

Stability of vortex solitons in a photorefractive optical lattice

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Abstract. Stability of on- and off-site vortex solitons with unit charge in a photorefractive optical lattice is analysed. It is shown that both types of vortex solitons are linearly unstable in the high- and low-intensity regimes. In the high-intensity regime, the vortex appears as a familiar ring vortex, and it suffers oscillatory instabilities. In the low-intensity regime, the vortex suffers both oscillatory and Vakhitov–Kolokolov instabilities. However, in the intermediate-intensity regime, both types of vortices could be stable if the lattice intensity is moderate, and the applied DC field is above a certain threshold value. Under the same physical conditions, on-site vortices are more stable than off-site ones. Increasing the DC field stabilizes both types of vortex solitons.

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1. Introduction

Vortex solitons are ubiquitous in many branches of physics such as optics [1] and Bose–Einstein condensates [2]–[4]. In a homogeneous medium, bright vortex rings are unstable [5], and only dark vortex solitons are possible with defocusing non-linearity [1, 6]. However, in the presence of a periodic optical lattice, stable lattice vortices become possible due to the guiding properties of the lattice. Indeed, recent theoretical work [7, 8] has shown that in an optical lattice with Kerr non-linearity, both on-site vortices (vortices whose singularity is located on a lattice site) [7] and off-site vortices (vortices whose singularity is located between sites) [8] are stable within certain ranges of parameters. These theoretical studies are quickly followed by experiments in photorefractive crystals, where vortex lattice solitons have been observed very recently [9, 10]. See [11, 12] for a review of other nonlinear localized states in one- and two-dimensional (2D) periodic optical waveguides.

Stability of vortex lattice solitons in photorefractive crystals is clearly an important issue. This question was considered in [10], where the evolution of a particular on-site lattice vortex under random-noise perturbations was simulated. It was found that the on-site vortex was stable to very long distances. However, we know that lattice vortices in photorefractive crystals cannot all be stable. For instance, when the peak intensity (or power) of the vortex is high, the lattice is effectively weak, thus the lattice vortex would become the familiar ring vortex, which is known to be unstable (see figure 1(b)) [5]. The natural questions to ask then are: what lattice vortices are stable? If lattice vortices are unstable, what are the sources of their instability? So far, these questions have not been addressed comprehensively for either of the on-site and off-site lattice vortices.

In this paper, we numerically study both the on-site and off-site vortex solitons with unit charge and determine their stability properties in a 2D photorefractive optical lattice. We show that these vortices are not only unstable in the high-intensity regime, but also in the low-intensity regime. However, they can become stable in the intermediate-intensity regime if the lattice intensity is moderate and, in addition, the applied DC bias field is above a certain threshold value. Under the same physical conditions, on-site vortices are more stable than off-site ones, mainly because on-site vortices have larger hump separations than off-site ones. Increasing the DC bias field strongly stabilizes both types of vortices.

The mathematical model for light propagation in a photorefractive crystal has been known for some time [13]. Here we make the usual paraxial assumption, and the assumption that the photorefractive screening non-linearity acts isotropically along the two transverse directions, both of which are justified in many experiments. If the probe beam is extraordinarily polarized, while the lattice is ordinarily polarized, then the probe beam does not affect the linear lattice. In this case, the governing equation for the probe beam is [13]

$$iU_z + \frac{1}{2k_1}(U_{xx} + U_{yy}) - \frac{1}{2}k_0 n_e^3 r_{33} E_{sc} U = 0, \quad (1)$$

where U is the slowly varying amplitude of the probe beam, z the distance along the direction of the crystal, (x, y) are distances along the transverse directions, $k_0 = 2\pi/\lambda_0$ the wavenumber of the laser in the vacuum (λ_0 is the wavelength), n_e the refractive index along the extraordinary axis, $k_1 = k_0 n_e$, r_{33} the electro-optic coefficient for the extraordinary polarization, E_{sc} is the

space-charge field,

$$E_{sc} = \frac{E_0}{1 + I_l(x, y) + |U|^2}, \quad (2)$$

E_0 the applied DC field, and I_l the field intensity of the optical lattice. Here the intensities of the probe beam and the lattice have been normalized with respect to the dark irradiance of the crystal I_d . The dark irradiance is the background illumination used in experiments to fine-tune the non-linearity. Material damping of the probe beam is very weak in typical experiments since the crystals are fairly short (up to 2 cm), hence neglected in equation (1). If the lattice is periodic along the x and y directions (rectangular lattice), then I_l can be expressed as

$$I_l(x, y) = I_0 \sin^2 \frac{\pi}{D} x \sin^2 \frac{\pi}{D} y, \quad (3)$$

where I_0 is its peak intensity and D its spacing.

Equation (1) can be non-dimensionalized. If we measure the transverse directions (x, y) in units of D/π , the z direction in units of $2k_1 D^2/\pi^2$ and the applied bias field E_0 in units of $\pi^2/(k_0^2 n_e^4 D^2 r_{33})$, then equation (1) becomes

$$iU_z + U_{xx} + U_{yy} - \frac{E_0}{1 + I_0 \sin^2 x \sin^2 y + |U|^2} U = 0. \quad (4)$$

Consistent with the experiments [12], we choose physical parameters as $D = 20 \mu\text{m}$, $\lambda_0 = 0.5 \mu\text{m}$, $n_e = 2.3$, $r_{33} = 280 \text{ pm V}^{-1}$. Thus, in this paper, one x or y unit corresponds to $6.4 \mu\text{m}$, one z unit corresponds to 2.3 mm and one E_0 unit corresponds to 20 V mm^{-1} in physical units.

Lattice vortices of equation (4) are sought in the form $U = u(x, y)e^{-i\mu z}$, where μ is the propagation constant. The function $u(x, y)$ satisfies the non-linear equation

$$u_{xx} + u_{yy} + \left(\mu - \frac{E_0}{1 + I_0 \sin^2 x \sin^2 y + |u|^2} \right) u = 0. \quad (5)$$

We determined these vortices by a Fourier iteration method. The idea of this method was proposed in [14]. A modification of this method has been used to obtain fundamental and vortex solitons in a two-dimensional photonic lattice with Kerr non-linearity [8, 15]. Since this method has been described in detail in [15], it will not be repeated here.

With this iteration method, we have found both on- and off-site vortex solitons with unit charge (see also [7]–[10]). These solitons reside inside the first semi-infinite bandgap $-\infty < \mu < \mu_1$, where μ_1 is the edge of the bandgap. On-site vortices have their centres (singularities) on a lattice site, whereas off-site vortices have their centres between lattice sites. These two types of vortices and their stability properties will be studied separately below.

We would like to make a remark here. In our numerics, the control parameter is the propagation constant μ , and for each μ (under certain parameter regions), we could find a vortex soliton. However, from a physical point of view, this propagation constant is not as meaningful as the peak intensity I_p of the vortex soliton, which can be measured experimentally. Thus when we present our results, we would use the peak intensity I_p rather than the propagation constant μ .

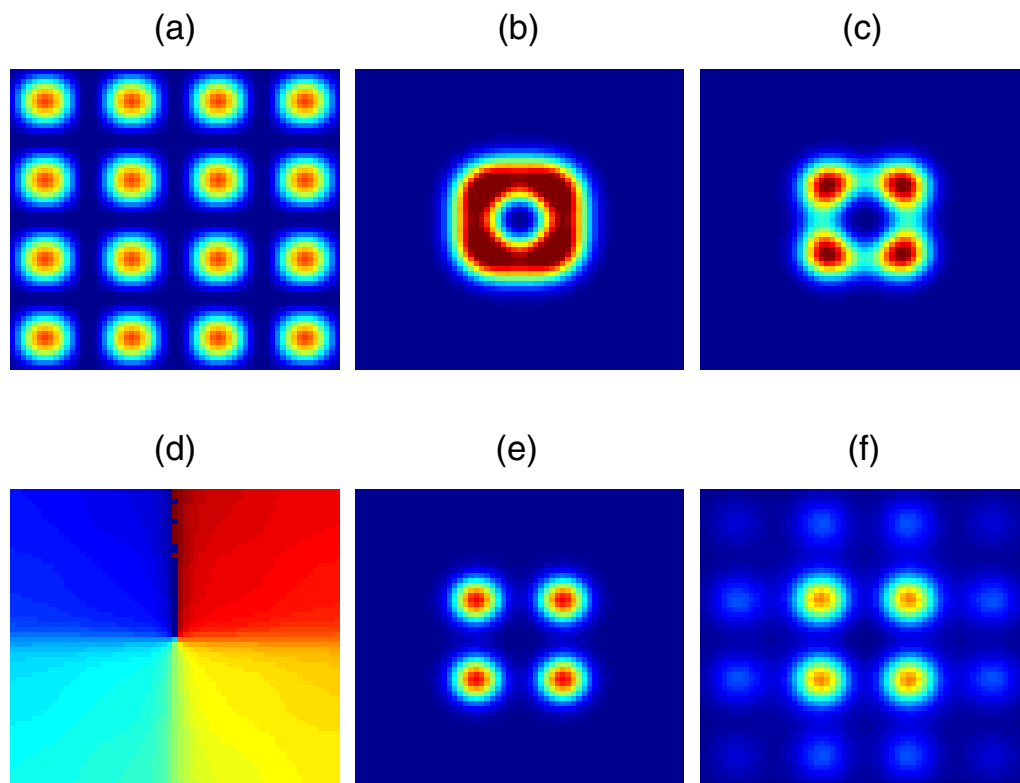


Figure 1. (a) Intensity field of the optical lattice with $I_0 = 2I_d$; (b, c, e, f) intensity fields of off-site lattice vortices with peak intensities $12, 6, 2$ and $0.5I_d$, respectively, under the applied bias field $E_0 = 7.5$; (d) phase structure of these vortices.

2. Off-site vortex solitons

First, we consider off-site vortex solitons with unit charge. These vortices are illustrated in figure 1 at $I_0 = 2I_d$ and $E_0 = 7.5$. We see that when the vortex's peak intensity I_p is high, the vortex becomes a familiar ring vortex (see figure 1(b)) since the optical lattice is relatively negligible in this case. As I_p decreases, the vortex develops four major lobes at four adjacent lattice sites in a square configuration (see figures 1(c) and (e)), and the vortex centre is between lattice sites. When I_p is low, the vortex spreads over to more lattice sites and becomes less localized (see figure 1(f)). The phase fields of all these lattice vortices, however, remain qualitatively the same as in a regular ring vortex (see figure 1(d)). An interesting fact we found is that, for given lattice intensity and applied bias field values, lattice vortices with I_p below a certain threshold $I_{p,c}$ do not exist. In the present case where $I_0 = 2I_d$ and $E_0 = 7.5$, this threshold value is $I_{p,c} \approx 0.48I_d$. In terms of the propagation constant μ , we found that lattice vortices disappear when $\mu > \mu_c \approx 4.88$. This fact indicates that, unlike fundamental lattice solitons, this family of lattice vortices do not bifurcate from infinitesimal Bloch waves at the edge of the bandgap.

As we have seen in figure 1, in the high-intensity regime, vortices in photorefractive optical lattices approach the lattice-free ring vortices. This is different from vortices in a Kerr medium, where they approach four singular spikes in the high-intensity limit [8]. The reason is that, in a Kerr medium, fundamental solitons become narrower and narrower when their peak intensities

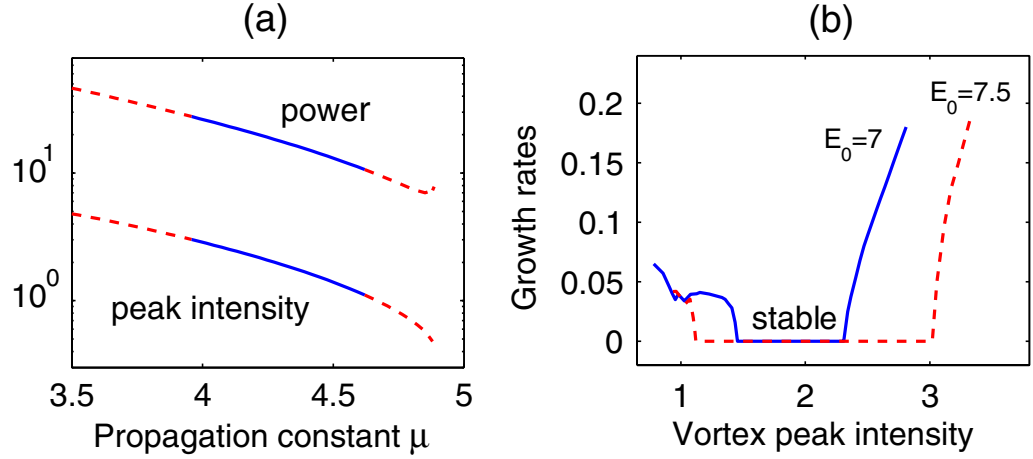


Figure 2. (a) Power and peak-intensity diagrams of off-site lattice vortices at $I_0 = 2I_d$ and $E_0 = 7.5$; —, stable vortices; - - -, unstable vortices. (b) Growth rates of off-site vortices versus their peak intensity at two applied bias fields $E_0 = 7$ and 7.5 .

get higher and higher. But in a photorefractive crystal where the non-linearity is saturable, fundamental solitons flatten out when their intensities become high (see figure 1(a) of [16]). Thus, in a photorefractive lattice, the four lobes of the vortex join together and form a ring vortex at high intensities, whereas in a Kerr lattice, the four lobes develop into four singular spikes at high intensities.

We can further determine the power and peak-intensity diagrams of these vortices versus the propagation constant μ . Here the power is defined as $P \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |u|^2 dx dy$. When $I_0 = 2I_d$ and $E_0 = 7.5$, the results are shown in figure 2(a). We see that the peak intensity is a monotone-decreasing function of μ , but the power is monotone-decreasing only when $\mu < 4.85$, or the peak intensity $I_p > 0.58I_d$. If $\mu > 4.85$, i.e. $I_p < 0.58I_d$, the power starts to increase with μ . This behaviour qualitatively holds also at other I_0 and E_0 values. A similar finding in the Kerr medium has been reported in [8].

Now we address the critical question of linear stability of these vortices in a photorefractive lattice. High-intensity lattice vortices clearly should be linearly unstable because they approach the regular ring vortex (see figure 1(b)) [5]. The instability is oscillatory (i.e. the unstable eigenvalues are complex). At low intensities, $dP/d\mu > 0$; hence the lattice vortices are expected to be linearly unstable as well according to the Vakhitov–Kolokolov (VK) criterion [17]. The VK instability is purely exponential (i.e. the unstable eigenvalues are purely real). How about the stability behaviours of vortices at intermediate peak intensities? To answer this question, we perturb these vortices as $U = \{u(x, y) + \tilde{U}(x, y, z)\}e^{-\mu z}$, where $\tilde{U}(x, y, z)$ is an infinitesimal perturbation. When this perturbed solution is substituted into equation (1), the linearized equation for perturbation $\tilde{U}(x, y, z)$ is

$$i\tilde{U}_z + \tilde{U}_{xx} + \tilde{U}_{yy} + \mu\tilde{U} - \frac{E_0}{(1 + |u|^2 + I_0 \sin^2 x \sin^2 y)^2} \{(1 + I_0 \sin^2 x \sin^2 y)\tilde{U} - u^2\tilde{U}^*\} = 0, \quad (6)$$

where the superscript * represents complex conjugation. Starting from random-noise initial conditions, we simulate this linearized equation for very long distances (hundreds of z units)

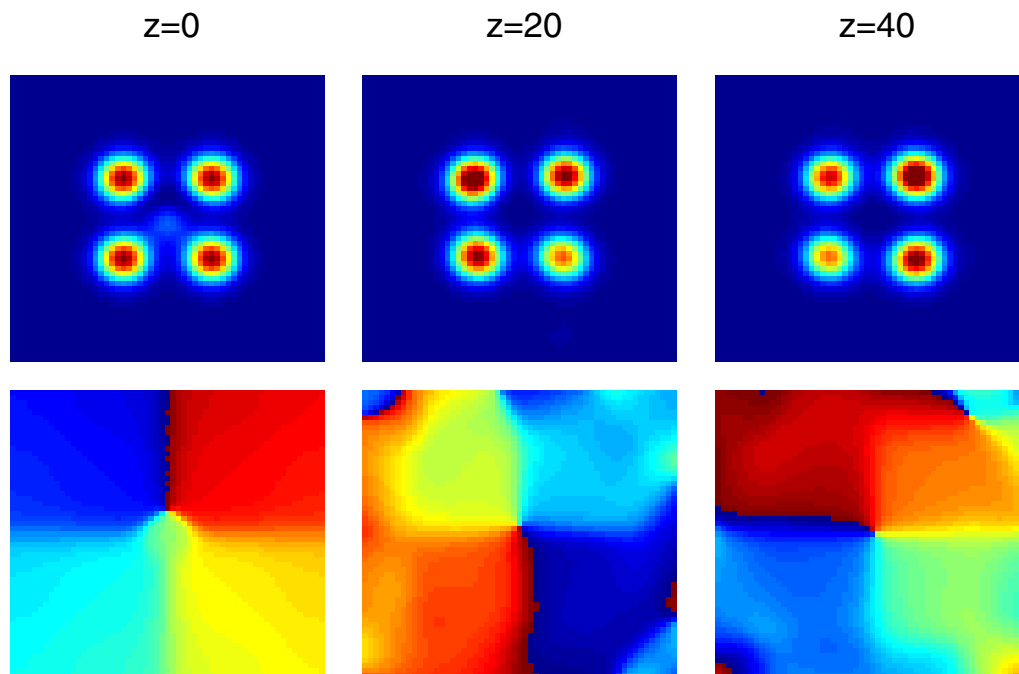


Figure 3. Stable propagation of an intermediate-intensity off-site vortex soliton with $I_0 = 2I_d$, $E_0 = 7.5$ and $I_p = 2I_d$ under random-noise perturbations. Top row: intensity; bottom row: phase.

by the pseudo-spectral method (fast Fourier transform to get \tilde{U}_{xx} and \tilde{U}_{yy} , and the fourth-order Runge–Kutta method to advance in z). If the solution grows exponentially, then the underlying vortex soliton is linearly unstable, and the growth rate can be calculated from the growing solution (the growth rate is the real part of the most unstable eigenvalue). Otherwise, the vortex is linearly stable. It is noted that this procedure can track down the leading instabilities of vortices if the vortices are linearly unstable.

Following the above numerical procedure, we have systematically determined the linear stability properties of off-site vortex solitons. At $I_0 = 2I_d$ and $E_0 = 7.5$, the growth-rate diagram is shown in figure 2(b). We see that lattice vortices are linearly unstable when $I_p > 3.01I_d$ and $I_p < 1.13I_d$, consistent with our expectations. In addition, the leading instabilities are all oscillatory, except at the low-intensity end where the VK instability can be stronger. In contrast with vortex lattice solitons in a Kerr medium [8], the vortices here are linearly unstable not only in the low-intensity regime, but also in the high-intensity regime.

A more important finding in figure 2(b) is that at $I_0 = 2I_d$ and $E_0 = 7.5$, vortex solitons with intermediate intensities $1.13I_d < I_p < 3.01I_d$ are linearly stable (the growth rates are zero for such vortices). This is an important result which is physically significant. Then a closely related question follows: are such vortices also non-linearly stable? To answer this question, we take a linearly stable off-site vortex soliton with peak intensity $I_p = 2I_d$, and perturb it by random noise. The noise has Gaussian intensity distribution in the spectral k -space with the full-width at half-maximum (FWHM) twice as large as the soliton FWHM spectrum. The noise power is 3% of the soliton's. Our simulation scheme is again the pseudo-spectral method as described before. The simulation result on the evolution of this perturbed vortex is shown in figure 3. We see that this vortex does propagate stably even after 40 units of distance z (corresponding to over 80 mm

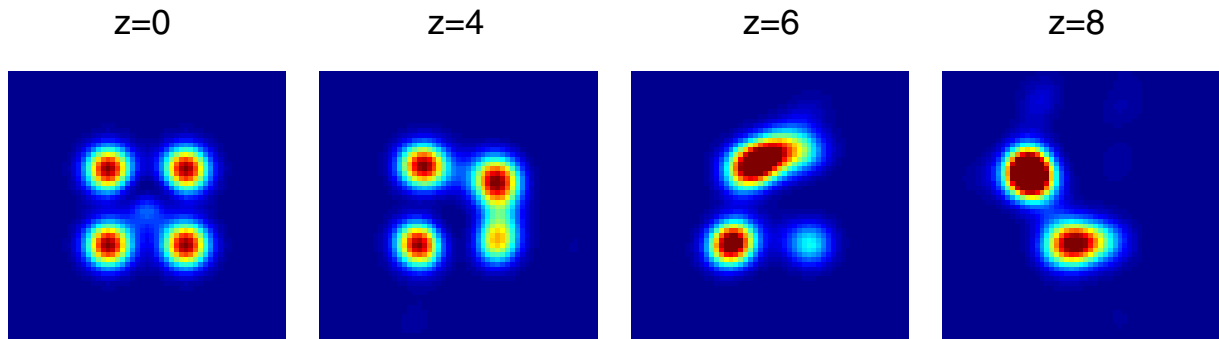


Figure 4. Fast break-up of a high-intensity off-site vortex soliton with $I_0 = 2I_d$, $E_0 = 7.5$ and $I_p = 3.5I_d$ under random-noise perturbations. The intensity fields at various distances are shown.

in physical units). In addition, its phase structure is maintained throughout the evolution. Our simulations were also performed for even longer distances, and the results remained the same. For other linearly stable lattices, we have repeated these simulations and found qualitatively similar results. These findings indicate that linearly stable vortex solitons are also non-linearly stable and they should be observable in experiments, as has been shown by Neshev *et al* [9] and Fleischer *et al* [10].

When lattice vortices are linearly unstable, how does this instability develop? What final state does it lead to? To address these issues, we first take a linearly unstable vortex soliton with a high peak intensity $I_p = 3.5I_d$ ($I_0 = 2I_d$, $E_0 = 7.5$), and perturb it by the random noise as described above. The simulation result is shown in figure 4. We see that this vortex quickly breaks up. On the other hand, if we perturb a linearly unstable vortex soliton with a low peak intensity $I_p = I_d$, the simulation result, plotted in figure 5, shows that this vortex takes a much longer distance to disintegrate. The phase diagram in figure 5 (bottom row) reveals that after the vortex breaks up, the right two spots become an out-of-phase dipole soliton with π phase difference, whereas the left two spots dim out or disappear altogether. These evolution results are consistent with the linear stability results in figure 2(b). In that figure, we see that the growth rates are quite large for unstable vortices with high peak intensities, whereas the growth rates are quite small for unstable vortices with low peak intensities. Thus unstable vortices with high intensities should break up much faster than vortices with low intensities, as seen in figures 4 and 5. Notice that since the growth rates of low-intensity vortices are quite small, they may be observed in experiments despite their instability. Indeed, the crystals used in experiments are typically short (under 20 mm), which may not be enough for weak instabilities to fully develop.

How does the applied DC field (voltage) affect the stability properties of vortex solitons? For this purpose, we lower the E_0 value from 7.5 to 7 (while keeping $I_0 = 2I_d$), and obtained the growth-rate diagram of vortices, which is also plotted in figure 2(b). At this lower E_0 value, we see that the stability region of vortex solitons shrinks by over a half. In other words, increasing the applied DC field significantly stabilizes vortex solitons. Physically, this can be understood as follows. The lattice-induced waveguide potential in equation (4) is $E_0/(1 + I_0 \sin^2 x \sin^2 y)$. At a fixed lattice intensity I_0 , if the applied DC field E_0 increases, this potential depth increases. Thus vortex solitons are more stabilized. A similar finding has been reported before for the Kerr medium [8].

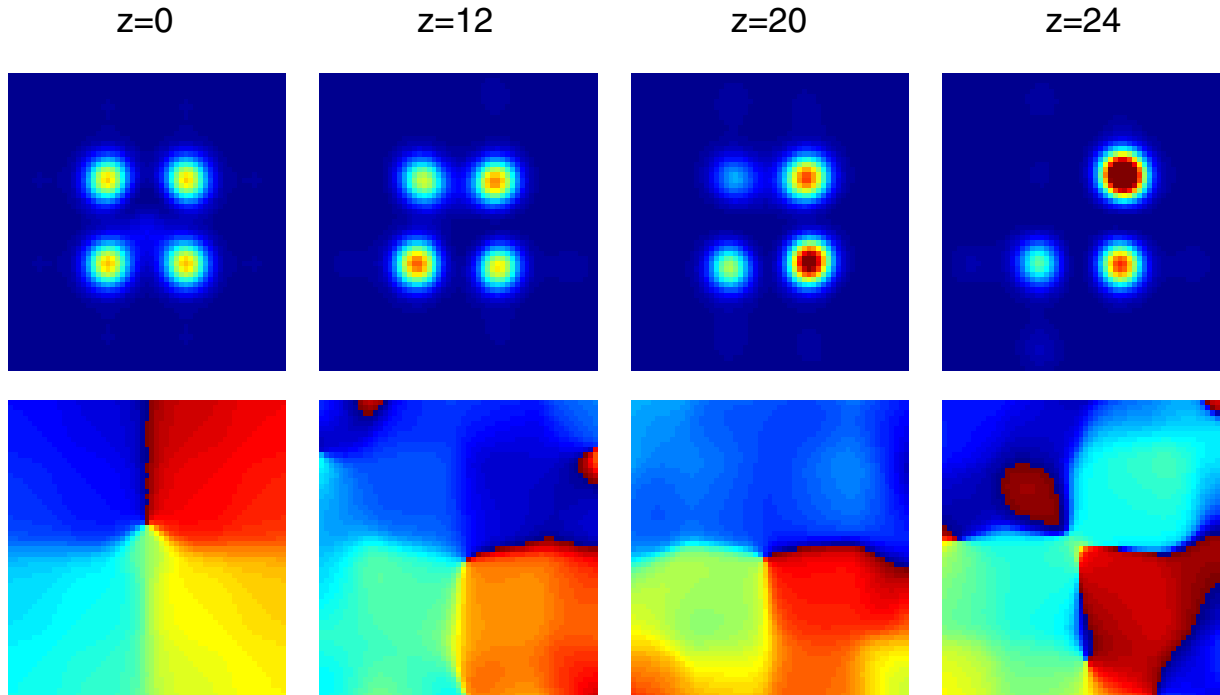


Figure 5. Slow break-up of a low-intensity off-site vortex soliton with $I_0 = 2I_d$, $E_0 = 7.5$ and $I_p = I_d$ under random-noise perturbations. Top row: intensity; bottom row: phase.

The effect of the lattice intensity on the stability of vortex solitons is a little more subtle. If the lattice intensity is very low, then the lattice can be considered absent. Obviously, the vortex solitons then become the familiar ring vortices which are known to be always unstable [5]. A less obvious fact is that if the lattice intensity is too high, vortices are always unstable as well. To demonstrate this phenomenon, we have systematically determined the linear stabilities of lattice vortices at various lattice intensity and vortex peak-intensity values at a fixed bias field $E_0 = 7.5$. The region of stable vortex solitons is presented in figure 6. We see that if the lattice intensity $I_0 < 0.9I_d$ or $I_0 > 4.1I_d$, vortex solitons are all linearly unstable no matter what their peak intensities are. Stable vortices (with intermediate peak intensities) appear only in the intermediate lattice-intensity region $0.9I_d < I_0 < 4.1I_d$. In experiments on vortex solitons [9, 10], the lattice was always kept at an intermediate-intensity level ($I_0 \approx 3I_d$), which is consistent with our finding above. For fundamental lattice solitons, we have checked that stable solutions exist at all lattice-intensity levels. Thus, the effects of the lattice intensity on fundamental lattice solitons and on vortex solitons are different.

When the applied bias field E_0 decreases, the stability region in figure 6 should shrink (see figure 2(b)). This is true indeed. At the lower value $E_0 = 7$, the stability region is also plotted in figure 6, which is much smaller than that at $E_0 = 7.5$. The stability region would shrink even further as E_0 continues to decrease. At a critical bias field $E_{0c} \approx 6.8$, the stability region disappears. In other words, when $E_0 < E_{0c}$, all vortex solitons are unstable no matter what the lattice- and vortex-intensity values are. For experiments, this implies that to observe vortex solitons, the applied DC field should be kept above a certain threshold value.

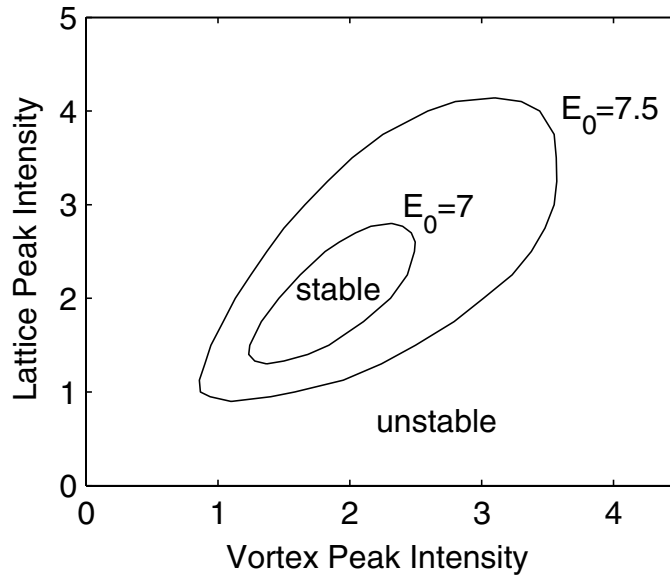


Figure 6. Stability boundaries of off-site vortex solitons in the (I_p, I_0) plane at two applied bias fields $E_0 = 7$ and 7.5 . Vortices inside the boundary are linearly stable.

3. On-site vortex solitons

In this section, we study on-site vortex solitons. These vortices are illustrated in figure 7 at $I_0 = 1.5I_d$ and $E_0 = 5.5$. Their power and intensity diagrams are plotted in figure 8(a). Generally speaking, behaviours of these vortices are qualitatively similar to off-site vortices (see figures 1 and 2(a)). The main differences are (i) the centres of these on-site vortices are at a lattice site; and (ii) the four lobes of these vortices have larger separations than those of off-site vortices (their separations are $\sqrt{2}$ of those in off-site vortices). Since these on-site vortices have larger lobe separations, they are expected to be more stable than off-site lattices. This is indeed the case (see below).

To determine the stability properties of on-site vortex solitons, we have repeated the above numerical procedure, with only off-site vortices replaced by on-site vortices. The growth-rate diagrams at $I_0 = 1.5I_d$ and two applied bias fields $E_0 = 5.3$ and 5.5 are plotted in figure 8(b). This figure is similar to figure 2(b). In other words, high- and low-intensity on-site vortices are also linearly unstable, whereas intermediate-intensity on-site vortices can be linearly stable. However, we should notice that even at this lower bias field $E_0 = 5.5$, the stability region of on-site vortices is already wider than that of off-site vortices at $E_0 = 7.5$. We know from figures 2(b), 6 and 7(b) that higher bias field stabilizes vortex solitons. Thus, under the same physical conditions (i.e. voltage and the lattice), on-site vortices are much more stable than off-site ones. This fact is further compounded in figure 9, where we have shown the stability boundaries of on-site vortices in the (I_p, I_0) space at two applied bias fields $E_0 = 5.3$ and 5.5 . Clearly, the stability domain of on-site vortices at the lower bias field $E_0 = 5.5$ is already much bigger than that of off-site vortices at the higher bias field $E_0 = 7$. Similar to off-site vortices, there is also a critical applied bias field value $E_{0c} \approx 5.2$, below which on-site vortices are all linearly unstable irrespective of what the lattice and vortex intensities are. Again, this critical value is lower

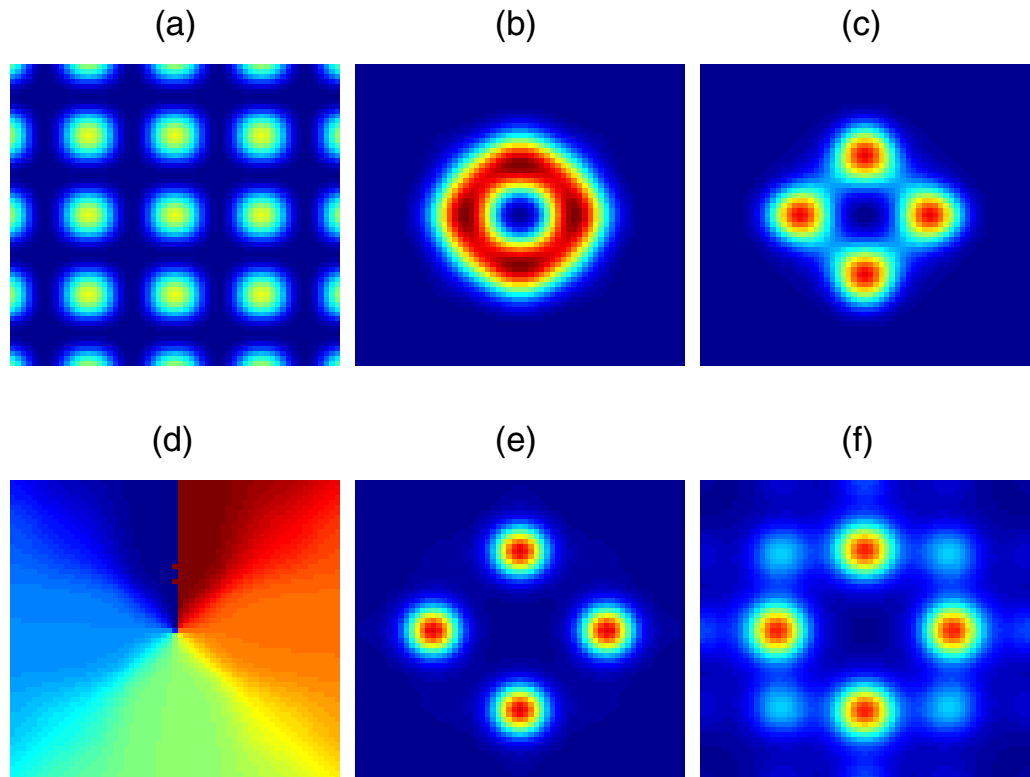


Figure 7. (a) Intensity field of the optical lattice with $I_0 = 1.5I_d$; (b, c, e, f) intensity fields of on-site lattice vortices with peak intensities 10, 5.5, 2 and $0.55I_d$, respectively, under the applied bias field $E_0 = 5.5$; (d) phase structure of these vortices.

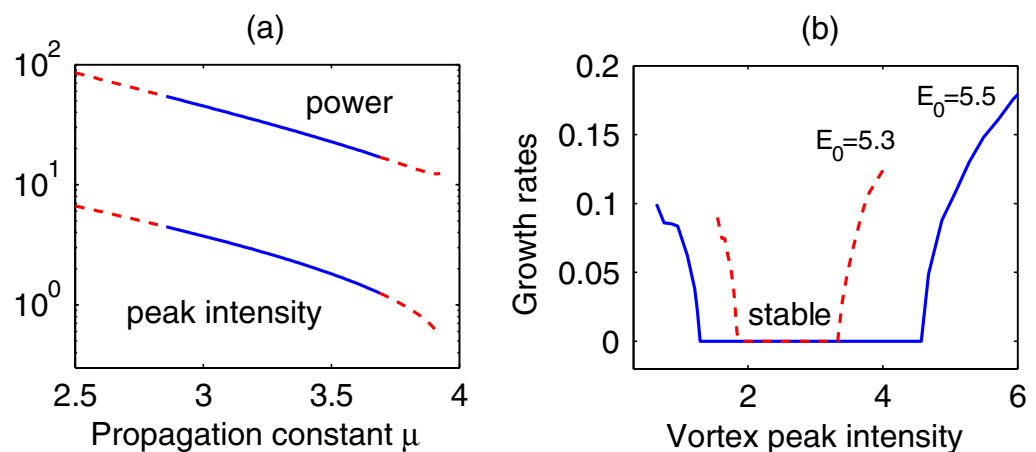


Figure 8. (a) Power and peak-intensity diagrams of on-site lattice vortices at $I_0 = 1.5I_d$ and $E_0 = 5.5$; —, stable vortices; - - -, unstable vortices. (b) Growth rates of on-site vortices versus their peak intensity at two applied bias fields $E_0 = 5.3$ and 5.5 .

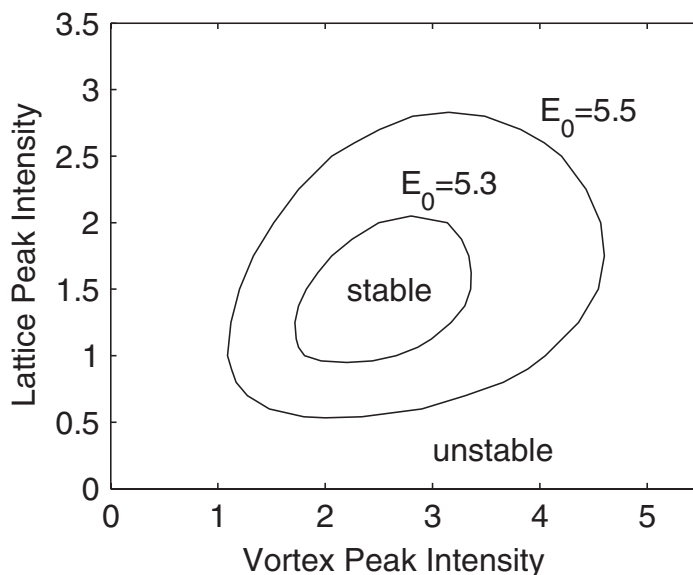


Figure 9. Stability boundaries of on-site lattice vortex solitons in the (I_p, I_0) plane at two applied bias fields $E_0 = 5.3$ and 5.5 . Vortices inside the boundary are linearly stable.

than that for off-site vortices, another sign that on-site vortices are generally more stable. Note that when $E_0 > E_{0c}$, even though stable on-site vortices exist, they have to be in the intermediate-intensity regime. In addition, the lattice intensity has to be in the intermediate value range too, similar to off-site vortices. Non-linear evolution of stable and unstable on-site vortex solitons under random-noise perturbations is qualitatively similar to that of off-site vortices (see figures 3–5), and will not be repeated here.

4. Summary

In summary, we have carried out a numerical stability analysis for both the on- and off-site lattice vortices with unit charge in a 2D photorefractive optical lattice. We have shown that high- and low-intensity lattice vortices of both types suffer oscillatory and VK instabilities, but intermediate-intensity vortices can be stable if the lattice intensity is moderate, and the applied DC field is above a certain threshold value. Under the same physical conditions, on-site vortices are more stable than off-site ones. Higher bias field stabilizes lattice vortices.

Acknowledgments

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