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A NOTE ON HOWELL DESIGNS OF ODD SIDE

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A *Howell design* $H(s,2n)$, with $n \le s \le 2n - 1$, is a square array of side s, where cells are either empty or contain an unordered pair of elements chosen from a set X of size $2n$ such that

(1) each member of X occurs exactly once in each row and column of the array,

(2) each pair of elements of X occurs in at most one cell of the array.

Although much progress has been made in the construction of Howell designs, the existence question has not been settled. The purpose of this note is to report some results we have obtained by computer. Combining our results with previous results we obtain the following.

THEOREM 1. *If* $s < 1000$ *is odd,* $n \ne s - 1$ *and* $(s,2n) \ne (5,6)$, *then there exists an* $H(s,2n)$.

The exception $(5,6)$ listed above does not exist [4]. The reader is referred to [4,5] for results concerning $H(s,2s - 2)$.

We employ a method described by Anderson [1,2] to construct Howell designs from strong starters.

A *strong starter* in an additive abelian group G of order $2t + 1$ is a set $S = \{\{s_i, t_i\}:1 \le i \le t\}$ such that

(1) $\{s_i, t_i:1 \le i \le t\} = G\backslash\{0\}$,

(2) $\{\pm(s_i - t_i):1 \le i \le t\} = G\backslash\{0\}$, and

(3) $s_i + t_i \ne s_j + t_j$ if $i \ne j$ with $s_i + t_i \ne 0$ for all i.

Given a strong starter S, define a digraph Δ_S as follows. Let Δ_S have vertices V_S and edges E_S, where $V_S = \{-s_i - t_i: 1 \le i \le t\}$ and $E_S = \{(-s_i - t_i, -2u): u = s_i \text{ or } t_i \text{ and } -2u \in V_S\}$.

It is easy to see that Δ_S is contrafunctional (each vertex has in degree one), so that each weak component has exactly one directed cycle. Let Δ_S have weak components $C_1,\ldots,C_\ell$. Let $c_i = |V(C_i)|$, $1 \le i \le \ell$, and let d_i be the length of the unique directed cycle of C_i, $1 \le i \le \ell$. The following is implicit in [2].

LEMMA 2. *Let* S *be a strong starter of odd order* s *and* Δ_S *the associated digraph. If* $e = 1 + \sum_{i=1}^{\ell} 2e_i$, *where* $0 \le e_i \le c_i - d_i$ *or* $e_i = c_i$, *for* $1 \le i \le \ell$, *then there esists an* H(s, s+e) *(note* s+e *is even).*

It can be easily checked that Δ_S contains no directed cycles of length less than three. Thus no H(s, 2s-2) or H(s, 2s-4) can be constructed by this method, and if Δ_S contains large cycles several other possibilities will be missed. Except for the class H(s, 2s-2), the difficulty will be remedied by the following result of Hung and Mendelsohn [4]. We state their result only for Howell designs of odd side.

LEMMA 3. *If* s *is odd and any one of the following conditions holds, then there exists an* H(s, 2s-2k).

(1) $k > 10$ *and* $s \ge 2k^2 - 6k + 10$;

(2) $k = 0$ *or* $3 \le k \le 10$, *and* $s \ge 2k + 1$;

(3) $k = 2$ *and* $s \ge 7$.

The condition (3) is caused by the non-existence of H(5,6).

Thus cycles in a Δ_S do not cause problems, provided they are not too large or too numerous. We make use of the following.

LEMMA 4. *Let* $0 \le d_i \le c_i$ *for* $1 \le i \le \ell$, *and let* $d = \Sigma d_i$, $c = \Sigma c_i$. *If* $c \ge 2d$, *and* $0 \le f \le c - \max\{d_i\}$, *then we can write* $f = \Sigma e_i$, $0 \le e_i \le c_i - d_i$ *or* $e_i = c_i$, *for* $1 \le i \le \ell$.

Proof. Induction on ℓ. If $\ell = 1$ the result is clear. For arbitrary positive ℓ, assume the result for $\ell - 1$. If necessary, permute the set $\{1,2,\ldots,\ell\}$ so that $c - c_\ell \ge 2(d-d_\ell)$ (this can be done, since $c \ge 2d$). Let $d_0 = \max\{d_i : 1 \le i \le \ell\}$ and let $d_0' = \max\{d_i : 1 \le i \le \ell-1\}$.

Let $f \le c - d_0$. If $f \le c - c_\ell - d_0'$, then $f = \sum_{i=1}^{\ell-1} e_i$ with the e_i's as required, by induction. If $c - c_\ell - d_0' \le f \le c - d_0' - d_\ell$, then $f = \sum_{i=1}^{\ell} e_i$ with $0 \le e_i \le c_i - d_i$ and $\sum_{i=1}^{\ell-1} e_i = c - c_\ell - d_0'$ (induction).

If $c_\ell \le f \le c - d_0'$, then $f - c_\ell = \sum_{i=1}^{\ell-1} e_i$ by induction, so $f = \sum_{i=1}^{\ell-1} e_i + c_\ell$. If $c - c_\ell \le f \le c - d_\ell$, then $f = \sum_{i=1}^{\ell-1} c_i + e_\ell$ with $0 \le e_\ell \le c_\ell - d_\ell$. Now $d_0 = \max\{d_0', d_\ell\}$ so both $c - d_0'$ and $c - d_\ell$ are at least $c - d_0$. Thus it suffices to check that $c - d_0' - d_\ell \ge \min\{c_\ell, c - c_\ell\}$. If not, then $2(c - d_0' - d_\ell) < c$, so $2(d_0' + d_\ell) > c$. But $d \ge d_0' + d_\ell$ and $2d \le c$, so we have a contradiction. This establishes the result. □

$$\text{Let } f(k) = \begin{cases} 2k^2 - 6k + 10 & \text{if } k > 10, \\ 2k + 1 & \text{if } k = 0 \text{ or } 3 \le k \le 10, \\ 7 & \text{if } k = 2. \end{cases}$$

We obtain the following.

LEMMA 5. *Suppose there exists a strong starter* S *of order* s, *with digraph* Δ_S *having largest cycle* d_0, *and* $c \ge 2d$.

If $f(d_0) \le s$, *then there exists an* $H(s,2n)$ *if* $n \le s \le 2n - 1$ *and* $n \ne s - 1$.

Proof. Let $2n = 2s - 2k$. Then $k \ne 1$. If $k < d_0$ then $H(s,2n)$ exists by Lemma 3, since $s \ge f(d_0)$ and f is a non-decreasing function. If $k \ge d_0$, we can write $\frac{s-1}{2} - k = \sum_{i=1}^{\ell} e_i$ with $0 \le e_i \le c_i - d_i$ or $e_i = c_i$, for $1 \le i \le \ell$, by Lemma 4. Then $e = 2n - s = s - 2k = 1 + \sum_{i=1}^{\ell} 2e_i$, and Lemma 2 implies the result. □

By computer we have constructed strong starters of all odd orders s with $53 \le s \le 999$ satisfying the hypotheses of Lemma 5. A description of our algorithm is given in [3].

By means of this strong starter construction and other methods it had already been established that Howell designs $H(s,2n)$, with s odd, $s \le 51$, $n \ne s - 1$, and $(s,2n) \ne (5,6)$ exist. For more details see [1,4].

Thus Theorem 1 is obtained. Due to space limitations, we cannot list all the strong starters here. We do however, list strong starters of orders 53-99, and 999 below, together with the directed cycles of the associated digraph. A complete listing is on file at the Ohio State University Mathematics Library.

STRONG STARTER OF ORDER 53

33, 34	46, 44	10, 7	43, 39	24, 19	11, 17	8, 1	47, 2	15, 6	13, 23
49, 38	30, 18	5, 45	36, 22	27, 42	28, 12	31, 48	3, 21	32, 51	9, 29
20, 41	4, 35	37, 14	40, 16	25, 50	26, 52				

CYCLES IN DIGRAPH

14 16 3 31 25 2 28 24 45
10 27 17 5

STRONG STARTER OF ORDER 55

49, 50	39, 37	42, 45	15, 19	8, 3	27, 33	47, 54	21, 29	31, 22	52, 7
48, 4	6, 18	28, 41	12, 53	11, 26	51, 35	40, 23	2, 20	32, 13	30, 10
25, 46	16, 38	1, 24	5, 36	9, 34	43, 14	17, 44			

CYCLES IN DIGRAPH

18 39 44 50 15 33

STRONG STARTER OF ORDER 57

51, 52	49, 47	37, 40	39, 43	27, 22	21, 15	26, 33	25, 17	6, 54	34, 44
19, 8	38, 50	11, 24	48, 5	16, 1	14, 30	13, 53	10, 28	12, 31	35, 55
9, 45	20, 42	7, 41	23, 56	4, 36	29, 3	32, 2	18, 46		

CYCLES IN DIGRAPH

50 23 15 21

STRONG STARTER OF ORDER 59

35, 36	23, 21	8, 5	51, 55	11, 6	43, 49	34, 27	28, 20	57, 48	14, 24
22, 33	38, 50	40, 53	17, 3	45, 30	58, 15	52, 10	13, 54	25, 44	7, 46
39, 18	41, 4	32, 9	26, 2	37, 12	42, 16	29, 56	47, 19	31, 1	

CYCLES IN DIGRAPH

47 42 30 49 46
15 4 13
56 27 1 33 51 14

STRONG STARTER OF ORDER 61

56, 57	41, 39	46, 49	54, 50	33, 38	6, 12	27, 20	24, 16	13, 22	9, 60
23, 34	28, 40	21, 8	29, 43	47, 1	19, 3	59, 15	55, 37	17, 36	25, 45
11, 51	14, 53	4, 42	7, 44	10, 35	26, 52	5, 32	30, 58	2, 31	18, 48

CYCLES IN DIGRAPH

9 44 42 54 4 48 30 27 8

STRONG STARTER OF ORDER 63

41, 42	14, 12	49, 46	56, 52	23, 28	61, 4	20, 13	47, 55	3, 57	29, 39
5, 16	22, 34	37, 50	38, 24	26, 11	43, 27	18, 1	53, 35	7, 51	45, 2
59, 17	9, 31	25, 48	58, 19	40, 15	32, 6	33, 60	36, 8	44, 10	21, 54
62, 30									

CYCLES IN DIGRAPH

9 56 38

STRONG STARTER OF ORDER 65

6, 5	26, 24	63, 60	16, 20	58, 53	43, 49	32, 39	4, 12	2, 11	25, 35
55, 44	64, 52	28, 15	59, 8	37, 22	18, 34	14, 31	51, 33	48, 29	54, 9
57, 13	62, 40	42, 19	45, 21	1, 41	46, 7	23, 50	38, 10	27, 56	17, 47
30, 61	36, 3								

CYCLES IN DIGRAPH

5 39 60

STRONG STARTER OF ORDER 67

55, 54	59, 57	42, 39	21, 17	7, 2	10, 16	32, 25	28, 20	40, 31	1, 58
53, 64	12, 24	33, 46	52, 38	4, 56	27, 43	19, 36	44, 62	66, 47	26, 6
14, 60	48, 3	11, 34	61, 37	5, 30	35, 9	23, 63	13, 41	22, 51	45, 8
65, 29	50, 18	15, 49							

CYCLES IN DIGRAPH

10 28 17
18 8 48 64 23 61 16

STRONG STARTER OF ORDER 69

63, 64	61, 59	15, 18	51, 55	32, 27	14, 8	4, 11	9, 1	31, 40	28, 38
43, 54	24, 12	49, 62	3, 17	21, 36	57, 41	13, 30	53, 35	16, 66	67, 47
58, 37	23, 45	19, 42	50, 26	46, 2	22, 48	10, 52	34, 6	20, 60	44, 5
56, 25	39, 7	29, 65	33, 68						

CYCLES IN DIGRAPH

11	44	24	40	27	12
18	58	67	59	20	

STRONG STARTER OF ORDER 71

48, 47	44, 42	59, 62	69, 65	2, 68	27, 21	29, 22	12, 4	16, 7	61, 51
35, 46	28, 40	43, 56	37, 23	39, 54	24, 8	30, 13	1, 19	5, 57	50, 70
67, 17	55, 6	41, 18	58, 11	45, 20	10, 36	26, 53	60, 32	38, 9	63, 33
34, 3	25, 64	14, 52	15, 49	31, 66					

CYCLES IN DIGRAPH

47	55	39	48
21	53	24	
9	45	28	

STRONG STARTER OF ORDER 73

18, 19	35, 33	61, 64	66, 62	54, 49	22, 16	36, 29	47, 39	6, 17	40, 50
2, 13	46, 34	72, 12	11, 70	38, 53	4, 20	24, 7	32, 14	44, 63	5, 58
52, 31	9, 60	26, 3	51, 27	42, 67	71, 45	10, 56	15, 43	57, 28	68, 25
6, 48	69, 37	1, 41	55, 21	65, 30	23, 59				

CYCLES IN DIGRAPH

30	10	53	7	5	66	56

STRONG STARTER OF ORDER 75

73, 74	41, 43	62, 59	24, 20	49, 54	12, 6	27, 34	45, 37	66, 57	7, 17
53, 42	33, 21	16, 29	52, 38	69, 9	72, 13	18, 35	47, 65	23, 4	56, 36
1, 22	14, 67	55, 32	61, 10	58, 8	28, 2	44, 71	68, 40	31, 60	50, 5
70, 26	51, 19	15, 48	30, 64	46, 11	39, 3	63, 25			

CYCLES IN DIGRAPH

47	69	33	21	14	42

STRONG STARTER OF ORDER 77

75, 76	51, 49	14, 17	64, 68	24, 19	3, 9	54, 47	27, 35	53, 44	38, 48
25, 36	40, 28	29, 16	71, 57	70, 8	26, 10	50, 67	72, 13	20, 1	42, 62
18, 74	4, 59	23, 46	32, 56	6, 58	63, 12	34, 61	41, 69	60, 31	45, 15
21, 52	39, 7	55, 11	65, 22	2, 37	30, 66	33, 73	43, 5		

CYCLES IN DIGRAPH

57	41	62	8	48
54	37	56		

STRONG STARTER OF ORDER 79

23, 22	42, 40	14, 11	74, 78	12, 7	30, 24	25, 32	20, 28	56, 47	75, 65
73, 62	38, 50	39, 52	49, 63	4, 68	51, 67	10, 27	66, 48	16, 35	41, 61
6, 64	76, 19	58, 2	70, 15	59, 34	55, 29	72, 45	46, 18	71, 21	33, 3
36, 5	37, 69	31, 77	60, 26	53, 9	8, 44	43, 1	54, 13	17, 57	

CYCLES IN DIGRAPH

25	42	19
60	46	55

STRONG STARTER OF ORDER 81

71, 70	34, 32	46, 49	55, 59	11, 16	48, 42	36, 43	30, 22	12, 3	17, 7
54, 65	72, 60	6, 19	8, 75	10, 25	15, 31	27, 44	13, 76	4, 66	33, 53
20, 80	62, 40	18, 41	26, 50	52, 77	56, 1	14, 68	79, 51	64, 35	67, 37
47, 78	9, 58	5, 38	57, 23	74, 39	73, 28	24, 61	45, 2	21, 63	29, 69

CYCLES IN DIGRAPH

5	38	60	32	43	56	76	2	62

STRONG STARTER OF ORDER 83

4, 3	65, 63	79, 76	25, 21	33, 28	55, 61	42, 35	12, 20	47, 38	68, 78	
34, 45	57, 69	81, 11	44, 30	32, 17	75, 59	27, 10	49, 31	73, 9	82, 19	
62, 41	66, 5	77, 54	8, 67	40, 15	48, 22	80, 53	29, 1	56, 2	7, 37	
43, 74	46, 14	39, 72	60, 26	36, 71	16, 52	70, 24	51, 13	6, 50	18, 58	
23, 64										

CYCLES IN DIGRAPH

79 25 53 28 40 38

STRONG STARTER OF ORDER 85

27, 28	50, 48	72, 75	51, 55	19, 14	10, 4	42, 35	32, 24	71, 62	47, 37
68, 57	58, 46	30, 43	17, 3	54, 69	34, 18	13, 81	79, 12	2, 21	9, 74
40, 61	38, 16	8, 70	77, 53	39, 64	23, 49	1, 59	82, 25	44, 73	6, 36
45, 76	84, 31	63, 11	80, 29	83, 33	56, 7	41, 78	5, 52	66, 20	60, 15
26, 67	22, 65								

CYCLES IN DIGRAPH

2 55 10 61 79 65 71 22
8 76 1

STRONG STARTER OF ORDER 87

67, 66	33, 31	72, 69	60, 64	40, 45	80, 86	77, 84	81, 2	37, 28	56, 46
63, 74	24, 12	35, 22	36, 50	78, 6	11, 82	42, 59	25, 7	13, 32	23, 3
34, 55	53, 75	41, 18	51, 27	17, 79	39, 65	47, 20	85, 26	4, 62	49, 19
52, 83	38, 70	14, 68	58, 5	9, 61	44, 8	21, 71	10, 48	15, 54	29, 76
16, 57	1, 43	30, 73							

CYCLES IN DIGRAPH

41 61 42
51 28 71 35 63

STRONG STARTER OF ORDER 89

48, 49	44, 46	38, 35	16, 12	7, 2	73, 79	76, 83	40, 32	62, 71	60, 70
10, 21	47, 59	6, 19	84, 9	52, 67	53, 69	17, 34	4, 75	50, 31	23, 3
22, 1	14, 36	80, 57	63, 87	61, 86	56, 30	51, 24	55, 27	28, 88	15, 45
68, 37	11, 43	72, 39	26, 81	20, 74	25, 78	29, 66	64, 13	58, 8	5, 54
82, 41	18, 65	85, 42	77, 33						

CYCLES IN DIGRAPH

81 10 85 80

STRONG STARTER OF ORDER 91

19, 20	70, 68	60, 57	5, 9	54, 49	21, 15	17, 10	58, 50	78, 87	45, 35
84, 4	11, 23	56, 43	41, 55	61, 76	53, 37	27, 44	75, 2	6, 25	28, 48
82, 12	85, 63	1, 69	83, 59	77, 52	89, 24	13, 40	66, 38	33, 62	18, 79
16, 47	67, 8	65, 7	31, 88	90, 34	3, 39	14, 51	36, 74	81, 42	86, 46
71, 30	64, 22	32, 80	73, 29	26, 72					

CYCLES IN DIGRAPH

52 19 72 86 15 78 53
55 85 49

STRONG STARTER OF ORDER 93

85, 86	38, 36	61, 64	73, 69	63, 68	54, 48	77, 84	11, 19	6, 15	33, 43
45, 34	1, 82	13, 26	51, 65	14, 29	62, 46	30, 47	39, 57	91, 72	87, 67
56, 35	89, 18	8, 78	21, 90	2, 27	66, 92	23, 50	31, 59	71, 42	83, 20
10, 41	44, 12	16, 76	3, 37	40, 75	22, 79	60, 4	80, 25	88, 49	81, 28
17, 58	32, 74	5, 55	9, 53	7, 52	24, 70				

CYCLES IN DIGRAPH

15 90 84 70 18 25 14

STRONG STARTER OF ORDER 95

49, 50	75, 77	19, 16	47, 51	84, 79	17, 11	54, 61	20, 28	5, 91	2, 12
83, 94	36, 48	86, 73	71, 57	40, 25	30, 14	81, 64	26, 8	29, 10	43, 23
45, 66	9, 82	35, 58	74, 3	53, 78	41, 67	1, 69	42, 70	60, 89	52, 22
24, 55	7, 39	59, 92	38, 72	93, 33	80, 21	32, 90	65, 27	37, 76	46, 6
85, 31	87, 34	56, 4	44, 88	18, 68	13, 62	63, 15			

CYCLES IN DIGRAPH

49	29	64	17
82	75	56	

STRONG STARTER OF ORDER 97

45, 44	84, 82	80, 77	73, 69	15, 20	47, 41	7, 14	75, 83	61, 70	56, 46
43, 32	8, 93	4, 88	68, 54	39, 24	11, 27	35, 18	34, 16	57, 38	37, 17
5, 81	52, 74	66, 89	25, 49	62, 87	23, 94	22, 92	90, 21	28, 96	86, 19
36, 67	85, 53	26, 59	65, 31	2, 64	42, 6	72, 12	71, 33	79, 40	58, 1
50, 91	13, 55	63, 9	51, 95	3, 48	76, 30	60, 10	78, 29		

CYCLES IN DIGRAPH

68	25	91	46		
12	53	80	2	70	45

STRONG STARTER OF ORDER 99

18, 17	88, 86	97, 94	80, 84	64, 69	60, 66	72, 65	93, 85	39, 48	22, 32
44, 33	43, 31	14, 27	42, 56	70, 55	12, 95	37, 20	83, 2	38, 57	78, 58
74, 53	68, 90	13, 36	4, 28	3, 77	26, 52	24, 96	62, 34	45, 16	21, 51
19, 50	8, 75	23, 89	81, 47	15, 79	10, 46	54, 91	35, 73	11, 71	1, 41
9, 67	30, 87	7, 63	6, 61	5, 59	29, 82	92, 40	25, 76	49, 98	

CYCLES IN DIGRAPH

72	29	90	53	86	1	51	78
58	73	50	71				

STRONG STARTER OF ORDER 999

165,166	245,243	101,104	936,932	32, 27	395,389	480,473	937,945	989,980	401,411
341,352	19, 31	29, 16	221,235	891,876	177,161	862,879	590,572	348,367	412,392
708,729	960,982	881,858	781,757	569,544	64, 38	946,973	530,558	720,691	261,291
844,813	130,162	159,126	751,717	317,282	501,465	686,723	56, 94	624,663	707,747
541,582	783,825	914,957	772,728	685,730	744,790	857,810	467,515	108, 59	189,139
119, 68	681,733	916,863	36, 90	330,275	560,616	249,306	956, 15	37,977	179,239
202,263	981,919	489,426	439,375	462,397	74,140	621,688	77,145	39,969	748,818
527,456	138, 66	852,779	983, 58	758,683	91,167	812,889	454,532	529,608	528,448
280,361	620,538	490,407	958, 43	147, 62	242,156	103,190	968,880	828,739	534,444
756,847	655,563	123,216	475,381	934, 30	433,337	619,716	568,666	427,328	111,211
555,656	776,674	357,254	549,653	461,356	472,366	634,741	463,571	646,537	593,703
474,585	520,408	304,191	799,913	54,938	861,745	89,206	82,200	193,312	240,360
840,719	824,702	592,715	622,498	920,795	342,468	820,693	41,169	774,645	244,374
761,892	898,766	855,722	718,584	258,393	930, 67	841,978	705,843	207,346	268,128
794,935	307,449	289,146	543,399	414,269	518,372	320,173	522,670	4,153	310,160
905, 57	791,943	874, 28	904,750	323,478	492,648	369,212	709,867	581,422	432,272
362,523	213, 51	322,485	615,451	418,253	420,586	984,817	955,124	801,632	771,601
964,793	782,954	546,373	692,866	789,614	589,765	402,225	865, 44	105,925	907,727

143,961	952,135	931,115	33,217	75,260	359,545	232, 45	684,872	755,944	525,335
521,712	561,753	976,170	759,565	292, 97	390,194	197,394	443,641	303,502	107,906
313,112	805,603	118,321	246,450	658,453	804, 11	740,533	995,787	636,845	281, 71
908,120	851,639	398,185	642,856	23,807	295, 79	505,288	816, 35	284, 65	99,878
329,550	987,210	854,631	495,271	875,650	383,157	700,927	821, 50	227,997	992,223
547,778	287,519	682,915	332,566	713,948	228,464	419,182	386,148	256, 17	671,431
150,391	102,859	668,425	110,354	388,633	382,628	986,234	204,452	80,830	762,512
237,488	224,971	912,659	731,985	711,966	929,673	704,447	73,814	923,664	6,266
270, 9	887,625	780,517	380,116	868,134	611,877	188,455	187,918	591,860	476,746
967,690	612,340	106,379	274,548	902,178	363, 87	410,687	587,309	993,714	378, 98
767, 49	95,377	326,609	811, 96	694,409	184,897	770,483	576,864	979,690	903,613
299, 8	457,749	924,218	554,848	440,735	933,230	180,882	297,998	535,834	836,137
255,556	424,122	466,163	93,788	63,368	404,710	651,344	441,133	333, 24	573,883
894,583	442,754	890,577	604,290	623,308	524,208	734, 52	257,575	962,643	293,972
48,726	672,350	154,477	175,850	285,610	286,959	311,638	370,698	928,599	837,507
815,484	796,129	552,219	842,508	965,301	823,487	203,540	497,835	657,318	471,131
127,785	580,238	732, 76	294,949	773,428	832,486	300,647	421,769	822,172	355, 5
13,661	630,278	400, 47	893,248	196,551	752,396	598,241	384,742	724,365	536,896
168,806	990,353	338,974	12,376	594,229	838,205	334,701	18,649	600,231	371, 1
132,760	939,567	171,797	499,125	61,436	42,665	803,181	888,267	562,183	886,506
607,226	315,697	117,500	618, 3	514,899	60,446	975,588	336,947	470, 81	626,236
988,597	706,314	283,676	40,434	121,516	100,496	695,298	689, 88	209,809	819,220
510,911	667,265	680,277	873,469	827,233	316,909	493, 86	942,351	14,423	784,195
460,871	25,437	415, 2	526,940	214,629	141,557	926,509	763,345	491, 72	679,259
416,994	479,901	578,155	20,595	279,853	839,413	736,164	438, 10	84,513	677,247
69,637	654,222	996,430	435,869	699,264	364,800	78,640	574,136	963,403	644, 85
885,327	186,743	385,941	768,324	114,559	826,273	895,343	725,174	504,953	503, 53
113,564	55,602	738,192	142,596	445,900	26,482	951,494	459,917	802,262	481, 21
570,109	606,144	662,199	605, 70	921,387	46,579	792,325	201,669	627,158	553, 83
358,829	250,777	305,831	617, 92	7,531	870,347	675,198	429,950	405,884	511,991
849,331	539, 22	635,152	833,349	252,737	764,251	458,970	660,149	808,319	786,296
542, 34	339,846	721,215	417,922	406,910	302,798	678,176	151,652	775,276	

CYCLES IN DIGRAPH

364 197 187 682 887 849 664 67 370 112 264 422 118 532 584 544
344 938 910 885 37 497 983 692 796 626 589 955 438 701 190
249 185 102 337 818 836 995 582 561 228 786
896 252 656

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