THE UNIVERSITY OF VERMONT
DEPARTMENT OF MATHEMATICS AND STATISTICS
SIXTY-THIRD ANNUAL HIGH SCHOOL PRIZE EXAMINATION
MARCH 10, 2020

THIS EXAMINATION IS TO BE ADMINISTERED ON WEDNESDAY, MARCH 10, 2020 BEGINNING BETWEEN 8:00 AM AND 1:00 PM. AN EXAMINATION GIVEN AT ANY OTHER TIME WILL BE DISQUALIFIED.

THE TIME LIMIT ON THIS EXAMINATION IS 2 HOURS.

INSTRUCTIONS TO THE CONTESTANTS:

Do not begin the examination until the examiner tells you to do so.

The answer sheet is on the reverse side of this page. Before beginning the examination, carefully print your full name, your address, the complete name of your school and the town/city in which your school is located on the appropriate lines of the answer sheet. Check the circle corresponding to your grade level in school.

Answers must be written on the answer sheet in pencil or ink. The answer sheets will be collected at the end of the examination. You may keep the examination questions. If you would like to retain a copy of your answers, record them on a separate piece of paper. You may work on problems in any order, but be sure that each answer is entered in the proper space on the answer sheet. (For example, if you solve number 12 first, make sure the answer is placed beside the 12 on the answer sheet.) All questions are weighted equally. Answer as many questions as you can in the allotted time. No contestant is expected to solve all of the problems.

CALCULATORS, COMPUTERS AND/OR ANY OTHER ELECTRONIC DEVICES ARE NOT PERMITTED.

UNLESS OTHERWISE INDICATED, ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST FORM.

A radical expression of index n is in simplest form if the radicand is not a fraction, denominators are rationalized and integer radicands do not have any factors that are nth powers of a prime. For example, $\sqrt{\frac{5}{12}}$ simplifies to $\frac{\sqrt{15}}{6}$.

Do NOT approximate the number π.

Do NOT approximate radicals.

The notation **log** is logarithm to the base 10.

The notation $\mathbf{log}_a$ is logarithm to the base a. The notation **ln** is logarithm to the base e.

The symbol **!** is the factorial symbol. For example, $3! = 3 \cdot 2 \cdot 1 = 6$.

The symbol ***i*** is the complex unit $\sqrt{-1}$.

All numbers are in base 10 unless otherwise indicated (e.g., 1001_2 is the base 2 representation of the decimal number 9).

Any answer which is a non-integer rational number must be expressed in the form $\frac{a}{b}$, where a and b are integers that have no common divisor other than 1.

ANSWER SHEET (2020)

PLEASE PRINT CLEARLY

STUDENT'S FULL NAME Answers

STUDENT'S ADDRESS

NAME OF SCHOOL

TOWN (OR CITY) OF SCHOOL

WHAT GRADE ARE YOU IN? ○ 9th ○ 10th ○ 11th ○ 12th ○ Other

1. 312
2. 1/3
3. 13 feet
4. 4 blue marbles
5. 16
6. 3/10
7. 9 medium pizzas
8. 25/8
9. 296_{12}
10. 19 percent
11. $80 + 40\sqrt{2}$ cm
12. 94 percent
13. 2/5
14. 7
15. 4096
16. 15 hours
17. 89 ways
18. 540/11 minutes
19. $\sqrt{401}$ cm
20. 29
21. 9
22. 27 miles per hour
23. 65 degrees
24. $(-\sqrt{5}, \infty)$
25. $10 - 14i$
26. 200π square units
27. -22/3
28. 1/3
29. (6,-2)
30. 30 ways
31. 19801 integer solutions
32. (7,-3,-1)
33. $3\sqrt{2}$
34. 800 integers
35. 12, -15/2
36. $\sqrt{109}$ units
37. 240
38. 16/25 square units
39. 399 solutions
40. 25/12
41. 14