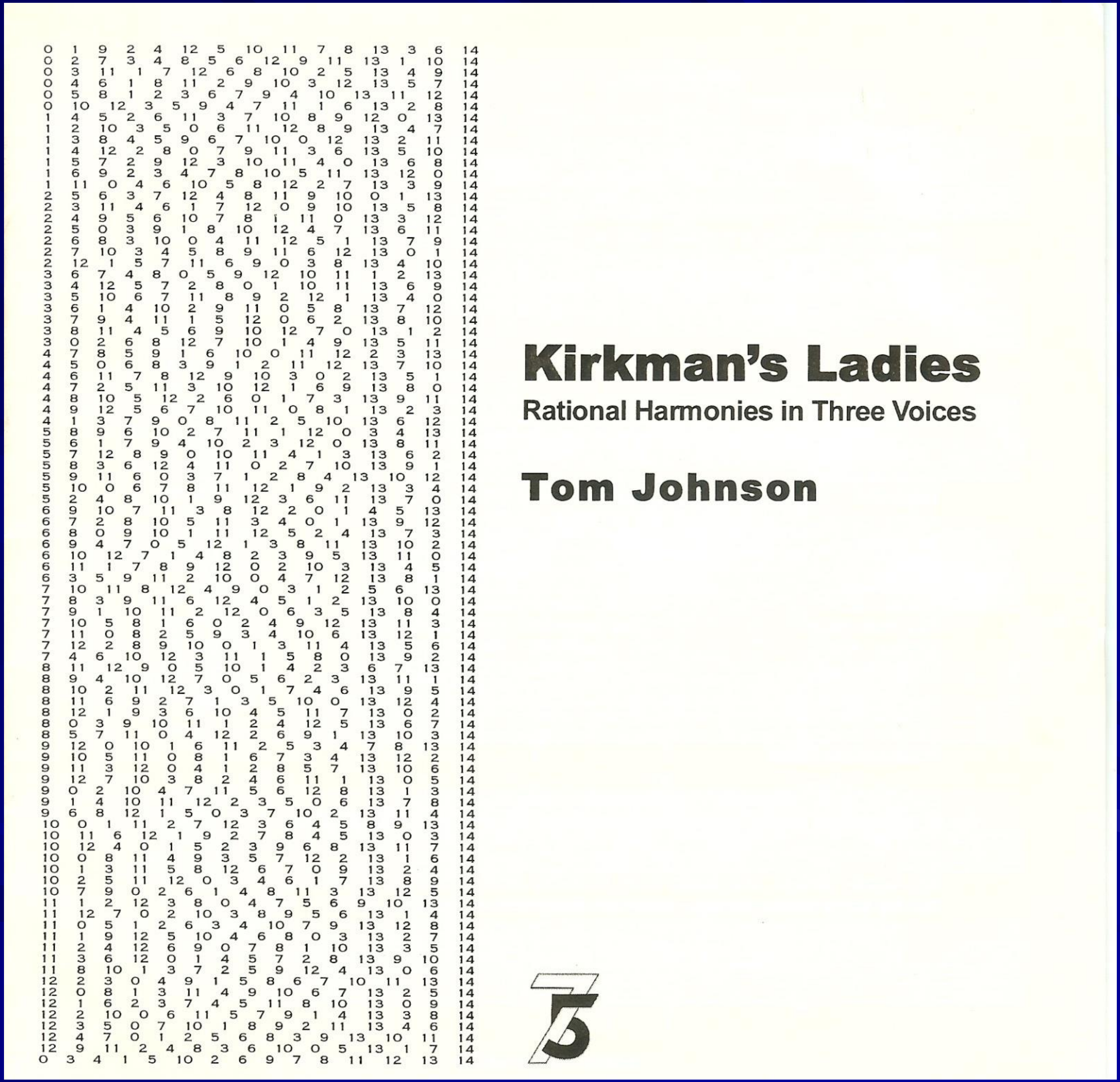
Is the number of disjoint pairs of STS(7) solutions equivalent to the number of 7 x 7 non-equivalent skew Room squares?

I calculate that there are 6 * 15 * 28 = 2520 ways to group the digits 1 – 8 into unordered pairs. Each group of 4 pairs can be ordered in 24 ways and 35 rhythms (xxxxooo, xxxoxoo, xxxooxo…) are possible with each. So there are 2520 * 24 * 35 = 2,116,800 possible lines in a 7 x 7 Room square. Would each of these fall 12 times in the 25,401,600 isomorphisms?

Finally, who was Room, and when did this all begin?

Many thanks,

PS Do you happen to know my old friend David Feldman in the Math Dept. at the Univ of NH?


Kirkman's Ladies
Rational Harmonies in Three Voices
Tom Johnson

Introduction

In 1847 Reverend Thomas Penyngton Kirkman, an English pastor who was also an amateur mathematician, proposed several solutions to this problem:

Fifteen young ladies in a school walk out three abreast for seven days in succession; it is required to arrange them daily so that no two shall walk twice abreast.
(*Ladies and Gentleman's Diary*, Query VI, p.48)

His work can be considered the first "block design," a subject that was to become a serious study in combinatorial mathematics. Of course, the discussion quickly grew to include all sorts of investigations of the possible combinations of sub-groups within larger groups, and even the original 15-ladies problem did not end with Kirkman, as mathematicians began to wonder whether it would be possible for the ladies to continue their daily walks for a complete semester of 13 weeks, so as to include all 455 possible three-lady combinations, once each. It was not until 1974 that R. H. F. Denniston of the University of Leicester published a solution, probably the only one, and thanks to him, I can now give you the music. Each lady/note appears once in the daily phrases of five chords, each pair of ladies walks together once a week, and by the end of the 13 weeks, all 455 possible trios of women have passed by, as have the 455 chords that represent them. I am particularly indebted to Paul Denny, a computer scientist at the University of Aukland, whose correspondence helped me to find this information and to understand it.

The main reason I like following logical sequences and mathematical processes is that they produce orderly music that is often predictable. The strange thing about the chords in *Kirkman's Ladies* is that, while they spin out of a super-intelligent logic, they surprise me with every change. The music seems to come from some point where precise organization meets near chaos.

Performance Notes

I particularly like to imagine my three-note chords played by three flutes, or as a harp solo, though an interpretation with three oboes, three strings or one vibraphone might also be just fine, and since the music is really only notes and numbers, I would not want to prevent one from playing it on the piano or with some other instruments. We should leave the chords in this register though, and always keep the ladies clean and pretty. It seems safe to assume that the sun is always shining —otherwise they would not be taking walks.

Tom Johnson
Paris, February 2005

Kirkman's Ladies

Rational Harmonies in Three Voices

First Week

Tom Johnson

tom: Room Squares and Related Designs (94 of 99)

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Back to tom

Date: Fri, 29 Jul 2005 13:42:48 +0200

From: Tom Johnson <tom@johnson.org>

To: Jeff H Dinitz <dinitz@cems.uvm.edu>

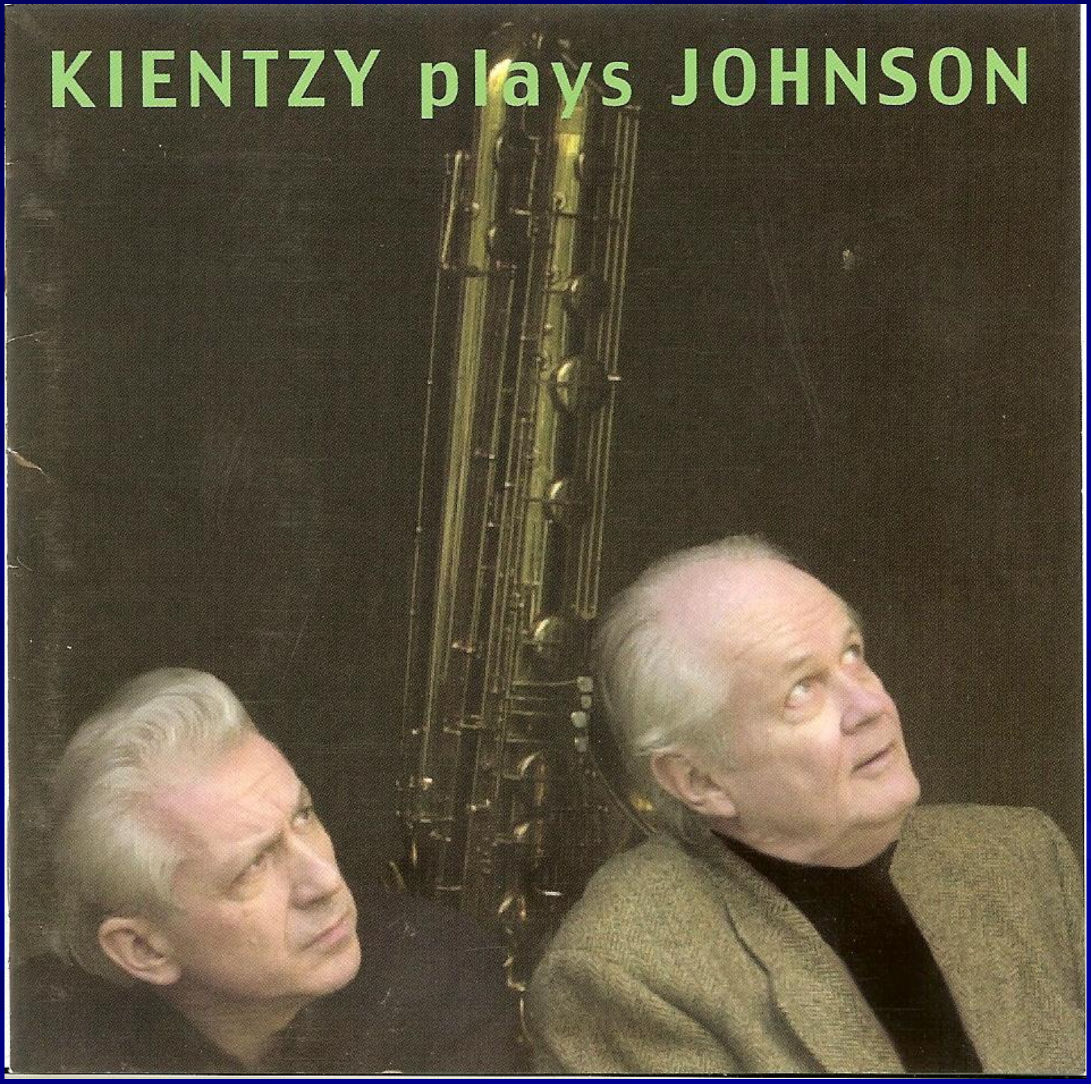
Subject: Room Squares and Related Designs

Dear Jeff,

I think there's only one copy of "Contemporary Design Theory" in Paris, but I found it and spent a long stimulating morning with "Room Squares and Related Designs." You said that this would give me some historical background, but in fact, it gave me much more. Strong starters, Howell Rotations. Martin Gardner references, starters in Z/13, the unique skew Room square of side 9, and even "houses" to go with the rooms. Of course, there's a lot I couldn't follow, but a lot that I could, and I'm sure the music will be richer because of the notes I took. Thanks. I'll send you some results as I get further along.

Glad you liked the CD.

Tom


KIENTZY plays JOHNSON

Date: Thu, 16 Nov 2006 11:02:26 +0100

From: Tom Johnson <tom@johnson.org>

To: dinitz@cems.uvm.edu

Subject: Progress report

Dear Jeffrey,

It is almost a year since our last correspondence, and I found your communications very useful, so I thought I'd drop a line.

I told you about the sound installations I was preparing. Six are ready to go, but they are kind of expensive to mount, and I haven't yet found a museum to do them. At the same time, many of these geometric arrangements seem quite interesting just to look at, and I am accumulating quite a few drawings. The Institut Poincaré found them lovely and wanted to show them, but they said it would be necessary to find a mathematician to explain the mathematics of all this, and the mathematicians I know all say: "But this is combination theory and graph theory and topology all at once and I don't know enough."

Meanwhile "Kirkman's Ladies" moves along. A harpist in London played them last month, and a very good flutist in Rome is making a three-track recording, but I don't have recordings of either.

I've studied Room Squares a lot, mostly thanks to your article, without ever finding a way to put them to music, but recently I looked back and now I see a possibility. But this leads me to one specific question. Do you think it would be possible to construct 15 7 X 7 Room squares (isomorphic or not) so that the 105 resulting lines would include once each the 105 possible rhythms : (1111000, 1110100, 1110010...)?

That's probably a question that has to wait for semester break.

Merry Christmas,

Tom

75 rue de la Roquette
75011 Paris
TEL 33 1 43 48 90 57
email: tom@johnson.org
www.tom.johnson.org

Date: Wed, 24 Oct 2007 16:44:08 -0400
From: Jeff Dinitz <jeff.dinitz@uvm.edu>
To: Tom Johnson <tom@johnson.org>
Cc: dinitz@cems.uvm.edu
Subject: Room squares of side 7

Hi Tom,
It's been a while -- hope you had a great summer.

I notice it has been nearly a year since you asked me the question below about finding 5 Room squares of side 7 that give all 35 possible rhythms. It is a nice question and I have had it on my back burner since you asked it. I now know the answer. It is not possible to find 5 such Room squares. If you want Room squares of side 7 which share no rhythms, the maximum number of these is 2, if you want to cover all the rhythms, it can be done in 6 Room squares. I have a masters student looking at these questions now and we can eventually send you her writeup, but I thought I'd answer your original question now. (Let me know if you want the proof -- it is pretty straightforward and uses no computers).

I will go out on the limb, however, and say that it may indeed be possible to do what you want with squares of side 11, namely find 42 Room squares of side 11 that give all (11 choose 6) possible rhythms. I don't have a proof of this, but I think we will eventually (I have the grad student looking at it so it may take longer than if I did it myself :-) If it exists -- is it music??

Hope this helps.

Best Regards,
Jeff

In her 2008 M.S. thesis Susan Janiszewski did indeed manage to find 42 Room squares of side 11 so that each one of the 462 possible arrangement of the 6 filled cells in a row occurs exactly once.

She found one nice one square and a permutation group which acts on it to give all the other squares.

A_0

2 7		0 3	9 a		1 5				4 6	8 b
		7 b	3 5		8 9	0 4	1 6	a 2		
a b		6 8		4 9			2 5	0 1	3 7	
9 0			7 1	5 8		2 b	3 4			6 a
				a 3	2 4	7 8		6 b	9 1	0 5
	3 8		2 6				9 b	5 7	a 0	1 4
3 6	0 b	1 2				5 a		4 8		7 9
4 5	a 1				3 b	6 9	7 0		8 2	
	4 7	5 9		1 b	6 0		8 a			2 3
8 1	5 6		4 b	0 2	7 a			9 3		
	9 2	a 4	8 0	6 7		1 3			5 b	

A_1

2 7		0 3	9 a	8 b		1 5				4 6
		7 b	3 5			8 9	0 4	1 6	a 2	
a b		6 8			4 9			2 5	0 1	3 7
9 0			7 1	6 a	5 8		2 b	3 4		
				0 5	a 3	2 4	7 8		6 b	9 1
	3 8		2 6	1 4				9 b	5 7	a 0
3 6	0 b	1 2		7 9			5 a		4 8	
4 5	a 1					3 b	6 9	7 0		8 2
	4 7	5 9		2 3	1 b	6 0		8 a		
8 1	5 6		4 b		0 2	7 a			9 3	
	9 2	a 4	8 0		6 7		1 3			5 b

A_2

2 7		0 3	9 a	4 6	8 b		1 5			
		7 b	3 5				8 9	0 4	1 6	a 2
a b		6 8		3 7		4 9			2 5	0 1
9 0			7 1		6 a	5 8		2 b	3 4	
				9 1	0 5	a 3	2 4	7 8		6 b
	3 8		2 6	a 0	1 4				9 b	5 7
3 6	0 b	1 2			7 9			5 a		4 8
4 5	a 1			8 2			3 b	6 9	7 0	
	4 7	5 9			2 3	1 b	6 0		8 a	
8 1	5 6		4 b			0 2	7 a			9 3
	9 2	a 4	8 0	5 b		6 7		1 3		

A_3

2 7		0 3	9 a		4 6	8 b		1 5		
		7 b	3 5	a 2				8 9	0 4	1 6
a b		6 8		0 1	3 7		4 9			2 5
9 0			7 1			6 a	5 8		2 b	3 4
				6 b	9 1	0 5	a 3	2 4	7 8	
	3 8		2 6	5 7	a 0	1 4				9 b
3 6	0 b	1 2		4 8		7 9			5 a	
4 5	a 1				8 2			3 b	6 9	7 0
	4 7	5 9				2 3	1 b	6 0		8 a
8 1	5 6		4 b	9 3			0 2	7 a		
	9 2	a 4	8 0		5 b		6 7		1 3	

A_4

2 7		0 3	9 a			4 6	8 b		1 5	
		7 b	3 5	1 6	a 2				8 9	0 4
a b		6 8		2 5	0 1	3 7		4 9		
9 0			7 1	3 4			6 a	5 8		2 b
					6 b	9 1	0 5	a 3	2 4	7 8
	3 8		2 6	9 b	5 7	a 0	1 4			
3 6	0 b	1 2			4 8		7 9			5 a
4 5	a 1			7 0		8 2			3 b	6 9
	4 7	5 9		8 a			2 3	1 b	6 0	
8 1	5 6		4 b		9 3			0 2	7 a	
	9 2	a 4	8 0			5 b		6 7		1 3

A_5

2 7		0 3	9 a				4 6	8 b		1 5
		7 b	3 5	0 4	1 6	a 2				8 9
a b		6 8			2 5	0 1	3 7		4 9	
9 0			7 1	2 b	3 4			6 a	5 8	
				7 8		6 b	9 1	0 5	a 3	2 4
	3 8		2 6		9 b	5 7	a 0	1 4		
3 6	0 b	1 2		5 a		4 8		7 9		
4 5	a 1			6 9	7 0		8 2			3 b
	4 7	5 9			8 a			2 3	1 b	6 0
8 1	5 6		4 b			9 3			0 2	7 a
	9 2	a 4	8 0	1 3			5 b		6 7	

A_6

2 7		0 3	9 a	1 5				4 6	8 b	
		7 b	3 5	8 9	0 4	1 6	a 2			
a b		6 8				2 5	0 1	3 7		4 9
9 0			7 1		2 b	3 4			6 a	5 8
				2 4	7 8		6 b	9 1	0 5	a 3
	3 8		2 6			9 b	5 7	a 0	1 4	
3 6	0 b	1 2			5 a		4 8		7 9	
4 5	a 1			3 b	6 9	7 0		8 2		
	4 7	5 9		6 0		8 a			2 3	1 b
8 1	5 6		4 b	7 a			9 3			0 2
	9 2	a 4	8 0		1 3			5 b		6 7

B_0

	4 6	8 b				9 a	1 5		2 7	0 3
a 2				1 6		3 5	8 9	0 4		7 b
0 1	3 7			2 5	4 9				a b	6 8
		6 a		3 4	5 8	7 1		2 b	9 0	
6 b	9 1	0 5			a 3		2 4	7 8		
5 7	a 0	1 4	3 8	9 b		2 6				
4 8		7 9	0 b					5 a	3 6	1 2
	8 2		a 1	7 0			3 b	6 9	4 5	
		2 3	4 7	8 a	1 b		6 0			5 9
9 3			5 6		0 2	4 b	7 a		8 1	
	5 b		9 2		6 7	8 0		1 3		a 4

B_1

	4 6	8 b		0 3			9 a	1 5		2 7
a 2				7 b	1 6		3 5	8 9	0 4	
0 1	3 7			6 8	2 5	4 9				a b
		6 a			3 4	5 8	7 1		2 b	9 0
6 b	9 1	0 5				a 3		2 4	7 8	
5 7	a 0	1 4	3 8		9 b		2 6			
4 8		7 9	0 b	1 2					5 a	3 6
	8 2		a 1		7 0			3 b	6 9	4 5
		2 3	4 7	5 9	8 a	1 b		6 0		
9 3			5 6			0 2	4 b	7 a		8 1
	5 b		9 2	a 4		6 7	8 0		1 3	

B_2

	4 6	8 b		2 7	0 3			9 a	1 5	
a 2					7 b	1 6		3 5	8 9	0 4
0 1	3 7			a b	6 8	2 5	4 9			
		6 a		9 0		3 4	5 8	7 1		2 b
6 b	9 1	0 5					a 3		2 4	7 8
5 7	a 0	1 4	3 8			9 b		2 6		
4 8		7 9	0 b	3 6	1 2					5 a
	8 2		a 1	4 5		7 0			3 b	6 9
		2 3	4 7		5 9	8 a	1 b		6 0	
9 3			5 6	8 1			0 2	4 b	7 a	
	5 b		9 2		a 4		6 7	8 0		1 3

B_3

	4 6	8 b			2 7	0 3			9 a	1 5
a 2				0 4		7 b	1 6		3 5	8 9
0 1	3 7				a b	6 8	2 5	4 9		
		6 a		2 b	9 0		3 4	5 8	7 1	
6 b	9 1	0 5		7 8				a 3		2 4
5 7	a 0	1 4	3 8				9 b		2 6	
4 8		7 9	0 b	5 a	3 6	1 2				
	8 2		a 1	6 9	4 5		7 0			3 b
		2 3	4 7			5 9	8 a	1 b		6 0
9 3			5 6		8 1			0 2	4 b	7 a
	5 b		9 2	1 3		a 4		6 7	8 0	

B_4

	4 6	8 b		1 5		2 7	0 3			9 a
a 2				8 9	0 4		7 b	1 6		3 5
0 1	3 7					a b	6 8	2 5	4 9	
		6 a			2 b	9 0		3 4	5 8	7 1
6 b	9 1	0 5		2 4	7 8				a 3	
5 7	a 0	1 4	3 8					9 b		2 6
4 8		7 9	0 b		5 a	3 6	1 2			
	8 2		a 1	3 b	6 9	4 5		7 0		
		2 3	4 7	6 0			5 9	8 a	1 b	
9 3			5 6	7 a		8 1			0 2	4 b
	5 b		9 2		1 3		a 4		6 7	8 0

B_5

	4 6	8 b		9 a	1 5		2 7	0 3		
a 2				3 5	8 9	0 4		7 b	1 6	
0 1	3 7						a b	6 8	2 5	4 9
		6 a		7 1		2 b	9 0		3 4	5 8
6 b	9 1	0 5			2 4	7 8				a 3
5 7	a 0	1 4	3 8	2 6					9 b	
4 8		7 9	0 b			5 a	3 6	1 2		
	8 2		a 1		3 b	6 9	4 5		7 0	
		2 3	4 7		6 0			5 9	8 a	1 b
9 3			5 6	4 b	7 a		8 1			0 2
	5 b		9 2	8 0		1 3		a 4		6 7

B_6

	4 6	8 b			9 a	1 5		2 7	0 3	
a 2					3 5	8 9	0 4		7 b	1 6
0 1	3 7			4 9				a b	6 8	2 5
		6 a		5 8	7 1		2 b	9 0		3 4
6 b	9 1	0 5		a 3		2 4	7 8			
5 7	a 0	1 4	3 8		2 6					9 b
4 8		7 9	0 b				5 a	3 6	1 2	
	8 2		a 1			3 b	6 9	4 5		7 0
		2 3	4 7	1 b		6 0			5 9	8 a
9 3			5 6	0 2	4 b	7 a		8 1		
	5 b		9 2	6 7	8 0		1 3		a 4	

C_0

			1 5	9 a	0 3	2 7		8 b		4 6
		0 4	8 9	3 5	7 b		1 6		a 2	
	4 9				6 8	a b	2 5		0 1	3 7
	5 8	2 b		7 1		9 0	3 4	6 a		
	a 3	7 8	2 4					0 5	6 b	9 1
3 8				2 6			9 b	1 4	5 7	a 0
0 b		5 a			1 2	3 6		7 9	4 8	
a 1		6 9	3 b			4 5	7 0			8 2
4 7	1 b		6 0		5 9		8 a	2 3		
5 6	0 2		7 a	4 b		8 1			9 3	
9 2	6 7	1 3		8 0	a 4					5 b

C_1

			1 5	4 6	9 a	0 3	2 7		8 b	
		0 4	8 9		3 5	7 b		1 6		a 2
	4 9			3 7		6 8	a b	2 5		0 1
	5 8	2 b			7 1		9 0	3 4	6 a	
	a 3	7 8	2 4	9 1					0 5	6 b
3 8				a 0	2 6			9 b	1 4	5 7
0 b		5 a				1 2	3 6		7 9	4 8
a 1		6 9	3 b	8 2			4 5	7 0		
4 7	1 b		6 0			5 9		8 a	2 3	
5 6	0 2		7 a		4 b		8 1			9 3
9 2	6 7	1 3		5 b	8 0	a 4				

C_2

			1 5		4 6	9 a	0 3	2 7		8 b
		0 4	8 9	a 2		3 5	7 b		1 6	
	4 9			0 1	3 7		6 8	a b	2 5	
	5 8	2 b				7 1		9 0	3 4	6 a
	a 3	7 8	2 4	6 b	9 1					0 5
3 8				5 7	a 0	2 6			9 b	1 4
0 b		5 a		4 8			1 2	3 6		7 9
a 1		6 9	3 b		8 2			4 5	7 0	
4 7	1 b		6 0				5 9		8 a	2 3
5 6	0 2		7 a	9 3		4 b		8 1		
9 2	6 7	1 3			5 b	8 0	a 4			

C_3

			1 5	8 b		4 6	9 a	0 3	2 7	
		0 4	8 9		a 2		3 5	7 b		1 6
	4 9				0 1	3 7		6 8	a b	2 5
	5 8	2 b		6 a			7 1		9 0	3 4
	a 3	7 8	2 4	0 5	6 b	9 1				
3 8				1 4	5 7	a 0	2 6			9 b
0 b		5 a		7 9	4 8			1 2	3 6	
a 1		6 9	3 b			8 2			4 5	7 0
4 7	1 b		6 0	2 3				5 9		8 a
5 6	0 2		7 a		9 3		4 b		8 1	
9 2	6 7	1 3				5 b	8 0	a 4		

C_4

			1 5		8 b		4 6	9 a	0 3	2 7
		0 4	8 9	1 6		a 2		3 5	7 b	
	4 9			2 5		0 1	3 7		6 8	a b
	5 8	2 b		3 4	6 a			7 1		9 0
	a 3	7 8	2 4		0 5	6 b	9 1			
3 8				9 b	1 4	5 7	a 0	2 6		
0 b		5 a			7 9	4 8			1 2	3 6
a 1		6 9	3 b	7 0			8 2			4 5
4 7	1 b		6 0	8 a	2 3				5 9	
5 6	0 2		7 a			9 3		4 b		8 1
9 2	6 7	1 3					5 b	8 0	a 4	

C_5

			1 5	2 7		8 b		4 6	9 a	0 3
		0 4	8 9		1 6		a 2		3 5	7 b
	4 9			a b	2 5		0 1	3 7		6 8
	5 8	2 b		9 0	3 4	6 a			7 1	
	a 3	7 8	2 4			0 5	6 b	9 1		
3 8					9 b	1 4	5 7	a 0	2 6	
0 b		5 a		3 6		7 9	4 8			1 2
a 1		6 9	3 b	4 5	7 0			8 2		
4 7	1 b		6 0		8 a	2 3				5 9
5 6	0 2		7 a	8 1			9 3		4 b	
9 2	6 7	1 3						5 b	8 0	a 4

C_6

			1 5	0 3	2 7		8 b		4 6	9 a
		0 4	8 9	7 b		1 6		a 2		3 5
	4 9			6 8	a b	2 5		0 1	3 7	
	5 8	2 b			9 0	3 4	6 a			7 1
	a 3	7 8	2 4				0 5	6 b	9 1	
3 8						9 b	1 4	5 7	a 0	2 6
0 b		5 a		1 2	3 6		7 9	4 8		
a 1		6 9	3 b		4 5	7 0			8 2	
4 7	1 b		6 0	5 9		8 a	2 3			
5 6	0 2		7 a		8 1			9 3		4 b
9 2	6 7	1 3		a 4					5 b	8 0

D_0

4 6			0 3		1 5	9 a		8 b		2 7
	0 4		7 b	a 2	8 9	3 5			1 6	
3 7			6 8	0 1			4 9		2 5	a b
	2 b					7 1	5 8	6 a	3 4	9 0
9 1	7 8			6 b	2 4		a 3	0 5		
a 0		3 8		5 7		2 6		1 4	9 b	
	5 a	0 b	1 2	4 8				7 9		3 6
8 2	6 9	a 1			3 b				7 0	4 5
		4 7	5 9		6 0		1 b	2 3	8 a	
		5 6		9 3	7 a	4 b	0 2			8 1
5 b	1 3	9 2	a 4			8 0	6 7			

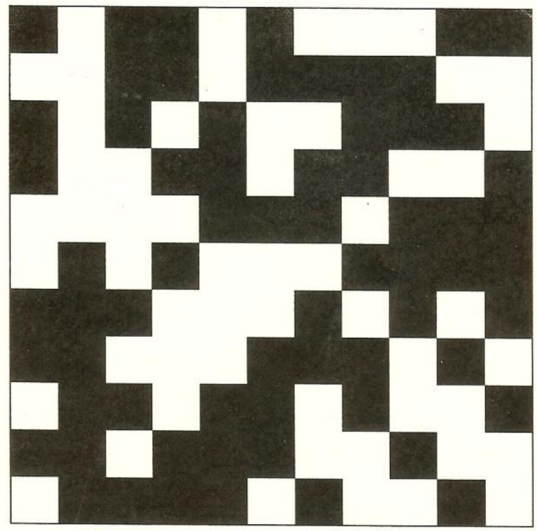
D_1

4 6			0 3	2 7		1 5	9 a		8 b	
	0 4		7 b		a 2	8 9	3 5			1 6
3 7			6 8	a b	0 1			4 9		2 5
	2 b			9 0			7 1	5 8	6 a	3 4
9 1	7 8				6 b	2 4		a 3	0 5	
a 0		3 8			5 7		2 6		1 4	9 b
	5 a	0 b	1 2	3 6	4 8				7 9	
8 2	6 9	a 1		4 5		3 b				7 0
		4 7	5 9			6 0		1 b	2 3	8 a
		5 6		8 1	9 3	7 a	4 b	0 2		
5 b	1 3	9 2	a 4				8 0	6 7		

D_2

4 6			0 3		2 7		1 5	9 a		8 b
	0 4		7 b	1 6		a 2	8 9	3 5		
3 7			6 8	2 5	a b	0 1			4 9	
	2 b			3 4	9 0			7 1	5 8	6 a
9 1	7 8					6 b	2 4		a 3	0 5
a 0		3 8		9 b		5 7		2 6		1 4
	5 a	0 b	1 2		3 6	4 8				7 9
8 2	6 9	a 1		7 0	4 5		3 b			
		4 7	5 9	8 a			6 0		1 b	2 3
		5 6			8 1	9 3	7 a	4 b	0 2	
5 b	1 3	9 2	a 4					8 0	6 7	

Vermont Rhythms



Tom Johnson

75 rue de la Roquette
75011 Paris

Two-Eighteen Press 12 Wolf Road
Croton- on-Hudson NY 10520

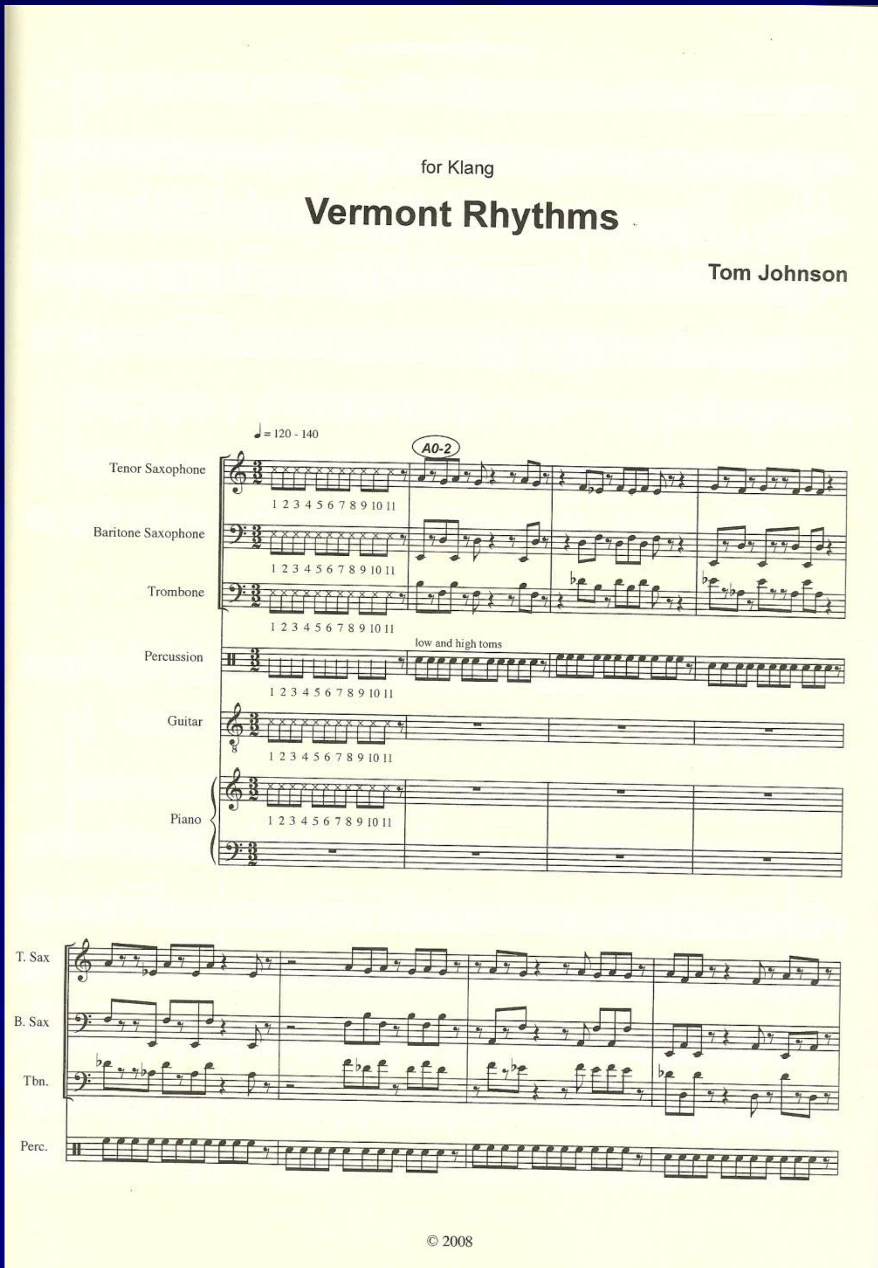
Introduction

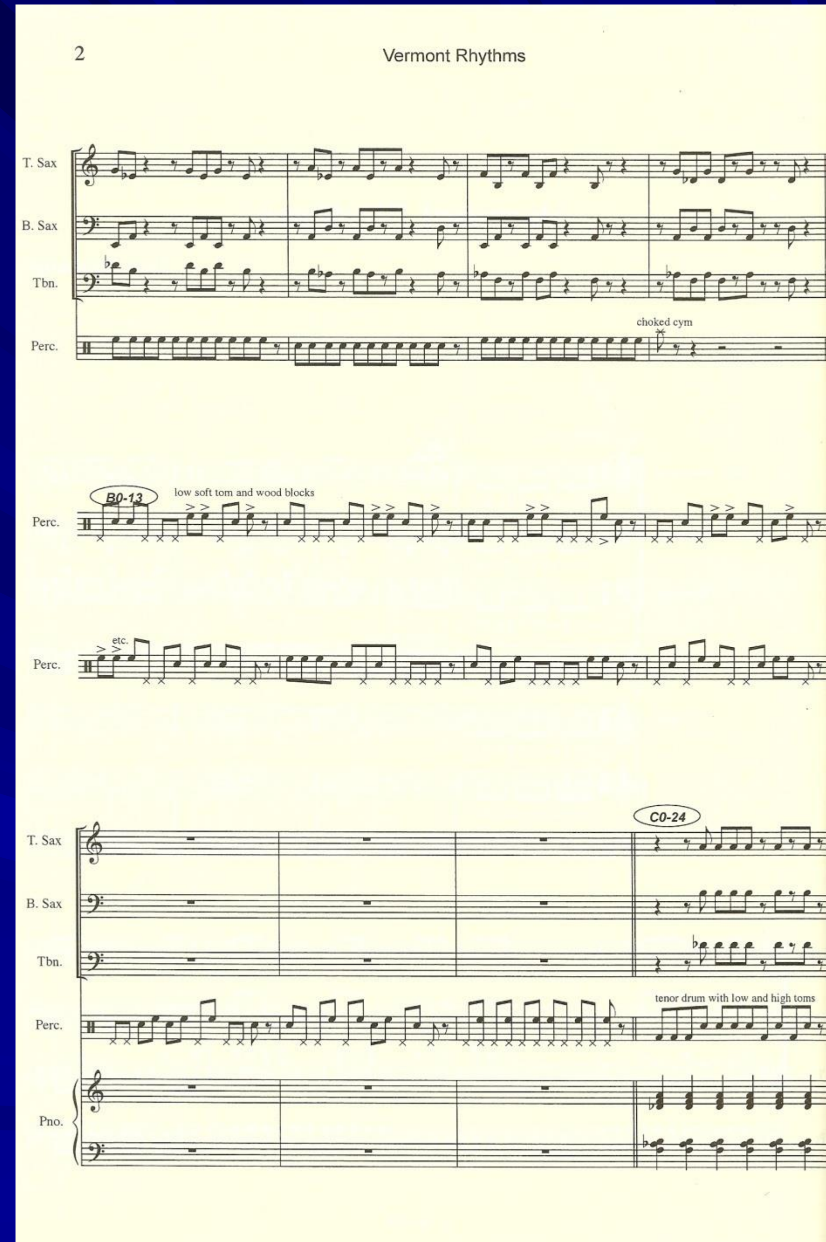
This piece is called *Vermont Rhythms* because it would never have been written without the cooperation of two Vermont mathematicians, working at the University of Vermont in Burlington. In answer to a question of mine, Susan Janiszewski, with her advisor, Professor Jeffrey H. Dinitz, constructed a remarkable list of all the 462 six-note rhythms possible in an 11-beat period. Their impressive list distributes the rhythms in 42 groups of 11, each group forming an 11 by 11 square. The first square, the first 11 measures of music, is shown on the cover, so that one can better appreciate the symmetry of these squares. All 42 squares contain six elements in each row and six elements in each column, giving maximal rhythmic variety within the 11 phrases of each square. Each six-note rhythm has exactly three beats in common with each of the 10 others, and mathematicians will appreciate additional symmetries in these configurations.

My primary interest was the 462 rhythms, but I soon realized that I could choose pitches by employing the 462 six-note chords possible on an 11-note scale at the same time, so I did that too. Of course, much of this organization will not be heard consciously, even by very astute listeners, but some of it will be quite clear to everyone, and it is satisfying to know that many unheard symmetries are also present, reflecting one another in the background.

As the piece became clearer in my mind, I realized it would be particularly effective played by Klang, an ensemble in The Hague that had recently done an amazing interpretation of *Narayana's Cows.* They agreed to premier the work, which explains why it is scored for two saxophones, trombone, guitar, percussion, and piano. The music has little to do with instrumental color, however, so the instrumentation may be varied somewhat to be more suitable for other ensembles.

Tom Johnson, Paris, December 2008


for Klang
Vermont Rhythms
Tom Johnson
♩ = 120 - 140
A0-2
Tenor Saxophone
Baritone Saxophone
Trombone
Percussion
Guitar
Piano
1 2 3 4 5 6 7 8 9 10 11
low and high toms
T. Sax
B. Sax
Tbn.
Perc.
© 2008


2
Vermont Rhythms
T. Sax
B. Sax
Tbn.
Perc.
choked cym
B0-13
low soft tom and wood blocks
etc.
C0-24
tenor drum with low and high toms
Pno.

click here to see our programmes for 2009-10

klang

• News Biography Reviews

Having just shaken off the jetlag from their latest American trip, Ensemble Klang are now busy working with the unique American composer Tom Johnson. He has written a substantial new work for the group, *Vermont Rhythms*, which will form part of a CD of his music the ensemble will record in January. The work will receive its premiere in England, at Kettle's Yard in Cambridge on 1st Feburary 2009, alongside works by Andrew Hamilton and Pete Harden.

The CD is the first in a new project the ensemble has initiated. Each CD-recording is an individual profile of a composer with whom the group has been working. This year the group will record CD profiles of, alongside Tom Johnson, Peter Adriaansz and Oscar Bettison. Other works on the Tom Johnson CD will include a selection of the *Rational Melodies* and *Narayana's Cows*. Watch this space for more info, and a special one-off CD launch concert!

The ensemble are also in preparations for a special and exciting concert of music by legendary composer Phill Niblock in *Het Paard van Troje*, The Hague. The event will form part of the March 14 edition of the *Dag in de Branding Festival*.

In November the group had both a productive and enjoyable time working with the post-graduate composition department at Princeton University, in the United States. They performed new works by Anne Hege, Jascha Narveson, Cameron Britt, Sean Friar and Lainie Fefferman alongside a movement from Oscar Bettison's *O Death* (to hear that movement click on the link on the right).

click here for the recording of Oscar Bettison's 'I believe I'm sinking down' for more music and photos see our Myspace

klang

Ensemble Klang (l-r): Anton van Houten, Tom Gelissen, Erik-Jan de With, Saskia Lankhoorn, Joey Marijs, Pete Harden, Heiko Geerts

The “Vermont Premiere” of

Vermont Rhythms

by

Tom Johnson

performed by

Klang

Thanks for coming!