Interchannel pulse collision in a wavelength-division-multiplexed system with strong dispersion management

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We develop a perturbation theory to analytically calculate the effects of complete and incomplete interchannel collisions of Gaussian pulses in a wavelength-division-multiplexed system with strong dispersion management. We show that, for complete collisions, the collision-induced frequency shift of a Gaussian pulse is negligible, whereas its position shift is significant and can be found in a simple analytical form. For strong dispersion management we find that incomplete collisions can be neglected, whereas for dispersion management of moderate strength the contribution of the incomplete collisions can be significant. The analytical predictions are in satisfactory agreement with numerical results. We also give an estimate of the limit imposed on the transmission distance by such collisions.

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A serious problem in the use of the wavelength-division-multiplexing (WDM) technology[1] is the timing jitter generated by collisions with pulses in other channels. It is known that the frequency shift (FS) generated by collisions in a WDM system with strong dispersion management[2] (DM) is considerably less than that predicted for sech pulses.[1,3,4] Here we extend the recent detailed analysis of the dynamics of a Gaussian pulse[5,6] (where fairly good agreement between the analytical and the numerical results was found) to collisions between pulses in different channels of a WDM system.

We show that, for complete collisions (collisions that begin and end with the pulses fully separated), the net FS is zero in first order, in agreement with recent numerical results.[4] Also, we show that the first-order collision-induced position shift (PS) is nonzero and that it can be obtained in a simple analytical form. Thus, for strong DM, the collision-induced jitter is caused mainly by PS's rather than by FS's. By contrast, for moderately strong DM, the incomplete collisions can become important contributors to this jitter, and we give analytical limits for such collisions. These analytical results are checked against numerical simulations, showing reasonable agreement. Very recently, the effects of the WDM collisions on Gaussian pulses were also studied by means of a semianalytical variational approximation.[7]

We start with a system of equations governing the propagation of electromagnetic field envelopes, $u(z,\ \tau)$ and $v(z,\ \tau)$, in two adjacent channels of an optical fiber, subjected to strong DM:

$$2i(u_z + cu_\tau) + D(z)u_{\tau\tau} + \epsilon[\overline{D}_u u_{\tau\tau} + 2(|u|^2 + 2|v|^2)u] = 0\,, \quad (1)$$

$$2iv_z + D(z)v_{\tau\tau} + \epsilon[\overline{D}_v v_{\tau\tau} + 2(|v|^2 + 2|u|^2)v] = 0\,, \quad (2)$$

where c is the inverse group-velocity difference between the channels; $D(z)$ is the main part of the dispersion (with a zero average); $\overline{D}_{u,v}$ are the residual, nonzero, average dispersions in the two channels; and the normalized nonlinear terms represent the usual self-phase modulation and cross-phase modulation (XPM).[8] The small parameter ϵ is introduced, as in Ref. 6, to indicate the perturbation terms in the strong-DM regime. The dispersions in the adjacent channels differ because of the third-order dispersion.[8]

Setting $\epsilon = 0$, one has an exact solution to Eq. (1) in the form of a Gaussian pulse moving at velocity c:

$$u_0(z,\tau) = \tau_0\left[\frac{P_u}{{\tau_0}^2 + 2i\Delta(z)}\right]^{1/2} \times \exp\left[-\frac{(\tau - cz)^2}{{\tau_0}^2 + 2i\Delta(z)} + i\phi_u\right], \quad (3)$$

where $\Delta(z) \equiv \int_0^z D(z')\mathrm{d}z' + \Delta_0$ and the constants P_u, τ_0, Δ_0, and ϕ_u determine, respectively, the Gaussian pulse's peak power, width, chirp, and phase. We assume equal widths, τ_0, of the pulses in the adjacent channels, while the values of Δ_0 are always to be taken as equal (see below). We now apply a Galilean boost to the above solution, Eq. (3), which gives

$$u(z,\tau) = u_0[z, \tau - T(z)]\exp\{-i\omega[\tau - T(z)] + i\psi(z)\}, \tag{4}$$

where ω is the FS, with the position and phase shifts governed by the equations $\mathrm{d}\psi/\mathrm{d}z = (1/2)D(z)\omega^2$ and

$$\mathrm{d}T/\mathrm{d}z = -\omega[D(z) + \epsilon\overline{D}_u]. \tag{5}$$

In the single-channel case, two relations among the parameters P_u, τ_0, and Δ_0 must be satisfied if the pulse's propagation is to remain stationary in the presence of weak self-phase modulation and average dispersion.[6] Assuming the usual two-step form of DM, $D(z) = D_1$ for $0 < z < L_1$ and $D(z) = D_2$ for $L_1 < z < L_1 + L_2$, with the dispersion-compensation condition $D_1L_1 + D_2L_2 = 0$, then, from Ref. 6, a stationary Gaussian pulse must satisfy the two conditions $\Delta_0 = -1/2\, S{\tau_0}^2$ and

$$2\sqrt{2}\,\overline{D}_u = -P_u\frac{{\tau_0}^2}{S}\Big[\ln\Big(\sqrt{1+S^2} + S\Big) - 2S(1+S^2)^{-1/2}\Big], \tag{6}$$

where $S = |D_1|L_1/{\tau_0}^2$ is our definition of the dimensionless DM strength parameter. (In Ref. 6 the units were such that ${\tau_0}^2 = 1/S$.) These conditions were verified in Ref. 6 with direct numerical simulations.

Fixing the parameters of the pulses in both channels according to Eq. (6), one can then treat the collision of pulses between channels by taking the XPM coupling as a second perturbation. Perturbative treatment of the collision is straightforward (neglecting radiative losses). One obtains for the final form of the first-order XPM-induced evolution equation for ω, defined in Eq. (4),

$$\frac{\mathrm{d}\omega}{\mathrm{d}z} = \frac{2^{3/2}\epsilon P_v{\tau_0}^4 cz}{[{\tau_0}^4 + 4\Delta^2(z)]^{3/2}}\exp\left\{-\frac{c^2{\tau_0}^2z^2}{[{\tau_0}^4 + 4\Delta^2(z)]}\right\}, \tag{7}$$

where P_v is the peak power of the pulse in the other channel. At the same order, the evolution of the position still given by Eq. (5).

Solving Eqs. (5) and (7) is facilitated by the fact that, because c is small, the function cz varies slowly in comparison with the rapidly oscillating $\Delta(z)$. In fact, in the strong-DM regime, colliding pulses pass each other many times before separating.[4] A technical result that we need for solving Eqs. (5) and (7) is the asymptotic formula

$$\int_{-\infty}^{+\infty}(\sqrt{2}\,z)^nF(z)\exp[-z^2f^2(z)]\mathrm{d}z \approx \sqrt{\pi}\,C_n\langle F(z)f^{-(n+1)}(z)\rangle, \tag{8}$$

where $f(z)$ and $F(z)$ are rapidly oscillating periodic functions; $\langle\rangle$ indicates an average over the period; $n = 0, 1, 2, \ldots$; and $C_n = 0$ if n is odd, $C_n = 1$ if $n = 0$, and $C_n = (n-1)!!$ if n is even. The derivation of relation (8) is a formal issue that is not considered here. A straightforward consequence of relations (7) and (8) is that $\int_{-\infty}^{+\infty}(\mathrm{d}\omega/\mathrm{d}z)\mathrm{d}z = 0$. Thus, in first order, a complete collision will produce a zero net shift FS. This is in agreement with the numerical results reported in Ref. 4.

Proceeding to the net collision-induced PS, δT_u, we integrate Eq. (5) by parts, casting the expression for δT_u into the form

$$\delta T_u \equiv \int_{-\infty}^{+\infty}\frac{\mathrm{d}T}{\mathrm{d}z}\mathrm{d}z = \epsilon\overline{D}_u\int_{-\infty}^{+\infty}z\frac{\mathrm{d}\omega}{\mathrm{d}z}\mathrm{d}z + \int_{-\infty}^{+\infty}\Delta(z)\frac{\mathrm{d}\omega}{\mathrm{d}z}\mathrm{d}z\,. \tag{9}$$

Next, with the use of relations (7) and (8), the second term in Eq. (9) vanishes, while the first term yields

$$\delta T_u = \sqrt{2\pi}\,(\epsilon^2\overline{D}_u/c^2)P_v\tau_0. \tag{10}$$

Because the collision length is large and a real WDM system will involve dozens of channels, it will be impossible to avoid having incomplete collisions at the line's input (i.e., the collisions that begin with the pulses overlapped). Such collisions will result in significant FS's. To estimate their size, we note that the maximum FS will occur when the two pulses exactly overlap at the input. We proceed as above and obtain

$$(\Delta\omega)_{\max} = \frac{\sqrt{2}\,\epsilon P_v}{cS}\ln\Big(S + \sqrt{1+S^2}\Big), \tag{11}$$

where S is as defined above. With Eq. (11) we can now find the maximum contribution of this FS to the net PS, using Eq. (5). For a sufficiently large propagation distance z, the corresponding contribution is

$$\delta {T_u}^{(\omega)} = -(\Delta\omega)_{\max}\epsilon\overline{D}_u z\,. \tag{12}$$

We now compare this PS generated by the incomplete WDM collision with Eq. (10). The ratio is

$$\left|\frac{\delta {T_u}^{(\omega)}}{\delta T_u}\right| = \sqrt{\frac{1}{\pi}}\,\frac{cz}{\tau_0 S}\ln\Big(S + \sqrt{1+S^2}\Big). \tag{13}$$

To estimate the relative importance of these terms, for example, we assume that we shall be operating near the point at which Eq. (6) predicts zero average dispersion in one channel, whereas in the other channel a nonzero average dispersion will exist. We take the pulse width to be 30 ps, with a DM period of 200 km, and a local fiber dispersion of 20 $\mathrm{ps}^2/\mathrm{km}$. We also take $\delta\lambda \sim 1$ nm and a realistic third-order dispersion coefficient of $|\beta_3| \sim 0.1\ \mathrm{ps}^3/\mathrm{km}$, from which we obtain the estimates of $|c| \sim 0.1$ ps/km and $z_{\mathrm{coll}} \sim 500$ km for the collision distance. Now, assuming a densely packed array with a minimum separation between the

Table 1. Comparison between the Analytically Predicted and Numerically Found Collision-Induced PS's of the Gaussian Pulse[a]

S	PS	
	$(T_u)_{\mathrm{num}}$	$(T_u)_{\mathrm{an}}$
0.500	0.216	0.2302
0.667	0.144	0.1314
1.000	0.080	0.0502
1.250	0.048	0.0289

[a]See the text for the values of the parameters. $S \approx 1$ corresponds, in the example case, to the physical pulse's width $\tau_0 \sim 30$ ps; the physical values of the PS can be rescaled accordingly.

pulse centers of $3\tau_0$ and taking z to be the distance per complete WDM collision, $z \sim 3z_{\mathrm{coll}} \sim 1500$ km, we have

$$\left| \frac{\delta T_u{}^{(\omega)}}{\delta T_u} \right| \sim \frac{\sqrt{2}}{S} \ln(S + \sqrt{1 + S^2})\,. \qquad (14)$$

The meaning of estimate (14) is very simple: In the case of a moderately strong DM, $S \approx 1$ (Ref. 9); then incomplete collisions can even be approximately twice as important as complete collisions. If S is sufficiently strong (say, $S \gtrsim 8$), the contribution of the incomplete collisions to the temporal jitter will be less important.

To verify the above results, we have solved Eqs. (1) and (2) numerically. The steadily propagating pulses in both channels were prepared by use of the results of Ref. 6. As an example, we take $\epsilon = 0.1$, $L_1 = 0.4$, $L_2 = 0.6$, $D_1 = 5/2$, $D_2 = -5/3$, $P_u = P_v = 1$ (these are the same values for which Fig. 1 of Ref. 6 was obtained), and $c = 0.3$. The average dispersion is taken from Eq. (6) for different values of the width τ_0, which is used as a control parameter. In all these simulations the collision-induced FS was found to be extremely small ($\lesssim 10^{-5}$). In Table 1 we show the numerically obtained and the analytically predicted values of the complete collision-induced PS's for various values of S. For narrow pulses (large S) the agreement is worst, whereas, as the pulse gets wider, the agreement becomes much better. We also numerically simulated the incomplete collisions of initially fully overlapped pulses and found good agreement between the numerically obtained FS and that obtained from Eq. (11). These complete and incomplete collisions impose a limitation on the wavelength separation $\delta\lambda$ between the channels: A small $\delta\lambda$ gives rise to a small c and, hence, to a large collision-induced PS. Using the same above-mentioned values of the physical parameters, we conclude that for $S \approx 1$ the shifts produced by multiple interchannel collisions, complete and incomplete, will not corrupt the information content of the signal for transmission distances of $\lesssim 15,000$ km. Using Eq. (10), one can easily determine how this distance scales with the change of the parameters. Our model does not include amplifiers and filters. The collision-induced timing jitter in a WDM–DM system with amplifiers was studied numerically in Ref. 10.

In conclusion, we have considered the effects of interchannel collisions on Gaussian pulses in WDM systems subjected to strong DM. We have shown that, for complete collisions, the collision-induced frequency shift of the Gaussian pulse is negligible, while the position shift dominates and is given in a simple analytical form. We have also placed upper limits on the shifts due to incomplete collisions and have shown that they are significant only for moderate values of S. The analytical predictions were checked against numerical simulations and showed reasonable agreement. For an example case of $S \approx 1$ we conclude that the limitation imposed by complete and incomplete collisions on the transmission of information is $\sim 15,000$ km.

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