Suppression of Manakov soliton interference in optical fibers

Jianke Yang*
Department of Mathematics and Statistics, University of Vermont, 16 Colchester Avenue, Burlington, Vermont 05401
(Received 2 July 2001; revised manuscript received 19 September 2001; published 11 February 2002)

In this paper, we study the interaction of two vector solitons in the Manakov equations that govern pulse transmission in randomly birefringent fibers. Under the assumptions that these solitons initially are well separated and having nearly the same amplitudes and velocities but arbitrary polarizations, we derive a reduced set of ordinary differential equations for both solitons' parameters. We then solve this reduced system analytically. Our analytical solutions show that, when two Manakov solitons have the same amplitude and phases, their collision distance steadily increases as their initial polarizations change from parallel to orthogonal. In particular, the collision distance at orthogonal polarizations is of the order of the square of the collision distance at parallel polarizations. When the Manakov solitons have different amplitudes, a quasiequidistant bound state can be formed. The degrees of position and amplitude oscillations in this bound state diminish as the initial polarizations change from parallel to orthogonal. With a combination of launching Manakov solitons along orthogonal polarizations and at unequal amplitudes, Manakov-soliton interference is almost completely suppressed. These theoretical results are in excellent agreement with our direct numerical simulations.

DOI: 10.1103/PhysRevE.65.036606 PACS number(s): 42.65.Tg, 05.45.Yv

I. INTRODUCTION

Soliton transmission in optical fibers has been theoretically predicted and experimentally demonstrated for over 20 years (see [1,2] and references therein). The early theoretical work primarily used the nonlinear Schrödinger (NLS) equation as the mathematical model. One of the impairments to soliton transmission systems is the interference of neighboring solitons that cause pulses to move away from their expected time slots. Within the NLS model, pulse interference has been comprehensively investigated [2–10]. It has been shown that when two solitons have the same phase and amplitude, they attract each other; when they have the opposite phase and same amplitude, they repel each other. If the solitons have different amplitudes, they could form a quasiequidistant bound state. In reality, optical fibers are birefringent, i.e., pulses along two orthogonal polarizations of the fiber travel at slightly different speeds. Over long distances, this birefringence is also random due to fiber bending, twisting, and environmental fluctuations. When this random birefringence is averaged and small perturbation terms such as polarization-mode dispersion neglected, pulse propagation is actually governed by the Manakov equations [11–15]. To reduce pulse-pulse interference in birefringent fibers, a technique called polarization-division multiplexing (PDM) has been proposed [14]. In a PDM system, adjacent solitons are launched along orthogonal polarizations of the fiber. Experiments have shown that this technique doubles the transmission rate compared to the launching of pulses along the same polarizations [14,16]. Analytically, orthogonal-soliton interactions in the Manakov system has been studied in [17] by the variational method. It was shown that the collision distance of initially orthogonal solitons is much longer than that of the parallel solitons, thus the benefit of the PDM technique was analytically demonstrated. When PDM is combined with wavelength-division multiplexing (WDM), it has been shown that collisions of solitons in different channels make neighboring solitons in the same channel nonorthogonal [18,19]. When polarization-mode dispersion (PMD) effect is included, it was pointed out in [20] that the use of PDM technique is only preferable to a copolarized pulse train only if the PMD is sufficiently weak. The interaction of Manakov solitons was also considered in [21], where it was conjectured that the soliton parameters satisfy the complex Toda chain. When frequency-filter perturbations are added to the Manakov system, it was shown in [22] that the collision distance of orthogonally polarized solitons substantially increase. Interaction of vector solitons in the non-Manakov coupled NLS equations was investigated in [23,24]. It was shown that stationary multivector-soliton bound states exist but are linearly unstable.

Despite the above-mentioned work on Manakov-soliton interactions, many important questions still remain open. For instance, the interaction of Manakov solitons at arbitrary polarizations has not been investigated. This study is desirable as in a WDM and PDM system, pulses in the same channel are generally nonorthogonal [18,19]. More importantly, with the NLS equation, it has been shown that launching adjacent pulses at different amplitudes is effective in suppressing soliton interactions [5,7–10]. How this technique affects the interference of Manakov solitons has not been carefully examined.

In this paper, we study the interaction of Manakov solitons at arbitrary polarizations. Our method is an extension of the Karpman-Solov'ev technique [3,4]. Assuming that the Manakov solitons are well separated and having nearly the same amplitudes and velocities but arbitrary polarizations, we derive the full dynamical equations for both solitons' amplitudes, velocities, positions, polarizations, and phases. We also solve these reduced equations analytically. Based on these results, we show that two Manakov solitons with the same amplitudes and phases attract each other. The collision distance increases as the initial polarizations change from

*FAX: 802-656-2552; Email address: jyang@emba.uvm.edu

 ©2002 The American Physical Society

parallel to orthogonal. In particular, the collision distance at orthogonal polarizations is of the order of the square of that at parallel polarizations. This explains why the PDM technique doubles the transmission capacity as reported in the experiment of [14]. More significantly, we show that two Manakov solitons with different amplitudes form quasiequidistant bound states, similar to the NLS solitons. But position and amplitude oscillations of individual Manakov solitons in the quasiequidistant state diminish as the initial polarizations change from parallel to orthogonal. Combining the above two results, we show that if Manakov solitons are launched along orthogonal polarizations and at unequal amplitudes, their interference is almost completely suppressed. We have also checked these analytical results against direct numerical simulations and good agreement is obtained.

II. DYNAMICAL EQUATIONS FOR MANAKOV-SOLITON INTERACTIONS

The Manakov equations for fiber communication systems are written as [14,15]

$$iA_z+\frac{1}{2}A_{tt}+(|A|^2+|B|^2)A=0, \tag{2.1}$$

$$iB_z+\frac{1}{2}B_{tt}+(|A|^2+|B|^2)B=0, \tag{2.2}$$

where A and B are amplitudes of electrical fields along the fiber's orthogonal polarizations, z is the propagation distance, and t is the retarded time. Manakov solitons are of the form

$$A(t,z)=\eta\cos\theta\,\mathrm{sech}\,\eta(t-T)\exp[iV(t-T)+i\gamma], \tag{2.3}$$

$$B(t,z)=\eta\sin\theta\,\mathrm{sech}\,\eta(t-T)\exp[iV(t-T)+i\Gamma], \tag{2.4}$$

$$T=Vz+\overline{T},\quad \gamma=\frac{1}{2}(\eta^2+V^2)z+\overline{\gamma},\quad \Gamma=\frac{1}{2}(\eta^2+V^2)z+\overline{\Gamma}, \tag{2.5}$$

where amplitude η, velocity V, polarization angle θ, position parameter $\overline{T}$, and phases $\overline{\gamma}$ and $\overline{\Gamma}$ are all constants. When two Manakov solitons are placed adjacent to each other, they would interfere through tail overlapping. To study this interference, the idea of Karpman and Solov'ev perturbation method [3,4] is to treat this interference as a small perturbation to each soliton. To leading order, the solution is simply a superposition of two Manakov solitons

$$A=A_1+A_2,\quad B=B_1+B_2, \tag{2.6}$$

where (A_k,B_k) are of the form (2.3) and (2.4), and all parameters have indices $k(k=1,2)$ and slowly vary over distance z. For convenience, we assign the left soliton with index $k=1$ and the right soliton with index $k=2$. Thus $T_2>T_1$. Picking up the dominant interference terms, each soliton is governed by the following perturbed Manakov equations:

$$iA_{k,z}+\frac{1}{2}A_{k,tt}+(|A_k|^2+|B_k|^2)A_k=F_k, \tag{2.7}$$

$$iB_{k,z}+\frac{1}{2}B_{k,tt}+(|A_k|^2+|B_k|^2)B_k=G_k, \tag{2.8}$$

where

$$F_k=-2|A_k|^2A_{3-k}-A_k^2A_{3-k}^*-A_kB_kB_{3-k}^*-A_kB_k^*B_{3-k}-|B_k|^2A_{3-k}, \tag{2.9}$$

$$G_k=-2|B_k|^2B_{3-k}-B_k^2B_{3-k}^*-B_kA_kA_{3-k}^*-B_kA_k^*A_{3-k}-|A_k|^2B_{3-k}. \tag{2.10}$$

The slow evolution of each soliton's parameters can be derived by a perturbation theory for the Manakov equations (2.7) and (2.8). Such a theory has been developed in [24–27]. To carry out the calculations, we assume that the two Manakov solitons have nearly the same amplitudes and velocities, and are well separated. But their polarizations are allowed to be arbitrary. Introducing notations

$$\eta\equiv\frac{1}{2}(\eta_2+\eta_1),\quad V\equiv\frac{1}{2}(V_2+V_1),\quad T\equiv\frac{1}{2}(T_2+T_1), \tag{2.11}$$

and

$$\Delta\eta\equiv\eta_2-\eta_1,\quad \Delta V\equiv V_2-V_1,\quad \Delta T=T_2-T_1, \tag{2.12}$$

our assumptions then are

$$|\Delta\eta|\ll\eta,\quad |\Delta V|\ll|V|,\quad \eta\Delta T\gg1,\quad |\Delta\eta\Delta T|\ll1. \tag{2.13}$$

It is noted that the Manakov system (2.1) and (2.2) is Galilean invariant. Thus it is always possible to choose a reference frame in which $V=0$. If this is done, then the condition on velocity difference ΔV is simply $|\Delta V|\ll1$. We also introduce the notations

$$\Delta\phi=\Delta\gamma-V\Delta T,\quad \Delta\Phi=\Delta\Gamma-V\Delta T, \tag{2.14}$$

where $\Delta\gamma\equiv\gamma_2-\gamma_1$ and $\Delta\Gamma\equiv\Gamma_2-\Gamma_1$. Under these assumptions and notations, and after some simple calculations, we obtain the following dynamical equations for the two Manakov solitons' parameters (the details are given in the Appendix):

$$\frac{d\eta}{dz}=\frac{dV}{dz}=0, \tag{2.15}$$

$$\frac{d(\Delta T)}{dz}=\Delta V, \tag{2.16}$$

$$\frac{d(\Delta\eta)}{dz} = 8\eta^3 e^{-\eta\Delta T}(\cos\theta_1\cos\theta_2\sin\Delta\phi + \sin\theta_1\sin\theta_2\sin\Delta\Phi), \tag{2.17}$$

$$\frac{d(\Delta V)}{dz} = -8\eta^3 e^{-\eta\Delta T}(\cos\theta_1\cos\theta_2\cos\Delta\phi + \sin\theta_1\sin\theta_2\cos\Delta\Phi), \tag{2.18}$$

$$\frac{d\theta_1}{dz} = 2\eta^2 e^{-\eta\Delta T}(\sin\theta_1\cos\theta_2\sin\Delta\phi - \cos\theta_1\sin\theta_2\sin\Delta\Phi), \tag{2.19}$$

$$\frac{d\theta_2}{dz} = -2\eta^2 e^{-\eta\Delta T}(\cos\theta_1\sin\theta_2\sin\Delta\phi - \sin\theta_1\cos\theta_2\sin\Delta\Phi), \tag{2.20}$$

$$\frac{d(\Delta\phi)}{dz} = \eta\Delta\eta + 2\eta^2 e^{-\eta\Delta T}\cos\Delta\phi \times\frac{\cos^2\theta_1\sin^2\theta_2 - \sin^2\theta_1\cos^2\theta_2}{\cos\theta_1\cos\theta_2}, \tag{2.21}$$

$$\frac{d(\Delta\Phi)}{dz} = \eta\Delta\eta - 2\eta^2 e^{-\eta\Delta T}\cos\Delta\Phi \times\frac{\cos^2\theta_1\sin^2\theta_2 - \sin^2\theta_1\cos^2\theta_2}{\sin\theta_1\sin\theta_2}, \tag{2.22}$$

$$\frac{d}{dz}\left(T + \frac{\Delta\eta}{4\eta^2}\right) = V. \tag{2.23}$$

The above equations are the leading-order evolution equations for soliton parameters in the Karpman-Solov'ev perturbation theory. The small parameter in this perturbation theory is $e^{-(1/2)\eta T_0}$, where $T_0 \equiv \Delta T(0)$ is the initial soliton separation. Indeed, in the generic case, the above equations indicate that ΔT_{zz}, $\Delta\phi_{zz}$, and $\Delta\Phi_{zz}$ are of the order $e^{-\eta T_0}$. Thus, soliton parameters evolve on the slow distance scale $e^{(1/2)\eta T_0}$. In the special case when the leading-order terms in the amplitude and velocity equations (2.17) and (2.18) vanish, which happens for initially orthogonal solitons (see below), soliton parameters will evolve on the slow distance scale $e^{\eta T_0}$ instead. In this case, higher-order terms in the above evolution equations will be needed in order for the perturbation theory to be asymptotically correct. We will not pursue such higher-order corrections in this paper. Lastly, we note that when $\theta_1 = \theta_2 = 0$, the above equations reduce to those of the NLS equation [2–4].

The above ordinary differential equations (ODEs) can be solved analytically. Similar to the ODEs for the NLS equation, these ODEs have a complex constant of motion that we denote as Λ^2 [2,3],

$$(\Delta V - i\Delta\eta)^2 - 16\eta^2 e^{-\eta\Delta T}(\cos\theta_1\cos\theta_2 e^{i\Delta\phi} + \sin\theta_1\sin\theta_2 e^{i\Delta\Phi}) \equiv \Lambda^2. \tag{2.24}$$

Utilizing this conserved quantity, the equation for $\Delta V - i\Delta\eta$ can be simplified as

$$\frac{d}{dz}(\Delta V - i\Delta\eta) = \frac{1}{2}\eta[\Lambda^2 - (\Delta V - i\Delta\eta)^2], \tag{2.25}$$

whose solution is

$$\Delta V - i\Delta\eta = \Lambda\tanh\left(\frac{1}{2}\eta\Lambda z + C_0\right), \tag{2.26}$$

where C_0 is a complex constant that is determined from the initial conditions of ΔV and $\Delta\eta$. When ΔV has been obtained from the above formula, ΔT can be given from Eq. (2.16) as

$$\Delta T = T_0 + \frac{2}{\eta}\ln\left|\frac{\cosh\left(\frac{1}{2}\eta\Lambda z + C_0\right)}{\cosh C_0}\right|. \tag{2.27}$$

Together with Eq. (2.23), the positions $T_{1,2}$ for both solitons can be obtained for any distance z. When solutions (2.26) and (2.27) are substituted into the constant of motion (2.24), we find that the polarization angles $\theta_{1,2}$ and phase differences $(\Delta\phi, \Delta\Phi)$ satisfy the following complex-valued relation:

$$\cos\theta_1\cos\theta_2 e^{i\Delta\phi} + \sin\theta_1\sin\theta_2 e^{i\Delta\Phi} = -\frac{\Lambda^2 e^{\eta T_0}}{16\eta^2|\cosh C_0|^2}\frac{\cosh\left(\frac{1}{2}\eta\Lambda^* z + C_0^*\right)}{\cosh\left(\frac{1}{2}\eta\Lambda z + C_0\right)}, \tag{2.28}$$

where superscript * denotes complex conjugation. This relation itself is not enough to determine polarization angles and phase differences individually. However, in some special but important cases, we have succeeded in obtaining the analytical formulas for polarization angles and phase differences as well (see Sec. III).

Several interesting facts are worth mentioning here. If the Manakov solitons (2.3) and (2.4) are written in the compact vector form

$$\begin{pmatrix} A \\ B \end{pmatrix} = \eta\,\mathrm{sech}\,\eta(t - T)e^{iVt}\mathbf{c}, \tag{2.29}$$

where

$$\mathbf{c} = \begin{pmatrix} \cos\theta\, e^{-iVT + i\gamma} \\ \sin\theta\, e^{-iVT + i\Gamma} \end{pmatrix} \tag{2.30}$$

is the (complex) polarization vector [11], then under our previous assumptions (2.13) and notations (2.14), the inner product of polarization vectors $\langle \mathbf{c}_1, \mathbf{c}_2 \rangle$ of two Manakov solitons is

$$\langle \mathbf{c}_1, \mathbf{c}_2 \rangle \equiv \mathbf{c}_1^* \cdot \mathbf{c}_2 = \cos\theta_1 \cos\theta_2 e^{i\Delta\phi} + \sin\theta_1 \sin\theta_2 e^{i\Delta\Phi}. \tag{2.31}$$

It is easy to show from Eqs. (2.15) to (2.22), $\Delta V - i\Delta\eta$ and $\langle \mathbf{c}_1, \mathbf{c}_2 \rangle$ satisfy the following equations:

$$\frac{d}{dz}(\Delta V - i\Delta\eta) = -8\eta^3 e^{-\eta\Delta T}\langle \mathbf{c}_1, \mathbf{c}_2 \rangle, \tag{2.32}$$

$$\frac{d}{dz}\langle \mathbf{c}_1, \mathbf{c}_2 \rangle = i\eta\Delta\eta\langle \mathbf{c}_1, \mathbf{c}_2 \rangle. \tag{2.33}$$

In addition, the constant of motion (2.24) becomes

$$(\Delta V - i\Delta\eta)^2 - 16\eta^2 e^{-\eta\Delta T}\langle \mathbf{c}_1, \mathbf{c}_2 \rangle = \Lambda^2. \tag{2.34}$$

Some important conclusions readily follow from Eqs. (2.32) and (2.33). Two Manakov solitons are said to be orthogonal if the inner product $\langle \mathbf{c}_1, \mathbf{c}_2 \rangle$ of their polarization vectors is zero. Equations (2.32) and (2.33) indicate that, if the Manakov solitons are initially orthogonal, they will remain orthogonal forever. In addition, their velocity and amplitude differences ΔV and $\Delta\eta$ will remain constant throughout evolution. We remind the reader that when two Manakov solitons remain orthogonal, their polarization angles $\theta_{1,2}$ may still vary (see Fig. 4).

Defining the complex variable

$$Y \equiv -\eta\Delta T + \ln\langle \mathbf{c}_1, \mathbf{c}_2 \rangle, \tag{2.35}$$

then it can be shown from Eqs. (2.15) to (2.33) that the equation for Y is closed,

$$\frac{d^2 Y}{dz^2} = 8\eta^4 e^Y. \tag{2.36}$$

When $\theta_1 = \theta_2 = 0$, this equation reduces to that of the NLS system [4]. It can be readily generalized to the interaction of an arbitrary number of Manakov solitons as well. It is noted, however, that Eq. (2.36), together with the constant of motion (2.34), can only give the evolution of amplitude difference $\Delta\eta$, velocity difference ΔV, and inner product of polarization vectors $\langle \mathbf{c}_1, \mathbf{c}_2 \rangle$. The evolution equations for individual polarization angles $\theta_{1,2}$ and phase differences $(\Delta\phi, \Delta\Phi)$ still need to be provided separately [see Eqs. (2.19) to (2.22)].

III. INTERACTION OF MANAKOV SOLITONS WITH EQUAL AMPLITUDES

In this section, we consider the interaction of two Manakov solitons with equal amplitudes. In particular, we focus on two special but important cases where the two solitons initially have the same velocities and the same or opposite phases. In these cases, we can give the solution formulas for all soliton parameters.

The first case is when the two Manakov solitons initially have the same velocities and phases. In this case, by polarization rotation and normalization, the initial conditions for the ODEs can be written as

$$\eta(0) = 1, \quad \Delta\eta(0) = 0, \quad V(0) = 0, \quad \Delta V(0) = 0,$$
$$\theta_1(0) = 0, \quad \theta_2(0) = \theta_0, \quad \Delta T(0) = T_0, \quad \Delta\phi(0) = 0, \tag{3.1}$$

where θ_0 is the initial polarization-angle difference and T_0 is the initial soliton separation. It is noted that since $\theta_1(0) = 0$, the initial phase difference $\Delta\Phi(0)$ can be arbitrary, and it does not affect the solution. When $\theta_0 = 0$, the two solitons are parallel; when $\theta_0 = \pi/2$ (90°), the solitons are orthogonal. Under the above initial conditions, the complete analytical solutions are found to be

$$\Delta\eta(z) = T(z) = 0, \tag{3.2}$$

$$\Delta T(z) = T_0 + 2\ln|\cos\beta z|, \tag{3.3}$$

$$\sin\theta_1(z) = \sin\theta_0 \sin(\cos^{-(1/2)}\theta_0 e^{-(1/2)T_0}\tan\beta z), \tag{3.4}$$

$$\sin\theta_2(z) = \sin\theta_0 \cos(\cos^{-(1/2)}\theta_0 e^{-(1/2)T_0}\tan\beta z), \tag{3.5}$$

$$\sin\Delta\phi(z) = \tan\theta_1(z)\tan\theta_2(z), \tag{3.6}$$

$$\Delta\Phi(z) = -\frac{\pi}{2}, \tag{3.7}$$

where

$$\beta = 2\sqrt{\cos\theta_0}\, e^{-(1/2)T_0}. \tag{3.8}$$

We see from these analytical solutions that the two solitons attract each other. At distance

$$Z_c = \frac{\pi}{2\beta} = \frac{\pi}{4}\cos^{-(1/2)}\theta_0 e^{(1/2)T_0}, \tag{3.9}$$

a singularity develops. In the literature, this distance sometimes is called the collision distance. At $\theta_0 = 0°$,

$$Z_c = \frac{\pi}{4}e^{(1/2)T_0}, \tag{3.10}$$

which has been derived before [2,3,28]. When θ_0 increases from 0° to 90°, the collision distance steadily increases. Thus neighboring-soliton interference is reduced. When $\theta_0 = 90°$, the analytical solutions become

$$\Delta\eta(z) = T(z) = 0, \quad \Delta T(z) = T_0, \tag{3.11}$$

$$\theta_1(z) = 2e^{-T_0}z, \quad \theta_2(z) = \frac{\pi}{2} - 2e^{-T_0}z, \tag{3.12}$$

$$\Delta\phi(z) = \frac{\pi}{2}, \quad \Delta\Phi(z) = -\frac{\pi}{2}. \tag{3.13}$$

Thus in this case, the two solitons do not move toward each other, but their polarizations linearly change over distance. If we define the collision distance Z_c here as the distance where soliton polarizations are changed by 90°, then

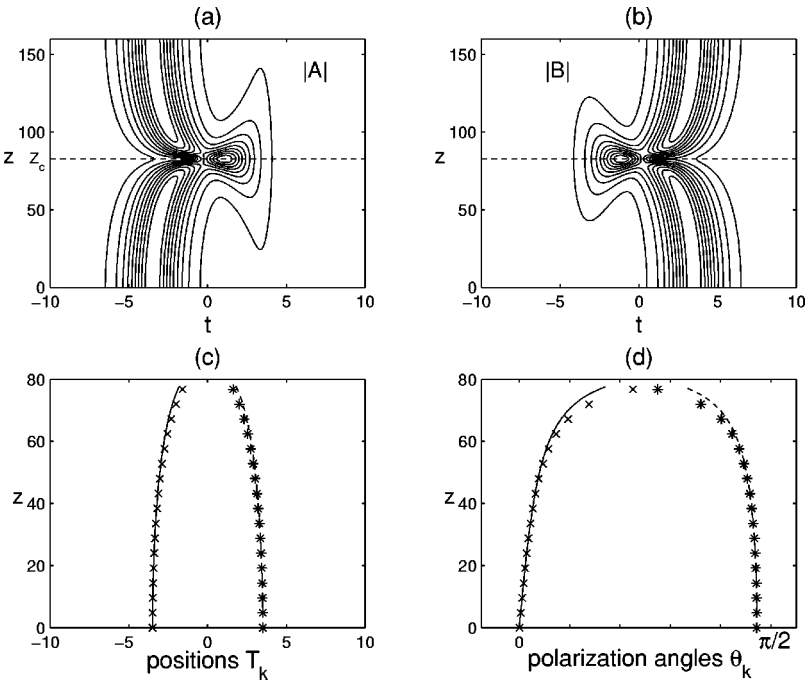

FIG. 1. Interaction of Manakov solitons initially with equal amplitudes, same phases but different polarizations. The initial condition is Eq. (3.1) where $\theta_0=85°$ and $T_0=7$. (a,b): contours of $|A|$ and $|B|$ solutions (numerical simulations); (c,d): soliton positions $T_{1,2}$ and polarization angles $\theta_{1,2}$. Crosses and stars are numerical values, and solid and dashed curves are analytical solutions (3.2)–(3.5).

$$Z_c=\frac{\pi}{4}e^{T_0}. \tag{3.14}$$

This distance is of the order of the square of the collision distance (3.10) at parallel polarizations when T_0 is large.

The case of initially orthogonal polarizations is quite special. In this case, the right-hand sides of the amplitude and velocity equations (2.17) and (2.18) identically vanish for all distances. Because of this, the higher-order terms $[O(e^{-2T_0})]$ become significant and included in these equations. Without them, we do not expect the leading-order equations derived above to be asymptotically accurate. However, since the soliton parameters in this case evolve on the distance scale e^{T_0} (see explanation above), the collision distance is naturally of this order as well. Thus our formula (3.14) is still qualitatively correct.

The second case is when the two solitons initially have the same velocities, but opposite phases. In this case, the initial conditions are the same as Eq. (3.1) except that $\Delta\phi(0)=\pi$ now. The complete ODE solutions for this case are

$$\Delta T(z)=T_0+2\ln\cosh(\beta z), \tag{3.15}$$

$$\sin\theta_1(z)=\sin\theta_0\sin(\cos^{-(1/2)}\theta_0 e^{-(1/2)T_0}\tanh\beta z), \tag{3.16}$$

$$\sin\theta_2(z)=\sin\theta_0\cos(\cos^{-(1/2)}\theta_0 e^{-(1/2)T_0}\tanh\beta z), \tag{3.17}$$

and solutions for the other variables as well as the constant β are the same as in the first case. In this second case, the two solitons always repel each other. When $\theta_0=0°$, the repulsion is the strongest. When θ_0 increases from 0° to 90°, the repulsion becomes weaker and the solitons separate slower.

To check these analytical results, we have simulated the Manakov equations (2.1) and (2.2) numerically using the pseudospectral method. First, we consider one particular case with initial condition (3.1) where the polarization angle $\theta_0=85°$, and initial separation $T_0=7$. The contour plots of $|A|$ and $|B|$ components are shown in Figs. 1(a,b), respectively. As expected from the analysis, the two solitons attract each other and form a bound state that is periodic in z. We define the collision distance in the simulation results as one-half of the bound-state's period. Then this distance for Figs. 1(a,b) is $Z_c=82.7$, which is marked in Fig. 1(a). The theoretical value from formula (3.9) is $Z_c=88.1$, which is quite close to the numerical value. In Figs. 1(c,d), we plotted the positions T_n and polarization angles $\theta_n(n=1,2)$ of both solitons at various distances (crosses and stars). The analytical solutions from Eqs. (3.2) to (3.5) are also plotted for comparison. As we can see, the agreement is very good until the two solitons get very close to each other.

Next, we consider the dependence of the collision distance Z_c on initial separation T_0 and initial polarization angle θ_0 in the initial condition (3.1). At three selected polarization angles $\theta_0=0°$, 85°, and 90°, we have numerically determined the collision distances at various initial separations. The results are plotted in Fig. 2 as stars, crosses, and triangles respectively. In the same figure, the analytical formulas (3.9) and (3.14) are also plotted for comparison (solid lines). We see that at $\theta_0=0°$ and 85°, the numerical and theoretical collision distances agree very well. This is expected, as our first-order perturbation theory is asymptotically accurate for $T_0\gg 1$ as long as $\theta_0\neq 90°$. In these two cases, the Z_c versus T_0 curve in log scale is a straight line with slope $\frac{1}{2}$ at large T_0 values, in agreement with formula (3.9). When θ_0 is very close but not equal to 90°, the numerical and analytical Z_c values start to diverge quantitatively at small and moderate T_0 values, but they still ap-

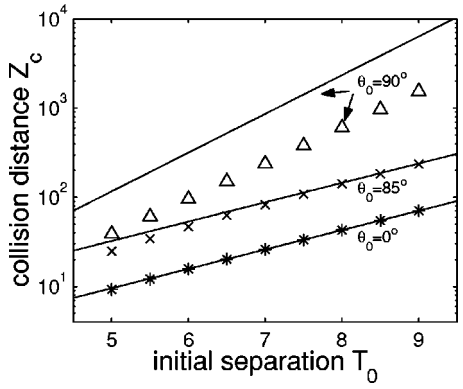

FIG. 2. Dependence of the collision distance Z_c on initial separation T_0 and polarization angle θ_0 in the interaction of initially equal-amplitude and same-phase Manakov solitons. The initial condition is given in Eq. (3.1). The stars, crosses, and triangles are numerical values for $\theta_0=0°$, 85°, and 90°, respectively. The solid lines are analytical formulas (3.9) and (3.14).

proach each other at large T_0 values. When $\theta_0=90°$, the analytical formula (3.14) predicts the $\log Z_c$ slope to be 1. Numerically, this slope is also approximately 1 (see Fig. 2). Thus the numerical and analytical values are in qualitative agreement. Note that the $\log Z_c$ slope at $\theta_0=90°$ is double of that at $\theta_0=0°$, i.e., the collision distance at orthogonal polarizations is of the order of the square of that at parallel polarizations. Thus, the PDM technique would double the transmission capacity compared to parallel-polarization pulse launching. This fact has been demonstrated in the experiment of [14] already. It is noted that when $\theta_0=90°$, quantitative disagreement between numerics and theory persists even at large T_0 values. The reason for this quantitative difference is due to the absence of higher-order terms in our ODE equations (2.17) and (2.18). This has been explained in the text above.

IV. INTERACTION OF MANAKOV SOLITONS AT UNEQUAL AMPLITUDES

In this section, we investigate how the amplitude difference of Manakov solitons affects their interactions. Here again, we consider two special but important cases where the initial solitons have the same velocities and the same or opposite phases.

In the first case, the two Manakov solitons have the same initial velocities and phases. By normalization, the initial conditions can be written as

$$\eta(0)=1, \quad \Delta\eta(0)=\eta_0, \quad V(0)=0, \quad \Delta V(0)=0,$$

$$\theta_1(0)=0, \quad \theta_2(0)=\theta_0, \quad \Delta T(0)=T_0, \quad \Delta\phi(0)=0, \tag{4.1}$$

where η_0 is the initial amplitude difference, θ_0 is the initial polarization difference, and T_0 is the initial soliton separation. In this case, the solution (2.26) for $\Delta V-i\Delta\eta$ becomes

$$\Delta V-i\Delta\eta=i\bar{\Lambda}\tanh\left(\frac{1}{2}i\bar{\Lambda}z+C_0\right), \tag{4.2}$$

where

$$\bar{\Lambda}=\sqrt{\eta_0^2+16e^{-T_0}\cos\theta_0},$$

$$C_0=-\operatorname{arctanh}\frac{\eta_0}{\sqrt{\eta_0^2+16e^{-T_0}\cos\theta_0}}. \tag{4.3}$$

Equation (4.2), together with Eq. (2.15), indicates that both the velocities $V_{1,2}$ and amplitudes $\eta_{1,2}$ of the two Manakov solitons are periodic functions of distance z with period

$$Z_p=\frac{2\pi}{\sqrt{\eta_0^2+16e^{-T_0}\cos\theta_0}}. \tag{4.4}$$

Naturally, the soliton positions $T_{1,2}$ are also periodic functions with the same period [see Eqs. (2.16), (2.23), and (2.27)]. Note that this period approaches $(2\pi/\eta_0)$ as T_0 approaches infinity. When amplitude difference η_0 goes to zero, this period approaches $2Z_c$, where Z_c is the collision distance given by Eq. (3.9).

Since the soliton positions $T_{1,2}$ now are periodic functions of distance z, these Manakov solitons form a quasiequidistant bound state. Recall that in the NLS case ($\theta_0=0$), solitons of different amplitudes also form quasiequidistant bound states [2–10]. Thus our present results show that nonparallel polarizations of Manakov solitons still preserve quasiequidistant bound states. Intuitively, the formation of quasiequidistant bound states by both the NLS solitons and Manakov solitons of different amplitudes is easy to understand. As is well known, if two NLS solitons have the same phase, they attract each other; if they have opposite phases, they repel each other. The same picture holds for Manakov solitons [23,24]. When two NLS or Manakov solitons of different amplitudes interact with each other, their phase differences roughly change linearly with distance z as $\eta\Delta\eta z$ [see Eqs. (2.21) and (2.22)]. Thus, the attracting and repelling forces experienced by these solitons when they have the same and opposite phases cancel out, hence quasiequidistant bound states are formed. In this quasiequidistant state, amounts of position and amplitude oscillations of individual solitons are important quantities. To minimize soliton interference, these oscillations should be as small as possible. A natural question we ask is how initial polarizations of Manakov solitons affect the degrees of such oscillations. To answer this question, we note from Eq. (2.27) that the soliton separation ΔT now is

$$\Delta T(z)=\Delta T_0+\ln\frac{\eta_0^2+8e^{-T_0}\cos\theta_0(1+\cos\bar{\Lambda}z)}{\eta_0^2+16e^{-T_0}\cos\theta_0}, \tag{4.5}$$

where constant $\bar{\Lambda}$ is given in Eq. (4.3). Thus the amount of position-separation oscillations is

$$\Delta T_{\max} - \Delta T_{\min} = \ln\left(1 + \frac{16e^{-T_0}\cos\theta_0}{\eta_0^2}\right). \tag{4.6}$$

The amount of average-position (T) oscillation can be calculated from Eqs. (4.2) and (2.23), and the result is

$$T_{\max} - T_{\min} = \frac{4e^{-T_0}\cos\theta_0}{|\eta_0|}. \tag{4.7}$$

The amount of oscillations in the amplitude difference $\Delta\eta$ can be derived from Eq. (4.2) as

$$\Delta\eta_{\max} - \Delta\eta_{\min} = \frac{16e^{-T_0}\cos\theta_0}{|\eta_0|}. \tag{4.8}$$

From these equations, the amounts of amplitude and position oscillations in individual Manakov solitons can be easily inferred. It is clear from the above three equations that as θ_0 increases from 0 to $\pi/2$, amount of position and amplitude oscillations diminish. Thus increasing the initial polarization difference of Manakov solitons reduces their interference. In particular, at orthogonal polarizations ($\theta_0 = \pi/2$), these oscillations are completely suppressed. So combining the PDM technique with amplitude unequalization technique can reduce Manakov-soliton interference drastically. When η_0 approaches zero (the equal-amplitude limit), soliton separation formula (4.5) reduces to Eq. (3.3) of the equal-amplitude case. Under this limit, Eq. (4.5) indicates that if the Manakov solitons initially are not orthogonal ($\theta_0 \neq \pi/2$), then soliton separation ΔT would decrease from its initial positive value T_0 to large negative values, thus the amount of position-separation oscillations approaches infinity [see Eq. (4.6)]. However, we need to keep in mind that one of the basic assumptions of our perturbation theory is that $\eta\Delta T \gg 1$ [see Eq. (2.13)]. So when ΔT decreases to zero or even further, that assumption breaks down, hence, care is needed to interpret the perturbation results. Direct numerical simulations of the Manakov equations show that solitons do approach each other and coalesce (ΔT decreasing to 0), then form a z-periodic bound state. But the amount of position-separation oscillations never goes to infinity as formula (4.6) suggests. Similar care is needed to interpret Eq. (4.8) under the limit $\eta_0 \to 0$. Recall that another assumption of our perturbation theory is that $\Delta\eta \ll \eta$. If η_0 is small, but $\Delta\eta_{\max} - \Delta\eta_{\min}$ from Eq. (4.8) is still much smaller than 1, then we have found that formula (4.8) agrees very well with the numerical simulation results. But when η_0 approaches zero so that $\Delta\eta_{\max} - \Delta\eta_{\min}$ from Eq. (4.8) becomes $O(1)$ or larger, then formula (4.8) breaks down.

How do polarizations of Manakov solitons vary in these quasiequidistant bound states? Unlike the amplitudes and positions, polarization angles of Manakov solitons are not periodic functions of distance z in general. On top of oscillations at the same frequency as amplitudes' and positions', the mean values of polarizations change as well. But both the oscillations and mean-value changes are rather weak compared to Manakov solitons at equal amplitudes (see Fig. 1). For instance, when $\eta_0 = 0.1$, $\theta_0 = 60°$, and $T_0 = 7$ in the initial condition (4.1), the changes of the mean values of θ_1 and θ_2 at $z = 300$ are just about 10^{-3} and 10^{-4}, respectively. The amplitudes of oscillations around the mean values are both approximately 0.016. As θ_0 approaches 90°, polarization functions $\theta_{1,2}(z)$ become strictly periodic, but their oscillations still remain weak (see Fig. 4). From these results, we conclude that in the quasiequidistant bound states of Manakov solitons at unequal amplitudes, variations of polarization angles are small and insignificant.

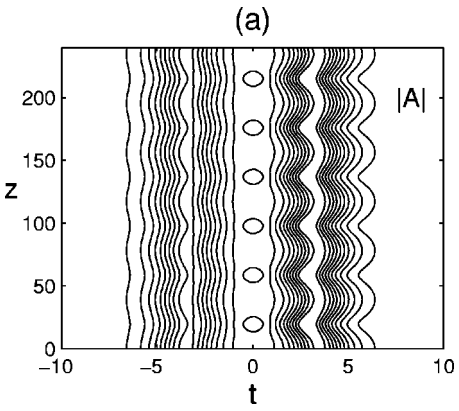

FIG. 3. Interaction of Manakov solitons initially with unequal amplitudes, same phases, and parallel polarizations (numerical simulation). The initial conditions are Eq. (4.1) where $\eta_0 = 0.1$, $T_0 = 7$, and $\theta_0 = 0°$.

Next, we compare the above analytical results with direct numerical simulations. Our main analytical result is that amplitude and position oscillations in the quasiequidistant bound states of unequal Manakov solitons are drastically reduced when the initial solitons become more orthogonal (θ_0 closer to 90°). To check this result, we have simulated the Manakov equations (2.1) and (2.2) with $\eta_0 = 0.1$, $T_0 = 7$ and various θ_0 values in the initial condition (4.1). First, we show in Fig. 3 the numerical results at $\theta_0 = 0°$ where initial Manakov solitons are parallel (scaler case). It is observed that a quasiequidistant bound state is formed. The position and amplitude oscillations of individual solitons in this state are significant. When the initial solitons are made orthogonal ($\theta_0 = 90°$), the numerical results are presented in Figs. 4(a,b). As one can see, in this case, the position and amplitude oscillations are almost invisible, thus the interference of Manakov solitons is drastically suppressed. These results are in good agreement with the above analytical predictions. In Figs. 4(c,d), we plotted the numerically obtained polarization-angle evolutions of individual solitons as crosses. These polarization functions are periodic in distance z, but their variations along distance are very small and quite insignificant. In the same figures, the analytical polarization angles obtained by numerically integrating the ODEs (2.16) to (2.22) are shown as solid curves for comparison. The agreement is quite reasonable.

In the following, we quantitatively investigate how the polarization angles θ_0 and amplitude difference η_0 affect the amount of position-separation oscillations of Manakov solitons, $\Delta T_{\max} - \Delta T_{\min}$. Here we fix the initial soliton separation $T_0 = 7$. At each of the three polarization angles θ_0

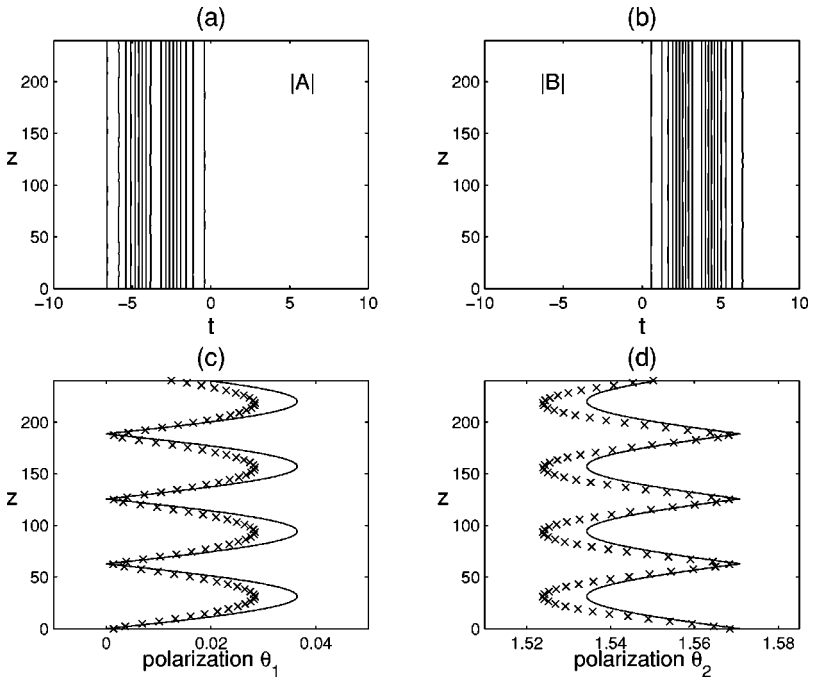

FIG. 4. Interaction of Manakov solitons initially with unequal amplitudes, same phases, and orthogonal polarizations. The initial conditions are Eq. (4.1) where $\eta_0=0.1$, $T_0=7$, and $\theta_0=90°$. (a,b): contours of $|A|$ and $|B|$ solutions (numerical simulations); (c,d): polarization angles θ_1 and θ_2. Crosses are numerical simulation results, and the solid lines are analytical predictions by integrating the ODE systems (2.16)–(2.22).

$=0°$, 60°, and 90°, and at various amplitude-difference values η_0, we have run numerical simulations and recorded positions of both Manakov solitons at each distance z. The difference between the largest and smallest separations, $\Delta T_{\max}-\Delta T_{\min}$, is recorded for each simulation. The results are shown in Fig. 5 as stars, crosses, and triangles for $\theta_0=0°$, 60°, and 90°, respectively. The analytical formula (4.6) for this quantity is also shown as solid curves for comparison. We see that the numerical results agree with the analytical formula quite well. In particular, it is confirmed numerically that orthogonal polarizations almost completely suppress the interference of unequal-amplitude Manakov solitons. Thus, a combination of launching Manakov solitons along orthogonal polarizations and at different amplitudes is most effective in reducing soliton-soliton interactions. Figure 5 also shows that at a fixed polarization angle θ_0, position oscillations of Manakov solitons decrease as amplitude difference η_0 increases. At moderate and large amplitude difference η_0, the quantitative disagreement between theory and numerics becomes more pronounced. This is understandable, as our analysis is based on the assumption that amplitudes of Manakov solitons are almost equal. When η_0 is moderate or large, that assumption breaks down, thus the analytical prediction becomes less accurate.

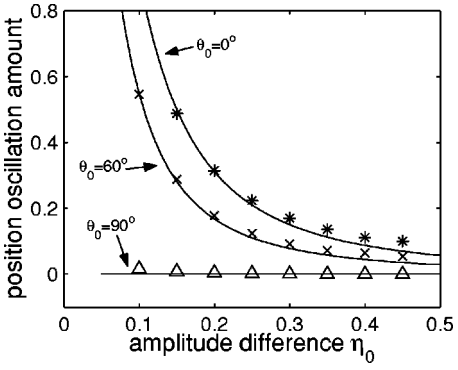

FIG. 5. Dependence of the amount of position oscillations on initial polarization θ_0 and amplitude difference η_0 in the interaction of two Manakov solitons initially with unequal amplitudes and same phases. Here the initial condition is Eq. (4.1) with $T_0=7$. The vertical axis is $\Delta T_{\max}-\Delta T_{\min}$—difference between the largest and smallest soliton separations. Stars, crosses, and triangles are this quantity at initial polarizations $\theta_0=0°$, 60°, and 90°, respectively. The solid lines are analytical formulas (4.6).

In the remaining section, we briefly discuss the interaction of Manakov solitons with unequal amplitudes but opposite phases. The initial condition for this case is Eq. (4.1) except that $\Delta\phi(0)=\pi$ now. In this case, the constant of motion (2.24) is

$$\Lambda^2=16e^{-T_0}\cos\theta_0-\eta_0^2. \tag{4.9}$$

When $|\eta_0|<\eta_c$ where the critical value η_c is defined as

$$\eta_c\equiv 4e^{-(1/2)T_0}\sqrt{\cos\theta_0}, \tag{4.10}$$

constant Λ is real, thus according to formula (2.27), the two Manakov solitons would repel each other. But when $|\eta_0|>\eta_c$, Λ is purely imaginary, hence the two solitons would again form a quasiequidistant bound state. These behaviors are analogous to those in the scaler NLS case. The main difference is that the critical value η_c now depends on the initial polarization angle θ_0. As θ_0 increases from 0° to 90°, the threshold η_c decreases, thus solitons are more likely to form quasiequidistant states. For fixed values of amplitude difference η_0 and initial separation T_0, larger polarization angles θ_0 reduce the escape velocities of solitons in the case of eventual separation, and reduce the position and amplitude oscillations in the case of quasi equidistant bound-state for-

mation. Thus, interference of Manakov solitons is always reduced when these solitons are more orthogonally polarized.

V. SUMMARY AND DISCUSSION

In this paper, we have studied the interaction of two Manakov solitons by the Karpman-Solov'ev perturbation method. Under the assumption that these solitons initially are well separated and having nearly the same amplitudes and velocities but arbitrary polarizations, we have derived a reduced set of ordinary differential equations for both solitons' parameters. We then solved this reduced system analytically. Our analytical solutions show that, when two Manakov solitons have the same amplitude and phases, their collision distance steadily increases as their initial polarizations change from parallel to orthogonal. When these solitons have different amplitudes, a quasiequidistant bound state can be formed. The degrees of position and amplitude oscillations in this bound state diminish as the initial polarizations change from parallel to orthogonal. With a combination of launching Manakov solitons along orthogonal polarizations and at different amplitudes, interference of Manakov solitons is almost completely eliminated. These analytical results are confirmed by our direct numerical simulations.

In real fiber communication systems, WDM technology is often utilized. When WDM is combined with PDM, collisions of solitons in different channels cause depolarizations to solitons in the same channel. Thus orthogonalization of neighboring solitons in the same channel cannot be maintained. But polarizations of neighboring solitons are still far from parallel in the general case. Since our results show that soliton interference is always the worst when their polarizations are parallel, we conclude that, as far as soliton-soliton interference is concerned, PDM technology still is beneficial even when it is combined with WDM technology. Recently, a popular technology in telecommunication systems is to launch pulses of adjacent channels along orthogonal polarizations (see [29] for instance). This technology is a modification of PDM. The pulses in such transmission systems generally are not solitons. But the idea of using orthogonal polarizations to reduce nonlinear pulse-pulse interference between different channels is still consistent with our analytical result that orthogonal polarizations strongly suppress nonlinear interactions of Manakov solitons. Another result of ours is that Manakov-soliton interference is almost completely eliminated when PDM is combined with amplitude nonequalization. It would be interesting to see how well this combination works in real transmission experiments. We also note that in communication systems, various perturbations to the Manakov system such as those due to polarization-mode dispersion and amplifier noise are present [1,2,15]. The effects of such perturbations on Manakov-soliton interactions can be analyzed by the same perturbation method as used in this paper. But this question falls outside the scope of the present paper.

ACKNOWLEDGMENTS

This work was supported in part by the Air Force Office of Scientific Research under Contract No. F49620-99-1-0174, and by the National Science Foundation under Grant No. DMS-9971712.

APPENDIX

In this appendix, we derive the dynamical equations (2.15)–(2.23) for the interaction of two Manakov solitons. These solitons have the form

$$A_k(t,z)=\eta_k\cos\theta_k\,\mathrm{sech}\,\eta_k(t-T_k)\exp[iV_k(t-T_k)+i\gamma_k], \tag{A1}$$

$$B_k(t,z)=\eta_k\sin\theta_k\,\mathrm{sech}\,\eta_k(t-T_k)\exp[iV_k(t-T_k)+i\Gamma_k], \tag{A2}$$

$$T_k=\int_0^z V_k dz+\overline{T}_k,\quad \gamma_k=\int_0^z\frac{1}{2}(\eta_k^2+V_k^2)dz+\overline{\gamma}_k,$$

$$\Gamma_k=\int_0^z\frac{1}{2}(\eta_k^2+V_k^2)dz+\overline{\Gamma}_k, \tag{A3}$$

where amplitudes η_k, velocities V_k, polarization angles θ_k, position parameters $\overline{T}_k$, and phases $\overline{\gamma}_k$ and $\overline{\Gamma}_k$ $(k=1,2)$ are all slowly varying over distance z. The slow evolution of these parameters can be determined by a perturbation theory for the Manakov equations (2.7) and (2.8), which has been developed in [24–27]. From this theory, the dynamical equations for each soliton's parameters can be readily written. After simple calculations of relevant integrals that account for tail interactions of Manakov solitons, the dynamical equations become

$$\frac{d\eta_k}{dz}=(-1)^k 4\eta^3 e^{-\eta\Delta T}(\cos\theta_1\cos\theta_2\sin\Delta\phi+\sin\theta_1\sin\theta_2\sin\Delta\Phi), \tag{A4}$$

$$\frac{dV_k}{dz}=(-1)^{k+1}4\eta^3 e^{-\eta\Delta T}(\cos\theta_1\cos\theta_2\cos\Delta\phi+\sin\theta_1\sin\theta_2\cos\Delta\Phi), \tag{A5}$$

$$\frac{d\theta_k}{dz}=(-1)^{k+1}2\eta^2 e^{-\eta\Delta T}(\sin\theta_k\cos\theta_{3-k}\sin\Delta\phi-\cos\theta_k\sin\theta_{3-k}\sin\Delta\Phi), \tag{A6}$$

$$\begin{aligned}\frac{d\gamma_k}{dz}=&\frac{1}{2}(\eta_k^2+V_k^2)+6\eta^2 e^{-\eta\Delta T}(\cos\theta_1\cos\theta_2\cos\Delta\phi\\&+\sin\theta_1\sin\theta_2\cos\Delta\Phi)\\&-2\eta V e^{-\eta\Delta T}(\cos\theta_1\cos\theta_2\sin\Delta\phi\\&+\sin\theta_1\sin\theta_2\sin\Delta\Phi)\\&+2\eta^2 e^{-\eta\Delta T}\tan\theta_k(\sin\theta_k\cos\theta_{3-k}\cos\Delta\phi\\&-\cos\theta_k\sin\theta_{3-k}\cos\Delta\Phi),\end{aligned} \tag{A7}$$

$$\frac{d\Gamma_k}{dz}=\frac{1}{2}(\eta_k^2+V_k^2)+6\eta^2 e^{-\eta\Delta T}(\cos\theta_1\cos\theta_2\cos\Delta\phi+\sin\theta_1\sin\theta_2\cos\Delta\Phi)-2\eta V e^{-\eta\Delta T}(\cos\theta_1\cos\theta_2\sin\Delta\phi+\sin\theta_1\sin\theta_2\sin\Delta\Phi)-2\eta^2 e^{-\eta\Delta T}\cot\theta_k(\sin\theta_k\cos\theta_{3-k}\cos\Delta\phi-\cos\theta_k\sin\theta_{3-k}\cos\Delta\Phi), \tag{A8}$$

$$\frac{dT_k}{dz}=V_k-2\eta e^{-\eta\Delta T}(\cos\theta_1\cos\theta_2\sin\Delta\phi+\sin\theta_1\sin\theta_2\sin\Delta\Phi), \tag{A9}$$

where $k=1,2$, and variables η, ΔT, $\Delta\phi$, $\Delta\Phi$ have been defined in Eqs. (2.11), (2.12), and (2.14). Using these equations, the dynamical equations (2.15)–(2.23) naturally follow.

[1] G.P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, San Diego, 1989).
[2] A. Hasegawa and Y. Kodama, *Solitons in Optical Communications* (Clarendon, Oxford, 1995).
[3] V.I. Karpman and V.V. Solov'ev, Physica D **3**, 142 (1981).
[4] K.A. Gorshkov and L.A. Ostrovsky, Physica D **3**, 428 (1981).
[5] J. Satsuma and N. Yajima, Prog. Theor. Phys. **55**, 284 (1974).
[6] Y. Kodama and K. Nozaki, Opt. Lett. **12**, 1038 (1987).
[7] C. Desem and P.L. Chu, IEE Proc.-J: Optoelectron. **134**, 145 (1987).
[8] C. Desem and P.L. Chu, in *Optical Solitons—Theory and Experiments*, edited by J.R. Taylor (Cambridge University Press, Cambridge, 1992), Chap. 5.
[9] V.S. Gerdjikov, I.M. Uzunov, E.G. Evstatiev, and G.L. Diankov, Phys. Rev. E **55**, 6039 (1997).
[10] V.S. Gerdjikov, E.G. Evstatiev, D.J. Kaup, G.L. Diankov, and I.M. Uzunov, Phys. Lett. A **241**, 323 (1998).
[11] S.V. Manakov, Zh. Eksp. Teor. Fiz **65**, 1392 (1973) [Sov. Phys. JETP **38**, 248 (1974)].
[12] C.R. Menyuk, IEEE J. Quantum Electron. **QE-23**, 174 (1987).
[13] P.K.A. Wai, C.R. Menyuk, and H.H. Chen, Opt. Lett. **16**, 1735 (1991).
[14] S.G. Evangelides, L.F. Mollenauer, J.P. Gordon, and N.S. Bergano, J. Lightwave Technol. **10**, 28 (1992).
[15] P.K.A. Wai and C.R. Menyuk, J. Lightwave Technol. **14**, 148 (1996).
[16] F. Matera, M. Romagnoli, and B. Daino, Electron. Lett. **31**, 1172 (1995).
[17] C.D. Angelis and S. Wabnitz, Opt. Commun. **125**, 186 (1996).
[18] L.F. Mollenauer, J.P. Gordon, and F. Heismann, Opt. Lett. **20**, 2060 (1995).
[19] J.P. Silmon-Clyde and J.N. Elgin, Opt. Lett. **23**, 180 (1998).
[20] X. Zhang, M. Karlsson, P.A. Andrekson, and Erling Kolltveit, IEEE Photonics Technol. Lett. **10**, 1742 (1998).
[21] V.S. Gerdjikov, e-print arXiv:nlin.SI/0009036.
[22] S. Wabnitz, Opt. Lett. **20**, 261 (1995).
[23] M. Haelterman, A.P. Sheppard, and A.W. Snyder, Opt. Lett. **18**, 1406 (1993).
[24] J. Yang, Phys. Rev. E **64**, 026607 (2001).
[25] T.I. Lakoba and D.J. Kaup, Phys. Rev. E **56**, 6147 (1997).
[26] M. Midrio and S. Wabnitz, Phys. Rev. E **54**, 5743 (1996).
[27] V.S. Shchesnovich and E.V. Doktorov, Phys. Rev. E **55**, 7626 (1997).
[28] K.J. Blow and N.J. Doran, Electron. Lett. **19**, 429 (1983).
[29] E.A. Golovchenko, A.N. Pilipetskii, and N.S. Bergano, in *Optical Fiber Communication Conference*, OSA Technical Digest (Optical Society of America, Washington, D.C., 2000).