Transverse instability of strongly coupled dark–bright Manakov vector solitons

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We show, by performing linear stability analysis and direct numerical simulations, that dark-soliton transverse instability is significantly reduced in Kerr media by strong coupling to a bright soliton. High instability suppression can be achieved by use of large-amplitude bright solitons. © 2001 Optical Society of America

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Symmetry-breaking instabilities are ubiquitous in nonlinear systems. They can lead to many fascinating phenomena, such as pattern formation, beam filamentation, self-focusing processes, and decay of low-dimensional solitary waves as a result of perturbations at higher dimensions.[1,2] Probably the best-known nonlinearity-induced instabilities are modulational instability (MI) and transverse instability (TI) of (1 + 1)D solitons. The latter occurs because perturbations in the transverse direction have nothing to restrain them from growing (driven by the nonlinearity), and as a result the soliton breaks up. Two well-known examples from spatial optical (1 + 1)D solitons are TI of scalar bright and dark solitons in which it was shown[3,4] that both cases are linearly unstable against long-wave perturbations. The long-term dynamics of this instability is that a bright stripe decays into a line of two-dimensional filaments,[5] whereas a dark stripe decays into a sequence of optical vortices.[6–9] TI puts severe limits on possible realization of either bright or dark strips in bulk media.

As a result, the commonly held belief has been that vector solitons are not observable in a higher dimension either. It has been believed that they should be even more unstable than scalar cases because of the mode interaction. For MI, this is indeed the case, as the cross-phase modulation of vector solitons can generate MI of otherwise stable waves.[10] However, for TI, this belief is not entirely true. For photorefractive materials in which the nonlinearity is saturable, vector solitons in three realizations, bright–bright, dark–dark, and dark–bright pairs, have been experimentally observed.[11–13] A theoretical explanation of these experiments is that nonlinearity saturation and incoherent mode interaction strongly suppress TI of vector solitons in all three realizations.[14] For Kerr nonlinearity (nonsaturable), is there a mechanism to suppress TI of vector solitons as well? If the vector soliton is made sufficiently incoherent along the transverse dimension, elimination of TI is possible (see Ref. 15 for TI elimination in the scalar case). If the vector soliton is coherent along the transverse dimension, one can reduce TI by increasing the bright-soliton component inside a dark–bright-soliton pair, at least in the regime of long transverse-wave perturbations.[14] This result is somewhat counterintuitive, as scalar bright solitons are themselves unstable to TI.[2] Since the result reported in Ref. 14 was obtained only for transverse long-wave disturbances, it remains unclear whether the same TI suppression holds for all transverse wavelengths, especially the wavelengths at which the instability growth rates are the largest.

In this Letter, we investigate the TI of strongly coupled dark–bright Manakov vector solitons for all transverse wavelengths. This case is very important from the physics standpoint because it provides what is believed to be the first counterexample in nonlinear waves in which coupling leads to substantial suppression of TI far below its scalar limit. Other cases such as dark–dark and bright–bright vector solitons are physically less interesting. We demonstrate numerically that the bright component significantly reduces the snakelike TI growth rate of the dark–bright-soliton pair to far below its scalar value over the entire transverse-wavelength spectrum. For instance, when the bright-soliton amplitude is 0.8 and the dark-soliton background is normalized to be 1 (strongly coupled regime), the maximum TI growth rate is reduced over 50% from the scalar case when the bright-soliton component is zero. This reduced growth rate implies that the physical distance that it takes for TI to develop will double from the scalar case. Thus nonlinear coupling here stabilizes the soliton instead of destabilizing it as one might expect. Our method is to perform a linear stability analysis and determine the unstable eigenvalues as well as to perform direct numerical simulations.

We start from the normalized equations

$$i\frac{\partial U}{\partial z} + \frac{1}{2}\nabla_\perp{}^2 U - (|U|^2 + |V|^2)U = 0\,,$$

$$i\frac{\partial V}{\partial z} + \frac{1}{2}\nabla_\perp{}^2 V - (|U|^2 + |V|^2)V = 0\,, \qquad (1)$$

where U and V are the envelopes of the two incoherently interacting beams and $\nabla_\perp{}^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian. Equations (1) describe two coupled beams in a self-defocusing optical Kerr

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medium. Such an interaction can form vector solitons that consist of two components that are mutually self-trapped in a nonlinear medium. Stationary solutions to Eqs. (1) in the form of a bright strip, $U(z,x,y) = u_s(x)\exp(i\mu z)$, and a dark strip, $V(z,x,y) = v_s(x)\exp(i\nu z)$, are given by[16–18]

$$u_s(x) = \sqrt{1 - a^2}\,\mathrm{sech}(ax), \qquad v_s(x) = \tanh(ax), \quad (2)$$

where the dark soliton's background is normalized to be 1, the propagation constants $\mu = -(1 - a^2/2)$, $\nu = -1$, and a ($|a| \le 1$) characterizes the amplitude of the bright component. To study the linear stability of the solution presented above, we write weakly perturbed solutions of Eqs. (1):

$$U(z,x,y) = [u_s(x) + \epsilon U_1(z,x,y)]\exp(i\mu z),$$

$$V(z,x,y) = [v_s(x) + \epsilon V_1(z,x,y)]\exp(i\nu z), \quad (3)$$

where each perturbation is expressed as a superposition of plane waves with wave number q and frequency ω:

$$U_1 = \phi_1(x)\exp(i\omega z + iqy) + \phi_2{}^*(x)\exp(-i\omega^* z - iqy),$$

$$V_1 = \psi_1(x)\exp(i\omega z + iqy) + \psi_2{}^*(x)\exp(-i\omega^* z - iqy).$$

By writing $\phi_{1,2} = 1/2(\phi^+ \pm \phi^-)$ and $\psi_{1,2} = 1/2(\psi^+ \pm \psi^-)$, we arrive, to the first order in ϵ, at the following linear eigenvalue problem:

$$\frac{1}{2}\frac{\mathrm{d}^2\phi^+}{\mathrm{d}x^2} - \left(R_\mu + \frac{q^2}{2}\right)\phi^+ - \omega\phi^- - 2u_s v_s \psi^+ = 0,$$

$$\frac{1}{2}\frac{\mathrm{d}^2\psi^+}{\mathrm{d}x^2} - \left(R_\nu + \frac{q^2}{2}\right)\psi^+ - \omega\psi^- - 2u_s v_s \phi^+ = 0,$$

$$\frac{1}{2}\frac{\mathrm{d}^2\phi^-}{\mathrm{d}x^2} - \left(\mu + \frac{q^2}{2} + u_s{}^2 + v_s{}^2\right)\phi^- - \omega\phi^+ = 0,$$

$$\frac{1}{2}\frac{\mathrm{d}^2\psi^-}{\mathrm{d}x^2} - \left(\nu + \frac{q^2}{2} + u_s{}^2 + v_s{}^2\right)\psi^- - \omega\psi^+ = 0, \quad (4)$$

where $R_\mu(x) = \mu + 3u_s{}^2 + v_s{}^2$ and $R_\nu(x) = \nu + 3v_s{}^2 + u_s{}^2$. Spectral problem (4) can be solved analytically only in the long-wave limit ($|q| \ll 1$), in which the perturbation scale is long in comparison with the soliton size. This means that the solution of system (4) may be found in the following asymptotic form:

$$\phi^\pm \simeq \phi_0{}^\pm + q\phi_1{}^\pm + q^2\phi_2{}^\pm + \cdots,$$

$$\psi^\pm \simeq \psi_0{}^\pm + q\psi_1{}^\pm + q^2\psi_2{}^\pm + \cdots,$$

$$\omega(q) = q\omega_1 + q^2\omega_2 + \cdots, \quad (5)$$

Substituting Eqs. (5) into system (4) and solving the corresponding equations at each order in q, we find that the leading-order unstable eigenvalue is given as

$$i\omega = \left[\frac{a^2(3 - a^2)}{3(a^2 + 1)}\right]^{1/2} q. \quad (6)$$

Result (6) indicates an unexpected feature of the dark–bright-soliton pair: A bright component embedded in a self-defocusing medium leads to effective suppression of the TI of a dark soliton. Now, the important question we want to ask is: Does this peculiar behavior occur for perturbations with any wave number? To answer this question, we have solved spectral problem (4) numerically by the shooting method. We first determine the asymptotic solution behavior at large $|x|$ values. Then we integrate Eqs. (4) from large $|x|$ values to $x = 0$. By demanding appropriate symmetry conditions on the solution at $x = 0$, we obtain the eigenvalue as well as the corresponding eigenfunction. We found that when $|q| < a$, the linearization operator has one unstable discrete eigenvalue $i\omega$ that is purely real. Figure 1 shows this eigenvalue as a function of q for different values of a. It is clear that the suppression of instability covers the whole spectrum of wave numbers. Moreover, when $q \ll 1$, our numerical results agree well with result (6). Remarkably, when the bright-soliton amplitude is increased from 0 ($a = 1$) to 0.8 ($a = 0.6$), the maximum-growth rate drops more than 50%. In Fig. 2, the most-unstable mode $[\phi_1(x), \phi_2(x), \psi_1(x), \psi_2(x)]$ is shown for $a = 0.6$. Note that (ϕ_1, ϕ_2) are antisymmetric, whereas (ψ_1, ψ_2) are symmetric, which implies that this instability is snakelike instead of necklike.

To confirm our findings and to support the linear stability theory, we have done a series of direct numerical simulations of Eqs. (1) in which we launch different initial conditions given by Eqs. (2) that correspond to

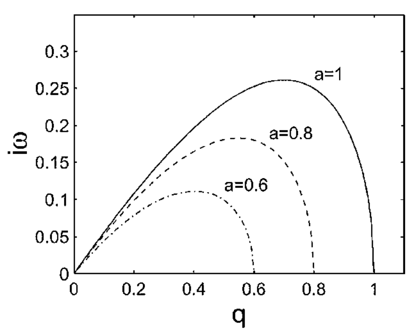

Fig. 1. Growth rate of the TI of a dark–bright-soliton pair for all transverse-wave numbers at three selected a values.

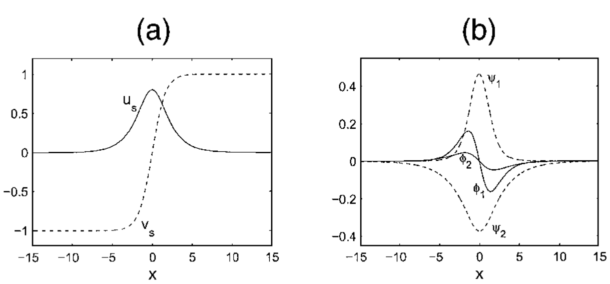

Fig. 2. (a) Dark–bright-soliton solution (2) at $a = 0.6$. (b) Most-unstable eigenfunctions $[\phi_1(x), \phi_2(x), \psi_1(x), \psi_2(x)]$ for the dark–bright-soliton in (a). Here, $a = 0.6$, $q = 0.4$, and unstable eigenvalue $i\omega = 0.1113$.

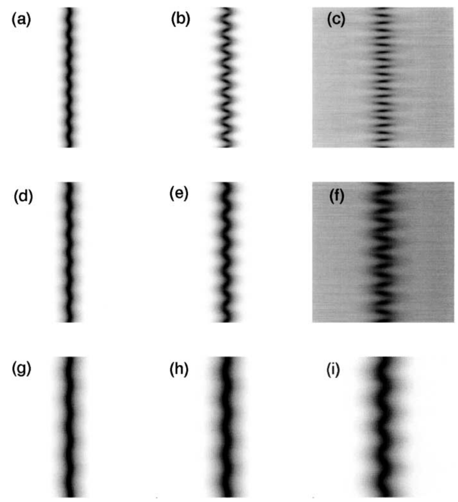

Fig. 3. Evolution of the dark-soliton TI (a)–(c) for the scalar case ($a = 1$), (d)–(f) in the presence of bright component with peak intensity 0.6 ($a = 0.8$), and (g)–(i) for higher bright-soliton amplitude 0.8 ($a = 0.6$). Snapshots are taken at $z = 8, 16, 24L_D$ for each a value. Black indicates low solution values, and white represents high solution values. The x and y scales are $-8 < x < 8$ and $-40 < y < 40$, respectively.

different values of a (or different bright peak intensities). A perturbation that is periodic along the y direction and localized along the x direction was added, with a transverse-wave number corresponding to the maximum-growth rate. Specifically, the initial condition was taken as

$$U(0,x,y) = u_s(x) + \epsilon \mathrm{d}u_s/\mathrm{d}x \cos(q_0 y),$$

$$V(0,x,y) = v_s(x) + \epsilon \mathrm{d}v_s/\mathrm{d}x \cos(q_0 y), \tag{7}$$

where q_0 is the maximum-growth wave number (see Fig. 1), and the perturbation amplitude $\epsilon = 0.1$. Figure 3 shows the dynamic evolution of the dark soliton in the (x, y) plane at three a values, $a = 1, 0.8, 0.6$. At each a value, snapshots of the dark-soliton component ($|V|$) are taken at distances $z = 8, 16, 24$. Here, the distance z is nondimensionalized by the diffraction length, L_D, which in terms of physical units is given by $L_D = \lambda n_0/(2\pi \Delta n_0)$, where λ is the wavelength of the laser beam, n_0 is the unperturbed refractive index, and Δn_0 is the maximum physical index change. For typical photorefractive materials, $\Delta n_0/n_0 \approx 2 \times 10^{-4}$, which gives (for $\lambda = 0.5\ \mu$m) $L_D \approx 0.4$ mm. As expected, when $a = 1$ (scalar dark soliton), the soliton undergoes a symmetry-breaking instability after $8L_D$ [see Fig. 3(a)] and after $16L_D$ snake instability sets in [Fig. 3(b)]. Eventually, after $24L_D$, the dark strip disintegrates into two-dimensional vortices [Fig. 3(c)]. This breakup was experimentally observed in sodium vapor.[5–8] To show the effect of the strong mode interaction on the dark soliton, we simulated Eqs. (1) with different values of a. Figures 3(d)–3(f) depict the evolution of the dark-soliton instability under the influence of the bright component, with peak amplitude 0.6 ($a = 0.8$). Comparing each evolution stage with the previous one ($a = 1$), we can clearly see instability suppression. The evolution when we increase the bright-soliton amplitude of 0.8 ($a = 0.6$) is shown in Figs. 3(g)–3(i), in which much sharper suppression of TI can be observed. Notice that, after $24L_D$, the dark component did not break up into vortices [Fig. 3(i)], as opposed to the scalar case [Fig. 3(c)]. It is noted that, although our linearization analysis predicts only the initial instability growth and the onset of snakelike instability, our numerical results in Fig. 3 also reveal the nonlinear development of this instability, which is the breakup of the dark–bright-soliton strip into vortices.

In conclusion, we have analyzed the transverse instability of dark–bright-soliton pairs in a self-defocusing Kerr nonlinear medium. By performing linear stability analysis and direct numerical simulations, we have demonstrated that the bright-soliton component strongly suppresses the TI of the soliton pair.

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