THE UNIVERSITY OF VERMONT
DEPARTMENT OF MATHEMATICS AND STATISTICS
SIXTY-SIXTH ANNUAL HIGH SCHOOL PRIZE EXAMINATION
TUESDAY, MARCH 14, 2023

THIS EXAMINATION IS TO BE ADMINISTERED DURING SCHOOL HOURS ON MARCH 14, 2023. AN EXAMINATION GIVEN AT ANY OTHER TIME WILL BE DISQUALIFIED.

<u>THE TIME LIMIT ON THIS EXAMINATION IS 2 HOURS.</u>

<u>INSTRUCTIONS TO THE CONTESTANTS:</u>

Do not begin the examination until the examiner tells you to do so.

The answer sheet is on the reverse side of this page. Before beginning the examination, carefully print your full name, your address, the complete name of your school and the town/city in which your school is located on the appropriate lines of the answer sheet. Check the circle corresponding to your grade level in school.

Answers must be written on the answer sheet in pencil or ink. You may work on problems in any order, but be sure that each answer is entered in the proper space on the answer sheet. (For example, if you solve number 12 first, make sure the answer is placed beside the 12 on the answer sheet.) All questions are weighted equally. Answer as many questions as you can in the allotted time. No contestant is expected to solve all of the problems.

<u>CALCULATORS, COMPUTERS AND/OR ANY OTHER ELECTRONIC DEVICES ARE NOT PERMITTED.</u>

<u>UNLESS OTHERWISE INDICATED, ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST FORM.</u>

A radical expression of index n is in simplest form if the radicand is not a fraction, denominators are rationalized and integer radicands do not have any factors that are nth powers of a prime. For example, $\sqrt{\frac{5}{12}}$ simplifies to $\frac{\sqrt{15}}{6}$.

Do NOT approximate the number π.

Do NOT approximate radicals.

The notation **log** is logarithm to the base 10.

The notation $\log_a$ is logarithm to the base a. The notation **ln** is logarithm to the base e.

The symbol **!** is the factorial symbol. For example, $3! = 3 \cdot 2 \cdot 1 = 6$.

The symbol i is the complex unit $\sqrt{-1}$.

All numbers are in base 10 unless otherwise indicated (e.g., 1001_2 is the base 2 representation of the decimal number 9).

Any answer which is a non-integer rational number must be expressed in the form $\frac{a}{b}$, where a and b are integers that have no common divisor other than 1.

ANSWER SHEET (2023)

STUDENT'S FULL NAME _____ Answer Key _____

STUDENT'S ADDRESS _____

NAME OF SCHOOL _____

TOWN (OR CITY) OF SCHOOL _____

WHAT GRADE ARE YOU IN?   O 9th   O 10th   O 11th   O 12th   O Other _____

1. _____ 9 / 10 _____

2. _____ 164 / 5 _____

3. _____ 2 / 9 _____

4. _____ $243\pi/2$ _____ minutes

5. _____ 11 _____ cows

6. _____ $n = 106$ _____

7. _____ 28 _____

8. _____ 2023 / 2022 _____

9. _____ $-4/7$ _____

10. _____ 1575 _____

11. _____ 56 _____ units

12. _____ $x = -6/11$ _____

13. _____ 1 / 630 _____

14. _____ 10 _____ squares

15. _____ 40 _____ times

16. _____ $\sqrt{6}$ _____

17. _____ $4x - 1$ _____

18. _____ $m = 3/5$ _____

19. _____ 5 / 13 _____

20. _____ 26 _____ degrees

21. _____ $x = -5/4$ and $1/8$ _____

22. _____ $x = 7/2$ $y = 5/2$ _____

23. _____ 20 _____

24. _____ 28 _____ square units

25. _____ 55 _____ minutes

26. _____ 486_9 _____

27. _____ 620 _____

28. _____ 36 _____ minutes

29. _____ 90 _____

30. _____ 11 _____ integers

31. _____ $-40/3$ _____

32. _____ 12 _____ lattice points

33. _____ 29 _____

34. _____ $(3 - \sqrt{21})/2$ _____

35. _____ $N = 3121$ _____

36. _____ 5 / 9 _____

37. _____ 159 _____

38. _____ $16\pi - 24\sqrt{3}$ _____

39. _____ 500 _____

40. _____ 4 / 9 _____ cm

41. _____ 9 / 20 _____