Collision-induced pulse timing jitter in a wavelength-division-multiplexing system with strong dispersion management

D. J. Kaup

Department of Mathematics and Computer Science and Department of Physics, Clarkson University, Potsdam, New York 13699-5817

B. A. Malomed

Department of Interdisciplinary Studies, Faculty of Engineering, Tel Aviv University, Tel Aviv 69978, Israel

J. Yang

Department of Mathematics and Statistics, The University of Vermont, Burlington, Vermont 05401

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We develop a perturbation theory to calculate analytically the effects of interchannel collisions on Gaussian pulses in a wavelength-division-multiplexed (WDM) system with moderate and strong dispersion management (DM). The losses are assumed to be balanced by the amplification and are not explicitly included into the model. We show that, for complete collisions, the collision-induced frequency shift of a Gaussian pulse is negligible, whereas for incomplete collisions (those with initially overlapped pulses) this shift is significant. We also show that, as the DM strength increases, the collision-induced position shift becomes more important than the frequency shift produced by the incomplete collision. Another result is that the collisional shifts depend on the DM strength and the path-average dispersion but not on the lengths of the two fiber segments in the DM cell. We check the fully analytical predictions against direct PDE simulations and find satisfactory agreement between them. We also give an estimate of the limit imposed on the transmission distance in the WDM soliton systems by the interchannel collisions. [S0740-3224(99)00410-5]

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1. INTRODUCTION

Two topics have recently attracted a great deal of interest in studies of optical pulse communications: dispersion management (DM), i.e., the use of periodically modulated dispersion in a fiber communication line,[1] and wavelength-division multiplexing (WDM), i.e., the use of several channels in the same fiber, with the carrier wavelengths separated by a difference of $\gtrsim 1$ nm.[2] It has been experimentally demonstrated that these techniques permit significant improvement of bit rate and quality of the signal transmission in optical communication lines over long distances; see, e.g., a recent report.[3]

One of the most serious problems in the use of WDM is the cross talk (equivalent to a random timing jitter) induced by collisions of pulses that belong to different channels. Recently the collision-induced frequency shift was considered in several theoretical publications[4,5] by a method developed in Ref. 2 for the sech pulses (see also an early paper[6] in which the cross-talk effect was considered by means of a perturbation theory for the sech solitons in the WDM system). However, comparison of these results with direct partial differential equation simulations in the case of strong DM, which has the most promising potential for applications, demonstrates that the actual frequency shift is much smaller than predicted.[5] To reconcile the difference, a phenomenological factor, based on the energy enhancement of a DM soliton, was introduced and shown to work quite well.[5]

However, in the regime dominated by DM, when the local dispersion length is much smaller than the nonlinearity length the pulse profile is closer to a Gaussian than to a sech[1,7] (although far tails of the pulses are always exponential rather than Gaussian[8]). Consequently, in this case the pulse propagates in a nearly linear regime. This is the most fundamental explanation for the advantages offered by DM, such as suppression of the soliton's jitter and interaction effects: In the strictly linear dispersion-compensated system, the Gaussian pulse (which is an exact solution in this limiting case) cannot interact at all with the noise and with other pulses. The control parameter for the linear part of the DM model is the DM strength, whereas the relative strength of the nonlinearity is accounted for by an independent parameter, the pulse's normalized power.[9]

Recently the dynamics of the nearly Gaussian pulses was studied in detail,[7,9–11] and good agreement between the analytical and the numerical results was found. The objective of the present study is to apply this technique to the interchannel collisions of pulses in the DM-dominated WDM system in the lossless approximation. The new perturbation to be added to those considered in Ref. 11 is the interchannel nonlinear coupling through cross-phase modulation (XPM). Using the variational method, we derive a full system of evolutional equations for the collision-induced frequency and position shifts of the pulse. We demonstrate that, for the complete collisions, i.e., for those with the pulses well separated before and after the collision, the net frequency shift is zero in the first order of the perturbation theory. This explains why the numerical values of this shift in Ref. 5 were so small. At the same time, the collision-induced position shift is shown to be nonzero and can be obtained in a simple analytical form. For incomplete collisions, i.e., those between initially overlapped pulses, the frequency shift is significant and is also given by a simple analytical formula.

In the general case, a pulse overlaps those in adjacent channels at the line's input. This pulse then undergoes a single initial incomplete collision, followed by many complete ones. For such a case we show that, as the DM strength increases, the frequency shift generated by the initial incomplete collision becomes less important than the net position shift produced by the multiple in-line complete collisions. However, in no case is the extra timing jitter generated by an incomplete collision ever much more than that which is due to the multiple complete collisions. These analytical results are checked against numerical simulations and show good agreement for both complete and incomplete collisions. Finally, we give an estimate of the limit of the transmission distance imposed by both the incomplete and the complete collisions (a conservative estimate proves to be ~15,000 km for a typical set of values of the parameters and with moderately strong DM).

Some results of this analysis have already been reported briefly in Ref. 12. In this paper we provide a more detailed description of the theoretical approach and give new results that provide for a comprehensive comparison of analytical and numerical results as well as of systematic dependences of the results on the control parameters, which is definitely necessary for applications. Recently the effects of the WDM collisions for Gaussian pulses were studied in Refs. 13 and 14 by means of a semianalytical variational method; it was concluded that for Gaussian pulses the effects are smaller than for sech pulses. However, the main emphasis in Refs. 13 and 14 was placed on the final frequency shifts rather than on the position shifts. Another recent relevant work is Ref. 15, in which an analytical approach was developed in the weak-DM limit, and, in particular, the statistics of multiple collisions in a multichannel WDM system were studied in detail.

2. ANALYTICAL APPROACH

A. General Consideration

We start with a system of equations that govern the propagation of electromagnetic field envelopes, $u(z, \tau)$ and $v(z, \tau)$, in two adjacent channels in a dispersion-managed optical fiber link:

$$i(u_z + cu_\tau) + {}^1\!/_2 D(z) u_{\tau\tau} + \epsilon[{}^1\!/_2 \bar{D}_u u_{\tau\tau} + \gamma(|u|^2 + 2|v|^2)u] = 0, \quad (1)$$

$$iv_z + {}^1\!/_2 D(z) v_{\tau\tau} + \epsilon[{}^1\!/_2 \bar{D}_v v_{\tau\tau} + \gamma(|v|^2 + 2|u|^2)v] = 0, \quad (2)$$

where c is the inverse group-velocity difference between the channels, $D(z)$ is the main part of the dispersion (with the zero average), $\bar{D}_{u,v}$ are the residual path-average dispersions in the two channels, and the nonlinear terms represent the self-phase modulation and the XPM.[17] We assume the usual two-step form of DM:

$$D(z) = \begin{cases} D_1 & 0 < z < L_1 \\ D_2 & L_1 < z \leqslant L_1 + L_2 \end{cases}, \quad (3)$$

where, by definition, $D_1 L_1 + D_2 L_2 = 0$, which is repeated with the period $L \equiv L_1 + L_2$.

In what follows, we adopt the normalization in which the self-phase modulation coefficient $\gamma \equiv 1$. The small parameter ϵ is introduced, as in Ref. 11, to label the terms that are to be treated as perturbations in the quasi-linear strong-DM regime. The dispersions in the adjacent channels differ because of the nonzero third-order dispersion.[16] If the wavelength separation between the channels, $\delta\lambda$, is small, it is sufficient to keep only the interchannel differences in the small values of $\bar{D}_{u,v}$, and small differences in the local dispersion $D(z)$ may be neglected.

In the zeroth-order approximation ($\epsilon = 0$), one has an exact solution to linearized equation (1) in the form of the Gaussian pulse moving at velocity c:

$$u_0(z, \tau) = \tau_0 \left[\frac{P_u}{\tau_0{}^2 + 2i\Delta(z)} \right]^{1/2} \times \exp\left[-\frac{(\tau - cz)^2}{\tau_0{}^2 + 2i\Delta(z)} + i\phi_u \right], \quad (4)$$

where $\Delta(z) \equiv \int_0^z D(z')\mathrm{d}z' + \Delta_0$ is the accumulated dispersion and the constants P_u, τ_0, Δ_0, and ϕ_u determine, respectively, the pulse's maximum peak power, minimum width, chirp, and phase. Although a more general case can also be handled, here the analysis is restricted to the case when the pulses in the adjacent channels have equal widths τ_0. The values of Δ_0 always prove to be equal (see below). However, the peak powers could be different, particularly if the difference between $\bar{D}_u$ and $\bar{D}_v$ were considerable.

To describe the dynamics of interacting pulses we start with a more general exact unperturbed solution, which is obtained from Eq. (4) by a Galilean boost:

$$u(z, \tau) = u_0[z, \tau - T(z)]\exp[-i\omega\tau + i\psi(z)], \quad (5)$$

where ω is the frequency shift and the position and phase shifts are governed by the equations

$$\mathrm{d}\omega/\mathrm{d}z = -(1/2)\omega^2[D(z) + \epsilon\bar{D}_u],$$

$$\mathrm{d}T/\mathrm{d}z = -\omega[D(z) + \epsilon\bar{D}_u]. \quad (6)$$

In the single-channel case, two relations among the parameters P_u, τ_0, and Δ_0 must be satisfied if the pulse's propagation is to remain stationary in the presence of weak self-phase modulation and small $\bar{D}_{u,v}$.[10,11] By means of the variational method one can obtain the two conditions mentioned above for the stationary Gaussian pulse in the following analytical form[11]:

$$\Delta_0 = -(1/2) S \tau_0^2, \tag{7}$$

$$2\sqrt{2}\bar{D}_u = -P_u \frac{\tau_0^2}{S}[\ln(\sqrt{1 + S^2} + S) - 2S(1 + S^2)^{-1/2}], \tag{8}$$

where we have defined the dimensionless DM strength:

$$S \equiv |D_{1,2}| L_{1,2}/\tau_0^2. \tag{9}$$

This definition is slightly different from that put forward in Ref. 5, whose authors took $\tilde{S} \equiv |D_1 L_1 - D_2 L_2|/(t_{\mathrm{FWHM}}^2)_{\min}$. For the Gaussian pulse [Eq. (4)], the minimum (over the DM cycle) of the squared pulse's FWHM is $(t_{\mathrm{FWHM}}^2)_{\min} = 2\ln 2\tau_0^2$. Hence, recalling that $D_1 L_1 + D_2 L_2 = 0$, we arrive at the relation between our S and that defined in Ref. 9 as $\tilde{S} \equiv S/\ln 2 \approx 1.44S$.

Usually DM is considered (moderately) strong when $\tilde{S} \gtrsim 1$. In fact, both extremely strong ($S > 30$) (Ref. 17) and moderately strong ($S > 0.8$) DM may find their applications; an asset of the latter case is that it provides for strong suppression of the interaction between the pulses inside one channel without loss of the advantages offered by strong DM.[18]

Conditions (7) and (8) for the stationary propagation of the Gaussian pulse were verified in Ref. 11 against direct numerical simulations, yielding a fairly good agreement over a broad range of parameters (see also Ref. 9). In particular, stationary pulse propagation for normal average dispersion ($\bar{D}_u < 0$) is predicted for $S > S_{\mathrm{cr}} \approx 3.3$ (or $\tilde{S} > \tilde{S}_{\mathrm{cr}} \approx 4.8$). This result was verified numerically in Ref. 11 and was first observed in the research reported in Ref. 19 (see also Ref. 20). Moreover, straightforward examination of Eq. (8) demonstrates that, for $\bar{D}_u < 0$, P_u, and $S > S_{\mathrm{cr}}$, there are two different solutions for τ_0 (whereas for $\bar{D}_u > 0$ the solution is always single). As was numerically demonstrated in Ref. 9, only the solution with the higher power is stable. Note that, although the stable pulse has a peak power larger than that of the unstable pulse, both are successfully predicted by quasi-linear ansatz (4).[11]

Once the parameters of the pulses in both channels are selected according to Eqs. (7) and (8), the next step is to consider their collision, treating the XPM coupling between the channels as another perturbation. The simplest way to study the collision effect is, again, to use the variational method (neglecting radiative losses, which are, in fact, higher-order corrections). To this end, we substitute the Galilean-boosted u pulse [Eq. (5)], along with a similar v pulse, into the Lagrangian of Eqs. (1) and (2). Applying the standard variational procedure to the frequency shift ω and the position shift T, one can derive evolution equations for them. These can be reduced to a simple pair of equations (see below), which can be solved analytically and are a simplification of the analysis developed in Ref. 13. Omitting further technical details, we display the final form of the first-order XPM-induced evolution equation for the frequency shift ω, defined as in Eq. (5) (Ref. 11; a similar approach was developed in Ref. 21):

$$\frac{\mathrm{d}\omega}{\mathrm{d}z} = \frac{2^{3/2}\epsilon P_v \tau_0^4 cz}{[\tau_0^4 + 4\Delta^2(z)]^{3/2}} \exp\left\{-\frac{c^2\tau_0^2 z^2}{[\tau_0^4 + 4\Delta^2(z)]}\right\}, \tag{10}$$

where P_v is the peak power of the pulse in the v channel. At the same order, the evolution of T is still governed by Eqs. (6).

Solving Eqs. (6) and (10) is facilitated by the fact that, because c is small, the function cz varies slowly in comparison with the rapidly oscillating accumulated dispersion $\Delta(z)$. Actually, the latter circumstance is equivalent to the known fact that, in the strong-DM regime, colliding pulses pass through each other many times before separating.[5] The condition for this to be true is

$$c^2 \ll L^{-2}(\tau_0^2 + 4\tau_0^{-2}\Delta^2) \tag{11}$$

(recall that L is the DM period). To verify that this condition holds in reality, we take the most relevant case, when one channel is close to the point at which Eq. (8) predicts $\bar{D}_u = 0$, while, in the other channel, a nonzero average dispersion is generated by third-order dispersion. We take, as sample values, the pulse width 30 ps, the (maximum possible) DM period 200 km,[16] and the fiber's local dispersion 20 ps^2/km. In physical units, condition (11) then amounts to $|c| \ll 1$ ps/km. On the other hand, assuming that $\delta\lambda = 1$ nm and a realistic third-order dispersion coefficient $|\beta_3| = 0.1$ ps^3/km, we obtain the following estimate for the interchannel inverse group-velocity mismatch: $|c| \sim 0.1$ ps/km. Thus condition (11) can be readily satisfied.

Simultaneously, we obtain an estimate for the collision distance, $z_{\mathrm{coll}} \sim 500$ km. Because the DM period is $\leqslant 200$ km,[22] z_{coll} is much larger than L, which accounts for the effective cancellation of the net frequency shift in the complete collision (see below). However, a complete collision could produce an appreciable frequency shift if the collision distance were smaller (see, e.g., Ref. 14). This would happen if one used a relatively large wavelength separation, such as $\delta\lambda = 8$ nm.

B. Complete Collision

To solve Eqs. (6) and (10) in the case of the complete collision we need to evaluate integrals of the form

$$I_n = \int_{-\infty}^{+\infty} z^n F(z) \exp[-z^2 f^2(z)] \mathrm{d}z, \tag{12}$$

where $f(z) > 0$ and $F(z)$ are rapidly oscillating periodic functions with a period T ($\ll 1$) and n is an integer. To see how one may evaluate this integral, note that it can be rewritten as

$$I_n = \sum_{m=-\infty}^{\infty} \oint_{T_m} z^n F(z) \exp[-z^2 f^2(z)] \mathrm{d}z, \tag{13}$$

where T_m is the mth period of the functions $f(z)$ and $F(z)$ and $\oint$ stands for the integral over this period. Inasmuch as $T \ll 1$, the variables z^n and z^2 in Eq. (13) change little over the small interval T_m. This suggests that, in the lowest-order approximation, we can treat z in z^2 and z^n as some constant Z within each period T_m and then approximately replace the summation over m in Eq. (13) by the integration over this *ad hoc* variable Z. Thus we arrive at the approximation

$$I_n \approx T^{-1}\int_{-\infty}^{\infty} \mathrm{d}Z Z^n \int_0^T \mathrm{d}z F(z)\exp[-Z^2 f^2(z)]\mathrm{d}z. \quad (14)$$

Interchanging the order of integrations in Eq. (14) and using the formula

$$\int_{-\infty}^{\infty} x^n e^{-x^2}\mathrm{d}x = 2^{-n/2}\sqrt{\pi}C_n, \quad (15)$$

where

$$C_n = \begin{cases} 0 & \text{if } n \text{ is odd} \\ 1 & \text{if } n = 0 \\ (n-1)!! & \text{if } n \text{ is even} \end{cases}, \quad (16)$$

we finally find that

$$I_n \approx 2^{-n/2}\sqrt{\pi}C_n\langle F(z) f^{-(n+1)}(z)\rangle, \quad (17)$$

where $\langle\,\rangle$ indicates the average over period T.

A consequence of Eq. (10) and relation (17) is that

$$\delta\omega = \int_{-\infty}^{+\infty} (\mathrm{d}\omega/\mathrm{d}z)\mathrm{d}z = 0; \quad (18)$$

i.e., in the first-order approximation the complete collision produces zero net change of the frequency, which is a characteristic feature (and advantage) of strong DM. (We note that implicit in the definition of moderate or strong DM is the assumption that the nonlinear length is much larger than the local dispersion length; i.e., XPM is a small perturbation indeed. At much higher power levels this would not be so.) This result is in agreement with the numerical observations reported in Ref. 5, which showed that the collision-induced frequency shift in a WDM system with strong DM was ~10 times smaller than the estimate based on the sech soliton. A nonzero frequency shift produced by the next-order correction to relation (17) (by taking into account the small change in the slow variable Z, within the period of the rapid oscillations) could also be found. Here we only give an estimate for it (D is the local dispersion):

$$\delta\omega \sim \epsilon P_v \tau_0^{\,3}/D^2 L. \quad (19)$$

We shall estimate the relative size of this term below.

Proceeding to the net collision-induced position shift δT_u, we integrate Eqs. (6) by parts, casting the expression for δT_u into the form

$$\delta T_u \equiv \int_{-\infty}^{+\infty} \frac{\mathrm{d}T}{\mathrm{d}z}\mathrm{d}z = \epsilon \bar{D}_u \int_{-\infty}^{+\infty} z\frac{\mathrm{d}\omega}{\mathrm{d}z}\mathrm{d}z + \int_{-\infty}^{+\infty} \Delta(z)\frac{\mathrm{d}\omega}{\mathrm{d}z}\mathrm{d}z. \quad (20)$$

Next, we substitute here Eq. (10) for $\mathrm{d}\omega/\mathrm{d}z$ and perform the integration with the use of relation (17). Then the second term in Eq. (20) vanishes, whereas the first term yields a simple final result:

$$\delta T_u = \sqrt{2\pi}\epsilon^2 \bar{D}_u P_v \tau_0/c^2. \quad (21)$$

The small frequency shift [relation (19)] makes an additional contribution, $(\delta T_u)_{\text{extra}}$, to the timing shift. Its relative size can be estimated as follows: $|(\delta T_u)_{\text{extra}}/\delta T_u| \sim \tau_0^{\,2} cT/D^2 L$, where T is the temporal separation between the pulses in the information-carrying array. Assuming a densely packed array with $T \sim 3\tau_0$, and taking the same physical parameters as used above, one can finally conclude that $|(\delta T_u)_{\text{extra}}/\delta T_u| \sim 0.05$. Thus the effect of the frequency shift is, strictly speaking, different from zero, but it is indeed weak compared with that of the direct position shift.

C. Incomplete Collision

Equation (10), which describes the evolution of the frequency shift in the course of the collision, played the central role in the above analysis. The result that the net frequency shift is zero in the first approximation is valid only for complete collisions, such that the two pulses are well separated both before and after the collision. Because the collision length is large and a real WDM system may involve dozens of channels, it is impossible to avoid incomplete collisions (i.e., the collisions that begin with the pulses overlapped) at the line's input. Such collisions can result in a significantly larger frequency shift. This problem can also be analyzed on the basis of straightforward integration of Eq. (10) (for sech solitons in a system without DM, the results of incomplete collisions were analyzed in Ref. 6).

The net frequency shift generated by the incomplete collision is

$$\delta\omega = \int_{z_0}^{+\infty} (\mathrm{d}\omega/\mathrm{d}z)\mathrm{d}z, \quad (22)$$

where z_0 is the point at which another soliton is launched into the adjacent channel with a temporal delay ΔT ($\equiv cz_0$). This delay determines the degree of the initial overlap between the solitons. Substituting Eq. (10) into (22), one can again separate the fast and slow variations, as was done in deriving relation (17). Then straightforward calculations yield the result:

$$\delta\omega = \sqrt{2}\epsilon P_v \tau_0^{\,2} c^{-1} \times \left\langle [\tau_0^{\,4} + 4\Delta^2(z)]^{-1/2} \exp\left[-\frac{\tau_0^{\,2}(\Delta T)^2}{\tau_0^{\,4} + 4\Delta^2(z)}\right]\right\rangle, \quad (23)$$

where, as above, $\langle\,\rangle$ stands for the averaging over the period of the rapidly varying function $\Delta(z)$. In a transmission line, the time delay ΔT is a random variable. Our aim is not to consider the statistics of the incomplete collisions; instead, we restrict our considerations to the largest frequency shift that can be generated by the incomplete collision, which occurs when $\Delta T = 0$. In this case,

$$(\delta\omega)_{\max} = \sqrt{2}\epsilon P_v \tau_0^{\,2} c^{-1}\langle[\tau_0^{\,4} + 4\Delta^2(z)]^{-1/2}\rangle. \quad (24)$$

For the two-step DM model [Eq. (3)] adopted in this study, the average value in Eq. (24) can be immediately calculated, yielding

$$(\delta\omega)_{\max} = \sqrt{2}\epsilon P_v (cS)^{-1}\ln(S + \sqrt{1+S^2}), \quad (25)$$

where S is the DM-strength parameter defined by Eq. (3). Once the frequency shift is found, the next natural step is to find its contribution to the net timing shift, using Eqs.

(6). For a sufficiently large propagation distance z, the corresponding contribution is

$$\delta T_u^{(\omega)} = -\delta\omega\epsilon\overline{D_u}z. \tag{26}$$

An issue of real interest is to compare the size of the position shift that is due to an initial incomplete WDM collision with the position shift [Eq. (21)] that comes from all the subsequent complete collisions. Note that the position shift [Eq. (26)] produced by an incomplete collision grows proportionally to z (because it is the result of a frequency shift). The net shift that is due to the complete collisions is proportional to the number of collisions, which also grows linearly with z. Thus we may compare these two shifts by calculating their ratio:

$$\left|\frac{\delta T_u^{(\omega)}}{\delta T_u}\right|_{\max} = \frac{1}{\sqrt{\pi}}\frac{cz_1}{\tau_0 S}\ln(S + \sqrt{1 + S^2}), \tag{27}$$

where z_1 is the average distance traveled between collisions. Here the largest possible value [Eq. (25)] of the frequency shift has been inserted. Proceeding to a real estimate, we take the same values as used above: $c \sim 0.1\,\mathrm{ps/km}$ and $\tau_0 \sim 30\,\mathrm{ps}$. Additionally, it is necessary to make an assumption about the frequency of the collisions. As it was already adopted above, we assume dense packing of the signals in the return-to-zero mode of the communication, with the temporal separation $3\tau_0$ between the centers of the adjacent pulses. This means that, in Eq. (27), one should take $z_1 = 3z_{\mathrm{coll}} \sim 1500\,\mathrm{km}$, where $z_{\mathrm{coll}} \sim 500\,\mathrm{km}$ is the proper collision distance estimated above. Thus we get

$$\left|\frac{\delta T_u^{(\omega)}}{\delta T_u}\right|_{\max} \approx \frac{\sqrt{2}}{S}\ln(S + \sqrt{1 + S^2}). \tag{28}$$

We make two points here. First, for all values of $S > 0$, the largest that this ratio can be is $|\delta T_u^{(\omega)}/\delta T_u|_{\max} = \sqrt{2}$. Hence the worst that an initial incomplete collision can generate is an extra temporal jitter, which is really not much stronger than that which is due to the sum of all multiple in-line complete collisions. Second, very strong DM ($S \gg 1$) can completely suppress this extra jitter.

3. NUMERICAL SIMULATIONS

To verify the above results we solved the system of Eqs. (1) and (2) numerically. We prepared the steadily propagating pulses in both channels by using the results of Ref. 11. In particular, we adopted the same normalizations as employed in Ref. 11, i.e.,

$$|D_{1,2}|L_{1,2} \equiv 1, \qquad L_1 + L_2 \equiv 1. \tag{29}$$

Note that, in this normalization, $S \equiv 1/\tau_0^2$.

We consider here only the case $\overline{D}_u = \overline{D}_v$; hence the peak powers of the colliding pulses are also equal. As an example, we take the normalized values $\epsilon = 0.1$, $L_1 = 0.4$, $L_2 = 0.6$, $D_1 = 5/2$, $D_2 = -5/3$, $P_u = P_v = 1$ (these are the same values as in Fig. 1 of Ref. 11), and $c = 0.3$. The value of $\overline{D}_u$ is always taken from Eq. (8) for various values of the DM strength S, which was used as a control parameter.

First we studied the complete collisions of the Gaussian solitons. For this purpose, the initial u and v pulses were placed far from each other. The simulations were run until the collision was completed and the pulses were again separated. In all these simulations with various values of DM strength S, the collision-induced frequency shift was found to be extremely small ($\lesssim 10^{-5}$), consistent with the above analytical results.

For a typical case of moderately strong DM, $S = 1$ (this case, which corresponds to the DM strength 1.44 in the notation of Ref. 9, is of interest as it also allows one to suppress the intrachannel interactions between the solitons[18]), we show in Fig. 1(a) $|v|$ versus z at $\tau = 0$ and in Fig. 1(b) the profiles of the v pulse before and after collision. If there had been no collision, the center of the v pulse would have stayed at $\tau = 0$. However, we see a position shift in the v pulse of $\delta T \approx 0.080$ after its collision with the u pulse, which took place at $z \approx 65$. The corresponding theoretical prediction for the same shift, given by Eq. (21), is 0.1049, which is in reasonable agreement with the numerical value.

In Fig. 2 we compare the analytical predictions and numerical values of the position shift for various DM strengths S. We see that the position shift decreases as the DM strength increases. For narrow pulses (S large), the agreement between numerical and analytical values is worse, whereas as the pulse gets wider, this agreement

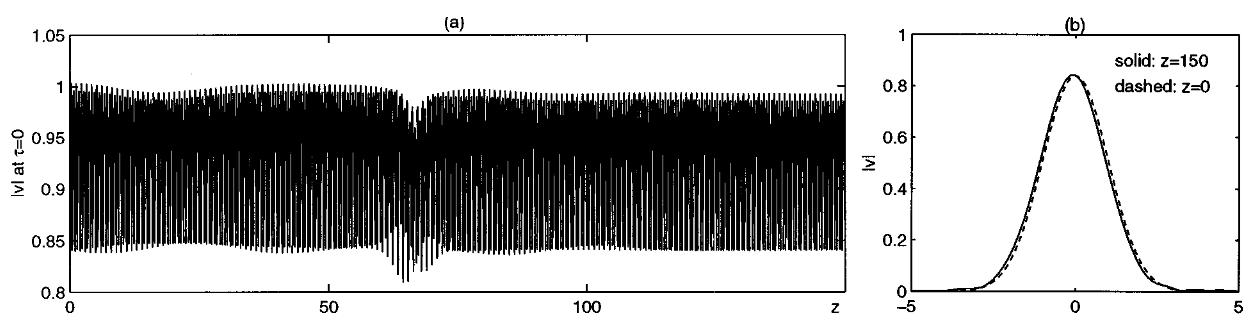

Fig. 1. Complete collision of two Gaussian pulses for $S = 1$. The other parameters are specified in the text. The normalizations are the same as in Ref. 11; in particular, $\tau_0 = 1$ corresponds, in the typical case, to the physical pulse's width $\tau_0 \sim 30\,\mathrm{ps}$. The physical values of the position shift can be rescaled accordingly. (a) Value of $|v|$ versus z at $\tau = 0$ (note that the collision takes place at $z \approx 65$). (b) Shapes of the $|v|$ pulse at $z = 0$ and $z = 150$. Note that, after the collision, the v pulse is slightly shifted.

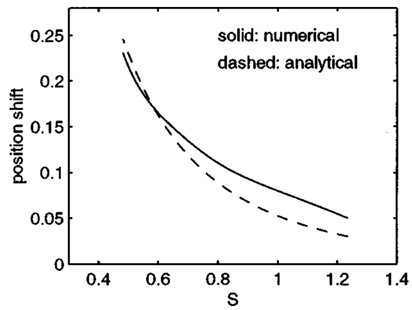

Fig. 2. Comparison of analytically predicted and numerically found position shifts of the Gaussian pulse induced by the complete collisions. All the parameters but the dispersion management strength S are fixed (see values in text).

becomes better. In any case, the analytical value does give a fairly acceptable estimate of the position shift.

Next we studied the incomplete collisions. For simplicity, we took the two initial Gaussian pulses to be fully overlapped ($z_0 = 0$) and ran the simulations until they separated far apart. A typical case with $S = 1$ is plotted in Fig. 3. Figure 3(a) shows $v(\tau = 0)$ versus z for this incomplete collision, and because the center of the pulse starts moving away from the point $\tau = 0$, we then see $v(\tau = 0)$ decreasing. By the end of the collision, because $v(\tau = 0)$ continues to decrease, we know that a finite frequency (velocity) shift has been created. Accordingly, the total position shift of the v pulse [see Fig. 3(b)] turns out to be much larger than in the case of the complete collision [cf. Fig. 1(b)]. Figure 4 compares the analytically predicted and the numerically observed frequency shifts for various values of S. Here again, satisfactory agreement between them is observed. Note that as the DM strength S increases, the frequency shift does considerably decrease, which demonstrates one of the advantages of strong DM.

It is interesting to note that, although the agreement between analytical and numerical results for the complete collisions is better for smaller S, for incomplete collisions the trend is the opposite. But also one should bear in mind that, for practical applications, only an estimate of the magnitude of the pulse's frequency and position shifts is really necessary to guarantee the error-free transmission of information. Thus the analytical results of Eqs. (21) and (25) are therefore useful beyond the limits of their formal validity. Both the analytical and the numerical dependences could easily be extended to larger values of S. However, when S becomes larger the position shift becomes so small that it becomes numerically more difficult to determine them accurately (see Fig. 2). Nonetheless, essential trends are clearly seen in Figs. 2 and 4.

The collision-induced timing shift, which we considered above, imposes a limitation on the wavelength separation $\delta\lambda$ between the channels: small $\delta\lambda$ gives rise to a small c and, hence, to large collision-induced frequency and position shifts. Using the same values of the physical parameters as before, choosing the moderate DM strength of $S \sim 1$, and assuming the same dense packing of the pulse stream $T = 3\tau_0$, as above, we conclude that the shifts produced by multiple interchannel complete and incomplete collisions will not corrupt the information content of the signal for transmission distances of $\lesssim$15,000 km. Using Eqs. (21) and (27), one can easily determine how this limit distance scales with the change of the parameters (in particular, it increases to ~50,000 km for larger S, where the effects of the incomplete collisions becomes less important).

These estimates concentrate on the role of the interchannel collisions and do not take into account the usual intrachannel Gordon–Haus (GH) jitter.[17] The GH jitter is determined by factors (such as the amplifiers' excess gain) other that those that control the interchannel collisions. For instance, there is no GH jitter in the lossless system, which still has the collisional jitter. Nevertheless, the GH jitter may be more dangerous, as its rms value grows to $\sim z^{3/2}$, whereas the interchannel collisions give rise to the linear growth law (see above). A well-known method for the suppression of the GH jitter is the use of optical filters, which (in the case of the fixed-frequency filters) lower this rms growth law to $\sqrt{z}$.[16] On the other hand, it has been demonstrated in many publications, starting from Ref. 23, that DM can also help to suppress the GH jitter. The best result may be provided by a proper combination of DM and filters.[24] In the WDM system the additional suppression of the jitter caused by the interchannel collisions can be provided by the so-called channel-isolating filters (which separate the channels from one another by means of a system of notch filters; see, e.g., Ref. 25). The effect of isolating filters on solitons, both by themselves and in a combination with DM, is discussed in Ref. 26.

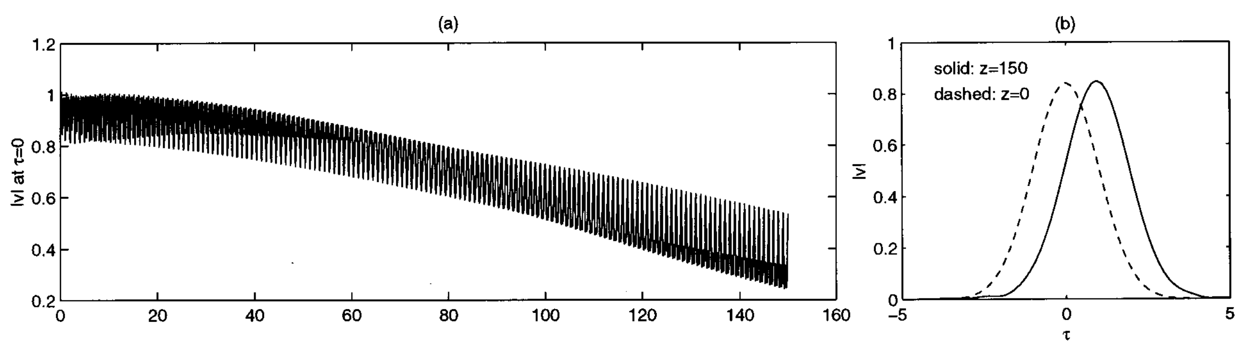

Fig. 3. Incomplete collision of two Gaussian pulses for $S = 1$. Initially the pulses fully overlap. The other parameters are specified in the text. (a) Value of $|v|$ versus z at $\tau = 0$. It can be seen that the v pulse experiences a velocity (frequency) shift after the incomplete collision. (b) Shapes of the $|v|$ pulse at $z = 0$ and $z = 150$. Note that, because of a nonzero frequency shift, the total position shift of the v pulse is much larger than in the case of the complete collision (see Fig. 1).

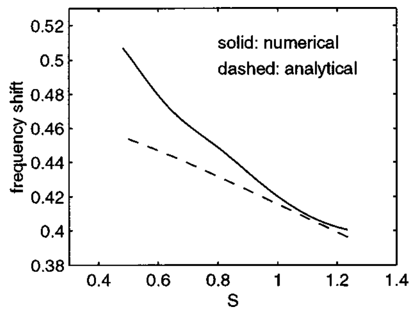

Fig. 4. Comparison of analytically predicted and numerically obtained frequency shifts generated by the collision of two initially fully overlapped Gaussian pulses in adjacent channels. All the parameters but S are fixed (see values in text).

The model considered here does not include amplifiers and losses. The justification for this limitation is as usual: Promising values of the DM period are 150–200 km,[22] which are much larger than the normal amplification spacing, $z_a \sim 30\,\mathrm{km}$. Hence the losses and amplification may be assumed to be locally compensated. The collision-induced timing jitter in a WDM–DM system with losses and amplifiers was studied numerically in Ref. 27, where it was shown (for weak DM) that DM performed better than the dispersion-decreasing and uniform-dispersion fibers. Other direct simulations of the WDM–DM models with losses and lumped amplifiers[14] demonstrate that, in the case of a relatively small collision distance (large wavelength separation $\delta\lambda$), the losses and amplifiers can render the results somewhat worse (in Ref. 14, $L = z_a$).

4. CONCLUSIONS

We have considered effects of interchannel collisions on Gaussian pulses in a WDM system with moderate and strong DM. We have shown that, for complete collisions, the collision-induced frequency shift of the Gaussian pulse is negligible, whereas the position shift is important and can be estimated by the fully analytical form of Eq. (21). For incomplete collisions the frequency shift is significant, and its maximum value is given by Eq. (25). The analytical predictions were checked against direct partial differential equation simulations and showed acceptable agreement. For typical values of the system's parameters and moderately strong DM ($S \gtrsim 1$) we conclude that the limitation imposed by the interchannel collisions on the transmission of information by a densely packed pulse stream exceeds the transoceanic distances.

Formulas (21) and (25) provide much insight and information on the design requirements for transmission lines with strong DM. They show that the timing shifts do not depend on the relative lengths of the two opposite-dispersion fiber segments, provided that the DM period L is much larger than the amplification spacing z_a (however, the situation may be different if $L = z_a$; see, e.g., Ref. 14). Thus, to take advantage of the existing communication network with the fibers that have negative dispersion at the carrier wavelength 1.54 μm, one could use a span of this fiber a few amplification spacings long, adding to this a short segment of a fiber (or a compact chirped-grating dispersion compensator) with a large positive dispersion. As long as the DM strength and the path-average dispersion are kept constant, other details of the fiber arrangements turn out to be of minor importance for the system's performance.

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D. J. Kaup's e-mail address is kaup@clarkson.edu.

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