

Scheduling the XFL

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1 Introduction

At a press conference in New York on February 3, 2000, World Wrestling Federation Entertainment, Inc. announced the formation of a new professional football league, the *XFL*. The first games of the league will be played in February 2001. As both co-authors of this paper are great sports fans and have some previous experience in tournament scheduling (JD has scheduled several golf and tennis leagues while DF has scheduled the Czech National Basketball League and Czech National Hockey League) we naturally became interested in the proposed (yet not announced) schedule of the league. We subsequently agreed that we should offer our services to the *WWF* to construct the schedule of play for the *XFL*. On March 2, we sent a letter to Rich Rose of the *XFL* describing our qualifications and proposing that we be hired to construct the schedule of play for the new league. On March 8, JD received a call from Rich Rose confirming that *XFL* was willing to hire us and that they were even willing to pay us for the schedule. It was agreed that Rich Rose would send a list of requirements for the league schedule and that we would construct the best possible schedule satisfying these requirements. In Appendix 1 we present a copy of the fax we received from Rich Rose which enumerated the requested Guidelines and Preferences.

One can indeed see that some of the Guidelines and Preferences are redundant. For instance, Preference 3 is included in Preference 2. Nevertheless,

the authors extracted the necessary information from the Guidelines and Preferences and started working.

2 First versions of the proposed schedule

In the 2001 season the *XFL* will consist of 8 teams divided into 2 divisions according to their geographical location: West and East. As can be seen from the Guidelines and Preferences in Appendix 1, each team will play 10 games, two *intradivisional games* against each team of its own division (one home, one away) and one *interdivisional game* against each team of the other division. Two of the interdivisional games for every team will be home games and two will be away games. In terms of graph theory, the schedule corresponds to a directed *1-factorization* of the multigraph K'_8 in which the edges of two disjoint subcliques of order 4 are doubled (the intradivisional games) and the edges between the subcliques are simple (the interdivisional games).

The authors believed that the Preference 1, *“Teams that have non-divisional games along opposite coast will play both non-divisional games in consecutive weeks to avoid scheduling competitive advantages”* was stated incorrectly, as the requirement seemed to be surprising and unnatural. Therefore they decided they would prepare at least two different versions of the schedule. One would obey the Preference 1, while a second one would obey the opposite requirement, which seemed to the authors more natural. Namely, Preference 1' *“Teams that have non-divisional games along opposite coast will not play both non-divisional games in consecutive weeks to avoid scheduling competitive advantages”*. In fact, we constructed three different schedules. These schedules are presented in Appendix 3. Notice that Schedule B obeys the original Preference 1 while Schedules A and C obey the Preference 1'. Schedule C is based on a schedule provided by Wal Wallis during the Thirty-first Southeastern International Conference on Combinatorics, Graph Theory, and Computing, where the authors presented their “work in progress” report.

Before we present the schedules and discuss their properties, we introduce some “measures” of their quality. Define a *home-away pattern* of a team T_i , denoted $HAP(T_i)$, as a sequence $(Z_1, Z_2, \dots, Z_{10})$, where $Z_j = H$ if team T_i plays a home game in round j and $Z_j = A$ if team T_i plays an away game in round j . The most desirable *HAP* has the property that for every $j = 1, 2, \dots, 9$ that $Z_j \neq Z_{j+1}$. One can easily observe that there can be at most two teams with such *HAPs*. Therefore we say that a team T_i has a

break in its schedule (or, alternatively, in $HAP(T_i)$) if $Z_j = Z_{j+1}$ for some j . Denote the number of breaks in $HAP(T_i)$ by $n(T_i)$ and the overall number of breaks in the schedule by N . A subsequence $Z_j, Z_{j+1}, \dots, Z_{j+k}$ is called *string* (of home or away games) if $Z_j = Z_{j+1} = \dots = Z_{j+k}$. We denote the length of the longest string of home and away games of T_i by $s_H(T_i)$ and $s_A(T_i)$, respectively. Then define $S_H = \max\{s_H(T_i) | i = 1, 2, \dots, 8\}$ and $S_A = \max\{s_A(T_i) | i = 1, 2, \dots, 8\}$.

We see that a good schedule will have a small number of breaks and that the longest string will be short. It is possible to construct a schedule satisfying all Guidelines and Preferences 1', 2, and 3 with 8 breaks and with $S_H = S_A = 2$. We present one such schedule in Appendix 2. However, the reader will notice that the games between the same intradivisional opponents are always played back to back, which is very unusual. In fact, the authors actually discussed such a possibility with the *XFL* and not too surprisingly the idea was rejected. Therefore, as another measure of the quality of a schedule one can take the minimum number of weeks between the two games that a team plays against the same opponent. We denote by $d(T_i)$ the minimum number of weeks between the repeated games of the team T_i and by D the minimum over all teams. Clearly a good schedule will try to maximize D . It is easy to see that D is always at most 7, but at the same time it is very difficult to get close to this value if we want to satisfy the Guidelines, especially guidelines number 6 and 7.

Before we analyze the proposed schedules, for convenience we introduce some notation. Both schedules A and B are based on "separate" factorizations of two copies of K_4 (intradivisional games) and a factorization of $K_{4,4}$ (interdivisional games). To follow the notation introduced by the original *XFL* schedules, we denote the teams of the West Division by $W1, W2, W3, W4$ and the teams of the East Division by $E1, E2, E3, E4$.

As there is only one 1-factorization of K_4 , we denote its respective factors by Y_i , where $Y \in \{E, W\}$ stands for division and i is the number of the team playing the team $Y4$ in that round. Furthermore, we use the "switching" 1-factorization of $K_{4,4}$. This can be described as follows: in X_1 , each team W_i plays E_i , in X_2 teams $W1$ and $W2$ interchange their opponents from the first round so that $W1$ plays $E2$ and $W2$ plays $E1$. Similarly, $W3$ and $W4$ interchange their opponents from the first round. Then in X_3 , W_i plays $E(i+2)$ (taken modulo 4) while in X_4 , $W1$ and $W2$ again interchange their opponents from round 3 as do $W3$ and $W4$. This does not describe the home and away patterns, these were worked out in conjunction with the 1-factor

sequence composing the schedule.

We can now take a close look at Schedules A and B. Below we give the sequence of factors that comprise these schedules. One point worth noting is that in each week under Schedule A, every game is either intradivisional or interdivisional. In Schedule B this is true for 7 of the 10 weeks. In week 4 of Schedule B there is one game each from W_3 and E_3 and two games from X_3 (denoted N_1 below); in week 6 are the two remaining games from X_3 along with two games from X_4 (denoted N_2 below); and in week 8 are the two remaining games from X_4 along with the remaining one game each from W_3 and E_3 (denoted N_3 below).

Schedule A

Week	1	2	3	4	5	6	7	8	9	10
1-Factor	$W_2 \cup E_2$	X_1	X_2	$W_1 \cup E_1$	$W_3 \cup E_3$	X_3	X_4	$W_2 \cup E_1$	$W_1 \cup E_2$	$W_3 \cup E_3$

Schedule B

Week	1	2	3	4	5	6	7	8	9	10
1-Factor	$W_3 \cup E_3$	$W_2 \cup E_2$	$W_1 \cup E_1$	N_1	X_1	N_2	X_2	N_3	$W_1 \cup E_1$	$W_2 \cup E_2$

Schedule C is different and is based directly on a cyclic factorization of the multigraph K'_8 rather than on separate factorizations of copies of K_4 and $K_{4,4}$. There are three types of factors. The first type has the starter M_0 with games $W1 - W2, E1 - E2, W3 - E4, E3 - W4$. The factors M_i for $i = 1, 2, 3$ are then defined by developing this starter block modulo 4. The other four rounds are based on the starter M'_0 with games $W2 - W1, E3 - E2, W4 - E4, E1 - W3$. The factors M'_i for $i = 1, 2, 3$ are again defined by developing the starter block modulo 4. While some intradivisional games appear twice in these rounds, games 1 - 3 and 2 - 4 do not appear at all there. Therefore there are two more rounds, N and N' , in which these games are played in both divisions. The rounds in Schedule C are ordered in the sequence $M_1, M_2, N, M_3, M_4, M'_1, M'_2, N', M'_3, M'_4$.

These three schedules were then mailed to the *XFL*. Attached with the schedules were three paragraphs which summarized the the pros and cons of each schedule. We also attached a table summarizing the properties satisfied by each schedule. The following is the summary of the schedules that was provided to the *XFL*.

Schedule A has the best home-away balance, there are only 6 home breaks and 6 away breaks. There is no string of three consecutive home games or three consecutive away games. No team starts the season with two home or two away games. Every week there are either just intradivisional games or just interdivisional games. Schedule A ends the season with one team (*E4*) having two consecutive away games and one team (*E2*) having two consecutive home games. This can easily be modified so that no team *ends* with two consecutive away or home games, but in this case two teams would *start* the season this way. After 5 weeks each team has played each intradivisional opponent once, and 2 of the 4 interdivisional opponents.

Schedule B satisfies the condition that both interdivisional games played by a team at the opposite cost are back to back. There are only two teams with a string of three home games and no team with a string of three away games. No team ends the regular season with two home or two away games. There are 5 weeks when only interdivisional games are played and 2 weeks when only intradivisional games are played. Three weeks are “mixed” with both intradivisional and interdivisional games. After 5 weeks each team has played each intradivisional opponent once, and 2 of the 4 interdivisional opponents.

Schedule C has the property that no team starts or ends the regular season with two home or two away games. The property that really differentiates this schedule from the others that in 8 of the 10 weeks, there are two intradivisional games and two interdivisional games. After 5 weeks 4 teams out of 8 have played each intradivisional opponent once, and 2 of the 4 interdivisional opponents.

3 Second versions of the proposed schedule

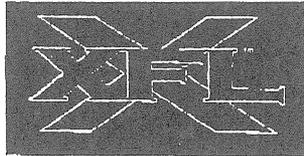
All schedules along with summaries were mailed to *XFL* on March 27, 2000. After receiving the first version of the schedules Rich Rose called again stating how pleased he was with our work. At that time he also asked JD whether it would be possible to incorporate into the schedules some additional requirements concerning some specific dates for home and/or away games of several teams. In particular, he asked that Chicago be away in weeks 1 and 2 (anticipating winter weather there that time of the year) and that Las Vegas play away in weeks 8 and 9. Without too much difficulty we were able to modify the current three schedules so that each in-

corporated the new requirements. Although the quality of the new versions (called A2, B2, C2) was little bit lower, the difference was not too significant. For instance, $N(A) = 12, N(B) = 28, N(C) = 20$, while $N(A2) = 14, N(B2) = 32, N(C2) = 18$. Also $S_H(A) = 2, S_H(B) = 3$ (two different teams), $S_H(C) = 2$, while $S_H(A2) = 2, S_H(B2) = 3$ (three different teams), $S_H(C2) = 3$ (one team). Finally, $D(A) = 4, D(B) = 3, D(C) = 3$, while $D(A2) = 3, D(B2) = 2, D(C2) = 3$.

The new versions of the schedules were faxed on April 17, 2000. To save space, we do not present them here, as they do not differ too much from Schedules A, B, and C. The final decision on which schedule will be adopted has not yet been made. It is expected to be decided upon at a meeting which will be attended by a representative of the *XFL*, *NBC* (broadcaster of 1/2 of the games), a second network which will be broadcasting the other 1/2 of the games, and some sponsors of the broadcasts. In addition, both authors are invited to participate in this meeting (and wouldn't miss it for the world).

We have already been told by the league that they appreciate our scheduling service and look forward to a long and fruitful relationship. In particular, they mentioned that for the 2002 season the *XFL* plans on expanding to 10 teams (two 5 team divisions) and they would like us to do a new schedule for that year. We are extremely pleased to be able to do "applied mathematics" that is so much fun.

APPENDIX 1



3/09/00

Divisions

East

New York
Washington D.C.
Miami
Orlando

West

LA
San Francisco
Las Vegas
Dallas

Guidelines

- Ten games
- Each team will play every other team in its own division twice
- Each team will play every other team in the other division once
- There will be five home games and five away games
- Each team will have one divisional home game and one divisional away game against every team in its division
- Each team will have two home games and two away games against the four teams from the other division
- There must be one Eastern standard time game and one Pacific standard time game each weekend

Preferences

- Teams that have non-divisional games along opposite coast will play both non-divisional games in consecutive weeks to avoid scheduling competitive advantages
- No three-week-long road trips
- Every team with at least 1 home game by the third week of season

Other

- Substitute Dallas for Houston on attached schedules

APPENDIX 2

Schedule with 8 breaks

WEEK 1 E1 @E4 E2 @E3 W1 @W4 W2 @W3	WEEK 2 E4 @E1 E3 @E2 W4 @W1 W3 @W2	WEEK 3 E1 @E3 E2 @E4 W1 @W3 W2 @W4	WEEK 4 E3 @E1 E4 @E2 W3 @W1 W4 @W2	WEEK 5 E3 @E4 E2 @E1 W3 @W4 W2 @W1
WEEK 6 E4 @E3 E1 @E2 W4 @W3 W1 @W2	WEEK 7 E1 @W1 E2 @W2 E3 @W3 E4 @W4	WEEK 8 W2 @E1 W3 @E2 W4 @E3 W1 @E4	WEEK 9 E1 @W3 E2 @W4 E3 @W1 E4 @W2	WEEK 10 W4 @E1 W1 @E2 W2 @E3 W3 @E4

Team Schedules

E1		
R	Home	Away
1		E4
2	E4	
3		E3
4	E3	
5	E2	
6		E2
7		W1
8	W2	
9		W3
10	W4	

E2		
R	Home	Away
1		E3
2	E3	
3		E4
4	E4	
5		E1
6	E1	
7		W2
8	W3	
9		W4
10	W1	

E3		
R	Home	Away
1	E2	
2		E2
3	E1	
4		E1
5		E4
6	E4	
7		W3
8	W4	
9		W1
10	W2	

E4		
R	Home	Away
1	E1	
2		E1
3	E2	
4		E2
5	E3	
6		E3
7		W4
8	W1	
9		W2
10	W3	

W1		
R	Home	Away
1		W4
2	W4	
3		W3
4	W3	
5	W2	
6		W2
7	E1	
8		E4
9	E3	
10		E2

W2		
R	Home	Away
1		W3
2	WE3	
3		W4
4	W4	
5		W1
6	W1	
7	E2	
8		E1
9	E4	
10		E3

W3		
R	Home	Away
1	W2	
2		W2
3	W1	
4		W1
5		W4
6	W4	
7	E3	
8		E2
9	E1	
10		E4

W4		
R	Home	Away
1	W1	
2		W1
3	W2	
4		W2
5	W3	
6		W3
7	E4	
8		E3
9	E2	
10		E1

APPENDIX 3

XFL 2001 Suggested Schedule – Schedule A

WEEK 1	WEEK 2	WEEK 3	WEEK 4	WEEK 5
E1 @E3	W1 @E1	E1 @W2	E3 @E2	E3 @E4
E2 @E4	W2 @E2	E2 @W1	E4 @E1	E2 @E1
W3 @W1	E3 @W3	W3 @E4	W1 @W4	W2 @W1
W4 @W2	E4 @W4	W4 @E3	W3 @W2	W4 @W3
WEEK 6	WEEK 7	WEEK 8	WEEK 9	WEEK 10
E1 @W3	W4 @E1	E2 @E3	E3 @E1	E4 @E3
E2 @W4	W3 @E2	E1 @E4	E4 @E2	E1 @E2
W1 @E3	E3 @W2	W1 @W3	W4 @W1	W1 @W2
W2 @E4	E4 @W1	W2 @W4	W2 @W3	W3 @W4

Team Schedules — Schedule A

E1		
R	Home	Away
1		E3
2	W1	
3		W2
4	E4	
5	E2	
6		W3
7	W4	
8		E4
9	E3	
10		E2

E2		
R	Home	Away
1		E4
2	W2	
3		W1
4	E3	
5		E1
6		W4
7	W3	
8		E3
9	E4	
10	E1	

E3		
R	Home	Away
1	E1	
2		W3
3	W4	
4		E2
5		E4
6	W1	
7		W2
8	E2	
9		E1
10	E4	

E4		
R	Home	Away
1	E2	
2		W4
3	W3	
4		E1
5	E3	
6	W2	
7		W1
8	E1	
9		E2
10		E3

W1		
R	Home	Away
1	W3	
2		E1
3	E2	
4		W4
5	W2	
6		E3
7	E4	
8		W3
9	W4	
10		W2

W2		
R	Home	Away
1	W4	
2		E2
3	E1	
4	W3	
5		W1
6		E4
7	E3	
8		W4
9		W3
10	W1	

W3		
R	Home	Away
1		W1
2	E3	
3		E4
4		W2
5	W4	
6	E1	
7		E2
8	W1	
9	W2	
10		W4

W4		
R	Home	Away
1		W2
2	E4	
3		E3
4	W1	
5		W3
6	E2	
7		E1
8	W2	
9		W1
10	W3	

XFL 2001 Suggested Schedule — Schedule B

WEEK 1	WEEK 2	WEEK 3	WEEK 4	WEEK 5
E4 @E3	E2 @E4	E4 @E1	E3 @E4	E1 @W1
E2 @E1	E1 @E3	E3 @E2	W1 @W2	E2 @W2
W3 @W4	W4 @W2	W1 @W4	W4 @E2	W4 @E4
W2 @W1	W3 @W1	W2 @W3	W3 @E1	W3 @E3
WEEK 6	WEEK 7	WEEK 8	WEEK 9	WEEK 10
E1 @W4	W1 @E2	E1 @E2	E1 @E4	E4 @E2
E2 @W3	W2 @E1	W4 @W3	E2 @E3	E3 @E1
W1 @E3	E3 @W4	E3 @W2	W4 @W1	W2 @W4
W2 @E4	E4 @W3	E4 @W1	W3 @W2	W1 @W3

Team Schedules — Schedule B

E1		
R	Home	Away
1	E2	
2		E3
3	E4	
4	W3	
5		W1
6		W4
7	W2	
8		E2
9		E4
10	E3	

E2		
R	Home	Away
1		E1
2		E4
3	E3	
4	W4	
5		W2
6		W3
7	W1	
8	E1	
9		E3
10	E4	

E3		
R	Home	Away
1	E4	
2	E1	
3		E2
4		E4
5	W3	
6	W1	
7		W4
8		W2
9	E2	
10		E1

E4		
R	Home	Away
1		E3
2	E2	
3		E1
4	E3	
5	W4	
6	W2	
7		W3
8		W1
9	E1	
10		E2

W1		
R	Home	Away
1	W2	
2	W3	
3		W4
4		W2
5	E1	
6		E3
7		E2
8	E4	
9	W4	
10		W3

W2		
R	Home	Away
1		W1
2	W4	
3		W3
4	W1	
5	E2	
6		E4
7		E1
8	E3	
9	W3	
10		W4

W3		
R	Home	Away
1		W4
2		W1
3	W2	
4		E1
5		E3
6	E2	
7	E4	
8	W4	
9		W2
10	W1	

W4		
R	Home	Away
1	W3	
2		W2
3	W1	
4		E2
5		E4
6	E1	
7	E3	
8		W3
9		W1
10	W2	

XFL 2001 Suggested Schedule — Schedule C

WEEK 1	WEEK 2	WEEK 3	WEEK 4	WEEK 5
E1 @E2	E2 @E3	E1 @E3	E3 @E4	E4 @E1
W1 @W2	W2 @W3	W1 @W3	W3 @W4	W4 @W1
E3 @W4	E4 @W1	E2 @E4	E1 @W2	E2 @W3
W3 @E4	W4 @E1	W2 @W4	W1 @E2	W2 @E3
WEEK 6	WEEK 7	WEEK 8	WEEK 9	WEEK 10
E3 @E2	E4 @E3	E3 @E1	E1 @E4	E2 @E1
W2 @W1	W3 @W2	W3 @W1	W4 @W3	W1 @W4
E1 @W3	E2 @W4	E4 @E2	E3 @W1	E4 @W2
W4 @E4	W1 @E1	W4 @W2	W2 @E2	W3 @E3

Team Schedules — Schedule C

E1		
R	Home	Away
1		E2
2	W4	
3		E3
4		W2
5	E4	
6		W3
7	W1	
8	E3	
9		E4
10	E2	

E2		
R	Home	Away
1	E1	
2		E3
3		E4
4	W1	
5		W3
6	E3	
7		W4
8	E4	
9	W2	
10		E1

E3		
R	Home	Away
1		W4
2	E2	
3	E1	
4		E4
5	W2	
6		E2
7	E4	
8		E1
9		W1
10	W3	

E4		
R	Home	Away
1	W3	
2		W11
3	E2	
4	E3	
5		E1
6	W4	
7		E3
8		E2
9	E1	
10		W2

W1		
R	Home	Away
1		W2
2	E4	
3		W3
4		E2
5	W4	
6	W2	
7		E1
8	W3	
9	E3	
10		W4

W2		
R	Home	Away
1	W1	
2		W3
3		W4
4	E1	
5		E3
6		W1
7	W3	
8	W4	
9		E2
10	E4	

W3		
R	Home	Away
1		E4
2	W2	
3	W1	
4		W4
5	E2	
6	E1	
7		W2
8		W1
9	W4	
10		E3

W4		
R	Home	Away
1	E3	
2		E1
3	W2	
4	W3	
5		W1
6		E4
7	E2	
8		W2
9		W3
10	W1	