Nonlinear Spectrum Reshaping and Gap-Soliton-Train Trapping in Optically Induced Photonic Structures

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We report the first theoretical prediction and experimental demonstration of gap soliton trains in a self-defocusing photonic lattice. Without *a priori* spectral or phase engineering, a stripe beam whose spatial power spectrum lies only in one transverse direction evolves into a gap soliton train with power spectrum growing also in the orthogonal direction due to nonlinear transport and spectrum reshaping. Our results suggest that, in nonlinear k-space evolution, energy can transfer not only between regions of normal and anomalous diffraction, but also from initially excited regions to initially unexcited regions.

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The formation of gap solitons is a fundamental phenomenon of wave propagation in nonlinear periodic media. It has been studied in various branches of science including biology [1], condensed matter physics [2], Bose-Einstein condensates [3], as well as nonlinear optics [4,5]. In optics, gap solitons are traditionally considered as a *temporal* phenomenon in one-dimensional (1D) periodic media such as an optical fiber with periodic refractive-index variations [4]. Recently, however, *spatial* gap solitons have been predicted and demonstrated in a number of experiments [5–7]. These gap solitons have their propagation constants located inside the first photonic band gap (between the first and the second optical Bloch bands) of the periodic structure, in contrast to discrete solitons in the semi-infinite gap [8]. Typically, these gap solitons were observed with *off-axis* excitations in which the probe beam was launched at an angle to match the edge of the first Brillouin zone (BZ). Such excitations led to the observation of spatial gap solitons in *self-defocusing* "backbone" lattices (bifurcating from the bottom of the first band at the BZ edge where diffraction is anomalous) [6] or in *self-focusing* lattices (bifurcating from the top of the second band at the BZ edge where diffraction is normal) [7]. While a host of novel phenomena have been demonstrated with a 2D photonic lattice (such as BZ spectroscopy, Bloch oscillations and Zenner-tunneling [9], and band gap guidance by defects [10], to name just a few), a simple *on-axis* excitation of a 2D fundamental gap soliton without *a priori* spectral or phase engineering [11] has not been realized. More importantly, gap soliton trains have never been demonstrated in any nonlinear system to our knowledge, and they could be considered as nonlinearity-induced line defects in photonic band gap structures [12].

In this Letter, we report the first prediction and experimental demonstration of spatial gap soliton trains by a single-beam excitation in a self-defocusing photonic lattice. While a narrow circular beam can evolve into a 2D gap soliton as long as its k-space spectrum covers the entire first BZ (or the four high-symmetry M points of the first Bloch band), self-trapping of a narrow stripe beam into a gap soliton is nontrivial. Surprisingly, we find that a stripe beam (whose Bloch momentum lies initially only in one transverse direction and spectrum covers only two opposite M points) can evolve into the gap soliton train whose k-space momentum grows also in the orthogonal transverse direction after nonlinear spectrum reshaping. This suggests that a gap soliton can arise from Bloch modes even if these modes are not initially excited or only weakly excited. We monitor the nontrivial staggered phase structure of the gap solitons, and find that nonlinear transport under a self-focusing and defocusing nonlinearity shows dramatically different behavior of spectrum reshaping.

Different from our earlier experiments with lattice solitons [13], here we employ a *self-defocusing* nonlinearity to induce a backbone waveguide lattice [6]. The lattice is established by sending a partially coherent light beam through an amplitude mask. Under appropriate negative bias conditions, the periodically modulated input intensity pattern [Fig. 1(a)] induces the lattice, which remains nearly invariant through a 10-mm long photorefractive crystal (SBN:61). The lattice beam is ordinarily-polarized, but the induced index variation (controlled by the lattice beam intensity, coherence, and the bias field) is adjusted to be high enough for opening the first gap [14]. A coherent Gaussian-like probe beam, splitting from the same laser but extraordinarily-polarized, is sent into the crystal and

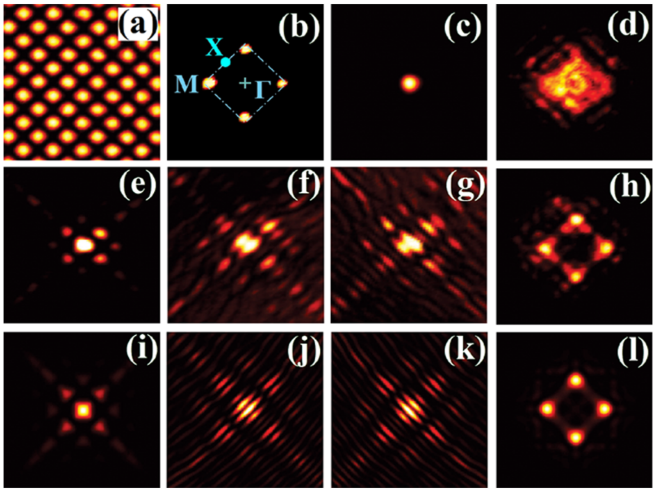

FIG. 1 (color online). Formation of a 2D gap soliton by single-beam on-axis excitation. Experimental results (a)–(h) show lattice pattern (a) and its spectrum with the first BZ and high-symmetry points marked (b), probe beam at input (c) and its linear output spectrum (d) through the lattice, output pattern of the gap soliton (e), its interferograms with a plane wave tilted from two different directions (f),(g) and its nonlinear output spectrum (h). Numerical results (i)–(l) show the gap soliton formation corresponding to (e)–(h).

propagates collinearly with the lattice beam. Linear and nonlinear transport of the probe beam is monitored simply by taking its instantaneous (before nonlinear self-action) and steady-state (after self-action) output patterns from the lattice. The spatial spectrum is obtained by using a lens to do Fourier transform and the far-field spectrum is recorded by a CCD camera positioned in its focal plane.

First, we summarize our results of *on-axis* excitation of a single 2D gap soliton in Fig. 1. Different from previous experimental observations in which either the probe beam was launched *off-axis* to match the edge of the first BZ [6] or its input phase or spectrum was engineered [11], we demonstrate on-axis excitation of a 2D gap soliton without *a priori* phase or spectral engineering. In fact, we show that nonlinear trapping of the probe beam leads to spectrum reshaping even though its initial spectrum is nearly uniform in the entire first BZ. The probe beam is focused into a 2D circular beam and launched into one of intensity minima (index maxima) of the backbone lattice (with about 23 μm spacing). The top panels of Fig. 1 show the input lattice beam, its Fourier spectrum, the input probe beam and its spectrum at output after linear propagation through the lattice, respectively. Under *linear* propagation, the probe beam experiences discrete diffraction, and its spectrum covers the first BZ with most of the power concentrating in the center. The middle panels show the formation of a 2D gap soliton under *nonlinear* propagation at a bias field of -1.3 kV/cm. The interferograms [Fig. 1(f) and 1(g)] show clearly that the gap soliton has a staggered phase structure (i.e., the central peak is out of phase with neighboring peaks), while its power spectrum [Fig. 1(h)] reshapes to have most of its power located in the four corners of the first BZ where diffraction is anomalous. These experimental results are corroborated with our numerical simulations using parameters close to those from experiment, as shown in the bottom panel of Fig. 1.

If the results of Fig. 1 are somewhat expected (although never reported before), our demonstration below on the formation of gap soliton trains is quite intriguing. Let us first illustrate theoretically the existence of such ‘‘staggered’’ gap soliton trains. Our theoretical model is a $(2+1)D$ NLS equation with a saturable self-defocusing nonlinearity and a periodic lattice potential [6,14,15]:

$$iU_z + U_{xx} + U_{yy} - \frac{E_0}{1 + I_L(x, y) + |U|^2} U = 0,$$

where U is the envelope function of the electric field, z is the distance of propagation, (x, y) are the transverse plane, E_0 is the applied dc field, and $I_L(x, y) = I_0\cos^2[(x + y)/\sqrt{2}]\cos^2[(x - y)/\sqrt{2}]$ is the lattice intensity pattern (with peak intensity I_0). All variables have been normalized [15]. For $I_0 = 10$, and $E_0 = -10$ (corresponding to -2.0 kV/cm), the band gap structure is shown in Fig. 2(a). Using the numerical method developed recently [16], we find a family of spatial gap soliton trains inside the first photonic gap, bifurcating from the right edge of the first band (corresponding to the lattice M-symmetry point). The power curve of these soliton trains is also plotted in Fig. 2(a), where the power is defined over one period along the train direction. At a point near the band edge, the gap soliton train is shown in Fig. 2(b). It exhibits several vertical stripes periodically modulated, and in the central high-intensity region the stripes form a checkerboard pattern as adjacent stripes are out of phase with each other while in phase along the stripe. This gap soliton train can be considered as a bound state of a sequence of single 2D gap solitons (as shown in the middle left panel of Fig. 1). The soliton k-space spectrum is displayed in Fig. 2(c), where the four spots correspond to the four M-symmetry points at the edge of the first BZ.

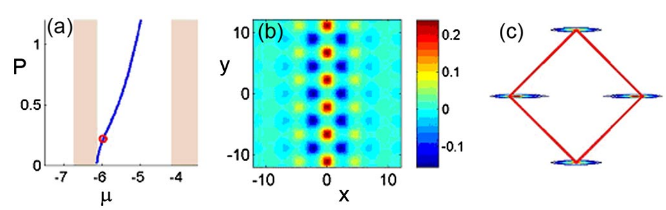

FIG. 2 (color online). Theoretical results showing formation of a gap soliton train. (a) the band gap structure (bands are shaded) and the soliton power curve; (b) a gap soliton train found at the red circle marked in (a), where the color bar shows the amplitude of the field; (c) the power spectrum of the gap soliton train in (b), and the red square marks the first BZ.

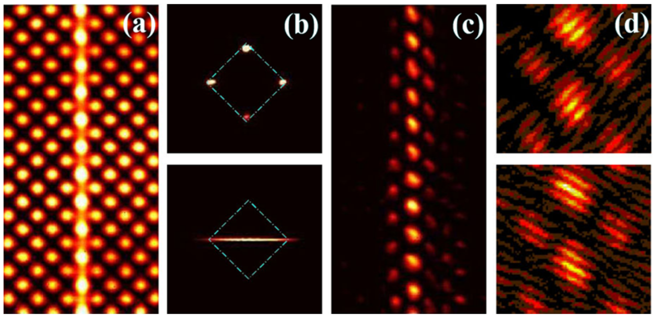

FIG. 3 (color online). Experiment results showing formation of a gap soliton train. (a) Superimposed pattern of a vertically oriented stripe beam and the lattice beam at input; (b) input spectrum of the lattice (top) and probe (bottom) beam with the first BZ marked by dotted line; (c) output pattern of the gap soliton train and its interferograms with a plane wave tilted from two different directions (d).

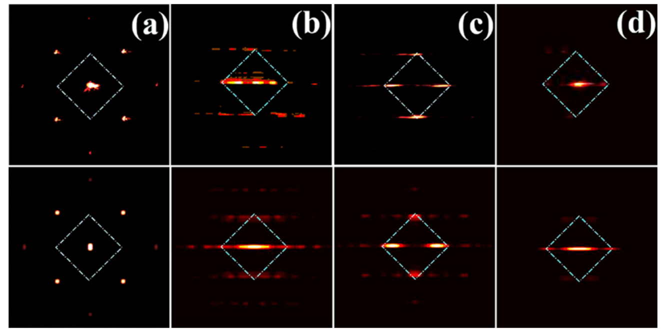

FIG. 4 (color online). Experimental (top) and numerical (bottom) results showing nonlinear spectrum reshaping. (a),(b) show the spectra of a 2D broad circular beam (a) and a 1D narrow stripe beam (b) under *linear* propagation (before self-action takes place) through the lattice. (c),(d) show the spectra of the stripe beam under *nonlinear* propagation (after self-action takes place) with self-defocusing (c) and self-focusing (d) nonlinearity.

Next, we show our experimental demonstration of a gap soliton train excited by a uniform stripe beam. The superimposed intensity pattern of the lattice and stripe beam at input is shown in Fig. 3(a). The vertically oriented stripe beam propagates collinearly with the lattice. Their power spectra are shown in Fig. 3(b), where the spectrum of the stripe beam forms nearly a horizontal line extended to two diagonal M points of the square lattice. Under nonlinear propagation at a bias field of -1.6 kV/cm, the stripe beam evolves into a gap soliton train [Fig. 3(c)], similar to that found in theory [Fig. 2(b)]. The staggered phase structure of the soliton beam is confirmed by its interferograms with a tilted plane wave [Fig. 3(d)], where the breaking and interleaving of interference fringes suggests the out-of-phase relation between the central stripe and two lateral stripes. Separate interferogram measurement shows the in-phase relation between peaks along the stripes in vertical direction.

The above observation poses a question. Different from a circular beam [Fig. 1(c) and 1(d)], a stripe beam has its initial k-space spectrum covering only a small portion of the first BZ, with no spatial frequency in regions near the two vertical M points [Fig. 3(b), bottom]. How does such a beam evolve into a gap soliton train? To answer the above question and better understand the linear and nonlinear transport of the stripe beam through the 2D lattice, we perform a series of experiments and numerical simulations under different conditions. We find that the power spectrum of the stripe beam becomes remarkably different from its initial spectrum after nonlinear propagation: not only does it break up in the horizontal direction into two spots near the two horizontal M points, but significant amount of power grows also near the other two vertical M points [Fig. 4(c)]. This indicates that the power spectrum of the probe beam has been drastically reshaped by nonlinear trapping into that of the exact gap soliton train found in Fig. 2(c).

Examining the spectra from linear and nonlinear propagation of a quasi-2D plane wave (a broad circular beam) and a quasi-1D plane wave (a narrow stripe beam) through the 2D lattice (Fig. 4, top panels), the picture of nonlinear spectrum reshaping becomes clear. According to the theory of Bloch-wave excitation in periodic structures [17], on-axis propagation of a 2D plane wave (with zero transverse k-vector) corresponds to the central high-symmetry Γ point of the first BZ [the brightest spot in Fig. 4(a)]. Because of lattice periodicity, it also excites the central points in the extended BZs of the transmission spectrum as shown in Fig. 4(a), but no excitation at the M points near the edge of the first BZ. With a broad vertically oriented stripe beam, the *linear* spectrum (before self-action takes place) remains the same, except that each bright spot becomes elongated in horizontal direction. When the stripe beam is narrow, the spectrum becomes highly anisotropic, and this elongation turns virtually the bright spots into lines which extend to regions close to the four M points of the first BZ [Fig. 4(b)], although the power is still concentrated in the central bright line. Conversely, the *nonlinear* spectrum (after self-action takes place) changes dramatically, in which the power grows in regions close to the four M points where diffraction is anomalous, but decays almost to zero in the center where diffraction is normal [Fig. 4(c)]. For comparison, the *nonlinear* spectrum for the same stripe beam with *self-focusing* nonlinearity is shown in Fig. 4(d), where a discrete soliton train arising from the central Γ point is formed in the semi-infinite gap [18]. These observations agree perfectly with our numerical results from beam-propagation simulations [Fig. 4, bottom panels].

Finally, the orientation of the stripe beam is changed from vertical to diagonal (i.e., along one of the principle axes of the square lattice) [Fig. 5]. When the stripe beam is launched straight into the lattice (on-axis excitation), new features such as embedded solitons related to excitation of

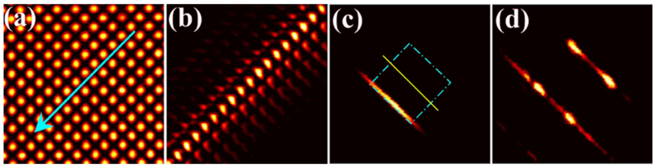

FIG. 5 (color online). Formation of a gap soliton train orientated along the lattice principle axis. (a) Lattice intensity pattern with an arrow illustrating the orientation and tilting direction of the probe beam; (b) output intensity pattern of the gap soliton train; (c) spectrum of the stripe beam tilted at the Bragg angle (yellow line marks spectrum location with no tilting angle); (d) spectrum of the gap soliton train.

Bloch modes at X points are observed, but a gap soliton train cannot arise from the M points since in this case the k-space spectrum of the probe beam covers X points [Fig. 5(c)] but provides no "seeding" momentum to the M points. However, once the stripe beam is tilted such that its spectrum "touches" one edge of the first BZ (off-axis excitation), a gap soliton train is realized again with modes arising from the four M points [Fig. 5(d)] due to Bragg reflection and spectrum reshaping. Interferograms reveal that, for this type of gap soliton train, adjacent spots are out of phase in directions both along and perpendicular to the initial stripe, characteristic of the Bloch modes close to M points of the first band [17] [see also Fig. 2(b)]. We have theoretically found such diagonally oriented (but tilted off-axis) gap soliton trains as well.

In summary, we have predicted and demonstrated the formation of gap soliton trains due to nonlinear transport and spectrum reshaping of a stripe beam in 2D induced lattices. The soliton trains arise from Bloch modes from the high-symmetry M points of the first photonic band, although some of these modes are initially not or only weakly excited from lattice scattering or diffraction. We note that in nonlinear spectrum reshaping (k-space evolution), while energy transfer between regions of normal diffraction and anomalous diffraction is not surprising [9], energy transfer from regions initially excited to regions initially unexcited is unexpected, as also mentioned in recent work with random-phase lattice solitons [19]. These results may have direct impact on the study of nonlinear Bloch-wave interaction and localization in periodic systems beyond optics such as condensed matter physics or Bose-Einstein condensates [2,3], where 2D gap solitons might arise from electronic or atomic Bloch modes weakly populated in the ground state by quantum fluctuations even if complete preparation of coherent wave packets at the corresponding band edge is not feasible.

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