



THE UNIVERSITY OF VERMONT
DEPARTMENT OF MATHEMATICS AND STATISTICS
SIXTY-FOURTH ANNUAL HIGH SCHOOL PRIZE EXAMINATION
WEEK OF MARCH 8, 2021

THIS EXAMINATION IS TO BE ADMINISTERED DURING THE WEEK OF MARCH 8, 2021. AN EXAMINATION GIVEN AT ANY OTHER TIME WILL BE DISQUALIFIED.

THE TIME LIMIT ON THIS EXAMINATION IS 2 HOURS.

INSTRUCTIONS TO THE CONTESTANTS:

Do not begin the examination until the examiner tells you to do so.

The answer sheet is on the reverse side of this page. Before beginning the examination, carefully print your full name, your address, the complete name of your school and the town/city in which your school is located on the appropriate lines of the answer sheet. Check the circle corresponding to your grade level in school.

Answers must be written on the answer sheet in pencil or ink. You may work on problems in any order, but be sure that each answer is entered in the proper space on the answer sheet. (For example, if you solve number 12 first, make sure the answer is placed beside the 12 on the answer sheet.) All questions are weighted equally. Answer as many questions as you can in the allotted time. No contestant is expected to solve all of the problems.

CALCULATORS, COMPUTERS AND/OR ANY OTHER ELECTRONIC DEVICES ARE NOT PERMITTED.

UNLESS OTHERWISE INDICATED, ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST FORM.

A radical expression of index n is in simplest form if the radicand is not a fraction, denominators are rationalized and integer radicands do not have any factors that are n th powers of a prime. For example, $\sqrt{\frac{5}{12}}$ simplifies to $\frac{\sqrt{15}}{6}$.

Do NOT approximate the number π .

Do NOT approximate radicals.

The notation **log** is logarithm to the base 10.

The notation **log_a** is logarithm to the base a . The notation **ln** is logarithm to the base e .

The symbol **!** is the factorial symbol. For example, $3! = 3 \cdot 2 \cdot 1 = 6$.

The symbol ***i*** is the complex unit $\sqrt{-1}$.

All numbers are in base 10 unless otherwise indicated (e.g., 1001_2 is the base 2 representation of the decimal number 9).

Any answer which is a non-integer rational number must be expressed in the form $\frac{a}{b}$, where a and b are integers that have no common divisor other than 1.

ANSWER SHEET (2021)

PLEASE PRINT CLEARLY

STUDENT'S FULL NAME _____

STUDENT'S ADDRESS _____

NAME OF SCHOOL _____

TOWN (OR CITY) OF SCHOOL _____

WHAT GRADE ARE YOU IN? 9th 10th 11th 12th Other _____1. 6/52. 50003. 56 cubes4. 5 hours5. 123/1246. 9/20 cubic inches7. 50/31 inches8. 1/7209. 25 percent10. 34 children11. 3812. 59213. 3414. 26 units15. 13 ways16. 206 responses17. 1/2418. 60 people19. 8 ounces20. 63 ways21. 9 and 21 units22. 1/99923. 1050 outfits24. 1+3i and 4+3i25. 12 square units26. 32 positive integers27. -66428. 14 - 4√10 units29. 5330. 7/431. 15/1632. 6√333. 294 paths34. 11/1635. 4/2536. 7/837. 630 unordered pairs38. 9 ordered pairs39. 50040. 1:341. 10π/3 square units