



THE UNIVERSITY OF VERMONT
DEPARTMENT OF MATHEMATICS AND STATISTICS
SIXTY-FIRST ANNUAL HIGH SCHOOL PRIZE EXAMINATION
MARCH 14, 2018

THIS EXAMINATION IS TO BE ADMINISTERED ON WEDNESDAY, MARCH 14, 2018 BEGINNING BETWEEN 8:00 AM AND 1:00 PM. AN EXAMINATION GIVEN AT ANY OTHER TIME WILL BE DISQUALIFIED.

THE TIME LIMIT ON THIS EXAMINATION IS 2 HOURS.

INSTRUCTIONS TO THE CONTESTANTS:

Do not begin the examination until the examiner tells you to do so.

The answer sheet is on the reverse side of this page. Before beginning the examination, carefully print your full name, your address, the complete name of your school and the town/city in which your school is located on the appropriate lines of the answer sheet. Check the circle corresponding to your grade level in school.

Answers must be written on the answer sheet in pencil or ink. The answer sheets will be collected at the end of the examination. You may keep the examination questions. If you would like to retain a copy of your answers, record them on a separate piece of paper. You may work on problems in any order, but be sure that each answer is entered in the proper space on the answer sheet. (For example, if you solve number 12 first, make sure the answer is placed beside the 12 on the answer sheet.) All questions are weighted equally. Answer as many questions as you can in the allotted time. No contestant is expected to solve all of the problems.

CALCULATORS, COMPUTERS AND/OR ANY OTHER ELECTRONIC DEVICES ARE NOT PERMITTED.

UNLESS OTHERWISE INDICATED, ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST FORM.

A radical expression of index n is in simplest form if the radicand is not a fraction, denominators are rationalized and integer radicands do not have any factors that are n th powers of a prime. For example, $\sqrt{\frac{5}{12}}$ simplifies to $\frac{\sqrt{15}}{6}$.

Do NOT approximate the number π .

Do NOT approximate radicals.

The notation **log** is logarithm to the base 10.

The notation **log_a** is logarithm to the base a . The notation **ln** is logarithm to the base e .

The symbol **!** is the factorial symbol. For example, $3! = 3 \cdot 2 \cdot 1 = 6$.

The symbol ***i*** is the complex unit $\sqrt{-1}$.

All numbers are in base 10 unless otherwise indicated (e.g., 1001_2 is the base 2 representation of the decimal number 9).

Any answer which is a non-integer rational number must be expressed in the form $\frac{a}{b}$, where a and b are integers that have no common divisor other than 1.

ANSWER SHEET (2018)

PLEASE PRINT CLEARLYSTUDENT'S FULL NAME _____ ANSWERS _____

STUDENT'S ADDRESS _____

NAME OF SCHOOL _____

TOWN (OR CITY) OF SCHOOL _____

WHAT GRADE ARE YOU IN? 9th 10th 11th 12th Other _____

1. _____ -32 _____

2. _____ 11/13 _____

3. _____ 106 _____ square units

4. _____ 40 _____ seats

5. _____ 22 _____ integers

6. _____ 20 _____

7. _____ 5 _____

8. _____ 1/11 _____

9. _____ 23 _____

10. _____ 14 _____ pounds

11. _____ 46 _____

12. _____ 1/2020 _____

13. _____ 5/32 _____

14. _____ 24 _____ mph

15. _____ 16 _____

16. _____ 204 _____ squares

17. _____ 12/5 _____

18. _____ 4 _____

19. _____ 60 _____ square cm

20. _____ 30 _____ integers

21. _____ 2/3 _____ square units

22. _____ $21\sqrt{10}$ _____ cubic units

23. _____ $1+3i, 4+3i$ _____

24. _____ -9/16 _____

25. _____ $(1 + \sqrt{2}, -1), (1 - \sqrt{2}, -1)$ _____

26. _____ 216 _____

27. _____ 360/11 _____ minutes

28. _____ -6, 9, 19 _____

29. _____ 27 _____

30. _____ -32/9 _____

31. _____ $\sqrt{15}$ _____

32. _____ 5998 _____ integers

33. _____ -3 _____

34. _____ 260 _____ paths

35. _____ 8/5 _____ units

36. _____ 14 _____

37. _____ 5/2 _____ units

38. _____ 96 _____ pairs

39. _____ 49/25 _____

40. _____ 177 _____

41. _____ 7/30 _____