A Singular Direct Product for Bicolorable Steiner Triple Systems

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Abstract.

A Steiner triple system has a bicoloring with m color classes if the points are partitioned into m subsets and the three points in every block are contained in exactly two of the color classes. In this paper we generalize the direct product theorem for bicolored Steiner triple systems given in [3] to a singular direct product theorem of the form $v \to 3(v-1)+1$. Our construction uses a generalization of the "forbidden latin squares" introduced in [3]. We also consider possible singular direct products of the form $v \to 3(v-w)+w$.

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1. Introduction and Background

Throughout this paper we use notation consistent with that found in [2]. Let $\mathcal{D} = (V, \mathcal{B})$ be a (v, k, λ)-design. A *coloring* of $\mathcal{D}$ is a mapping $\varphi : V \to C$. The elements of C are *colors*; if $|C| = m$, we have an *m-coloring* of $\mathcal{D}$. For each $c \in C$, the set $\varphi^{-1}(c) = \{x : \varphi(x) = c\}$ is a *color class*. For an extensive survey of results on coloring designs, the reader is referred to [8]. Here we consider a special class of colorings termed bicolorings. While a bicoloring is defined for any design, we examine only bicolorings of Steiner triple systems.

A coloring φ of $\mathcal{D}$ is a *bicoloring* if for all $B \in \mathcal{B}$, $|\ \varphi(B)\ | = 2$, where $\varphi(B) = \cup_{v \in B} \varphi(v)$. This definition implies that in a triple system every triple has two elements in one color class and one in another class, i.e., there are no monochromatic triples nor are there any triples receiving three colors.

An m-bicoloring is a bicoloring with m color classes, and a design admitting an m-bicoloring is m-bicolorable. A design is *m-bichromatic* if it is m-bicolorable but not $(m-1)$-bicolorable.

Example 1.1. A 3-bicolorable STS(13). First, construct an STS(13) by developing the base blocks $\{1,3,9\},\{2,5,6\}$ mod 13. The color classes are $\{0,1\},\{2,6,8,10,11\}$, $\{3,4,5,7,9,12\}$. □

In the context of strict colorings of hypergraphs defined recently by Voloshin [10], a bicoloring of an STS is a strict coloring of an STS in which all triples are both edges and also co-edges. In [4, 5], Milazzo and Tuza discuss several properties of strict colorings of Steiner triple systems. A second related topic is studied recently by Milici, Rosa, Voloshin [6]. In this paper the authors let S be a set of "color patterns" and define a coloring of type S as a coloring where every block has color pattern from S. They mainly study $(v,4,1)$-designs in that paper.

We summarize earlier results on bicolored Steiner triple systems. An easy counting argument [7] establishes that there exist no nontrivial 2-colorable STS (or 2-colorable triple systems of any index λ for $v>4$), and hence no 2-bichromatic triple systems. In [4, 5], Milazzo and Tuza discuss several properties of bicolorings of Steiner triple systems. In particular they prove that there is an infinite family of unbicolorable Steiner triple systems. They also prove a bound on the maximum number of colors in a bicolored Steiner triple system. Precisely, they prove that if there exists a t-bicolorable STS(v) with $v \le 2^k-1$, then $t \le k$. They also characterize those designs attaining this bound.

In [3] the authors concentrate on 3-bicoloring Steiner triple systems. In that paper, the following general necessary conditions are proven:

Proposition 1.2. *Let $(X,\mathcal{A})$ be an $m-$bicolorable triple system TS(v,λ) and assume that the m color classes have sizes $c_1,c_2,\dots,c_m$. Then*

1. $\sum_{i=1}^{m}\binom{c_i}{2}=\binom{v}{2}/3$.
2. *There do not exist c_i and c_j, $i\neq j$, with $c_i=c_j=2$ (no matter what the size of the other color classes are).*
3. *At most one of numbers $c_1,c_2,\dots,c_m$ can be odd.*
4. *Let $v\equiv 1,3 \pmod 6$. If there exists an m-bicolorable STS(v) with m-split $(c_1,\dots,c_k,d_1,\dots,d_{m-k})$ (with $0<k<m$), then the inequality*

$$\begin{aligned} 0 &\le \left[\sum_{i=1}^{k}\binom{c_i}{2}\right]-\frac{1}{2}\left[\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}c_ic_j\right] \\ &\le \frac{1}{2}\left[\sum_{i=1}^{k}c_i\right]\cdot\left[\sum_{i=1}^{m-k}d_i\right]-\ell\left(\sum_{i=1}^{m-k}d_i\right) \end{aligned}$$

holds, where

$$\ell(x) = \begin{cases} x/2 & \textit{if} \quad x \equiv 0,2 \pmod 6 \\ 0 & \textit{if} \quad x \equiv 1,3 \pmod 6 \\ (x+2)/2 & \textit{if} \quad x \equiv 4 \pmod 6 \\ 4 & \textit{if} \quad x \equiv 5 \pmod 6 \end{cases}$$

5. *If there exists a 3-bicolorable STS(v), then any prime p dividing v with $p \equiv 5 \pmod 6$ must have an even power in the prime factorization of v.*

Also in [3] is the following direct product theorem for 3-bicolorable STS.

Theorem 1.3. *If there exists a 3-bicolorable STS(u) and a 3-bicolorable STS(v), then there exists a 3-bicolorable STS(uv).*

In the remainder of this paper we will be concerned with modifying this construction to obtain a singular direct product theorem for 3-bicolorable STS. We are still unable to prove the following conjecture from [3], yet we also believe it to be true.

Conjecture 1.4. *For every $v \equiv 1,3 \pmod 6$, satisfying the condition (5) in Proposition 1.2 and for all 3-splits (a,b,c) for v satisfying conditions (1) and (2) of Proposition 1.2, there exists a 3-bicolorable STS(v) with color classes of sizes $a, b,$ and c.*

The following theorem from [3] gives the current state of knowledge concerning the existence of 3-bicolorable STS(v).

Theorem 1.5. *Let $v \equiv 1,3 \pmod 6$ and assume that in the prime factorization of v no prime congruent to 5 (mod 6) appears with an odd exponent. Further assume that all prime factors p congruent to 1 (mod 6) are less than 1000 and that all prime factors p congruent to 5 (mod 6) satisfy $p^2 < 1000$. Then there exists a 3-bicolorable STS(v).*

2. Forbidden Latin Squares

Underlying the singular direct product theorem is a special type of latin square termed a forbidden latin square. A special class of these latin squares was used in the proof of Theorem 1.3, but a more general definition is needed for the singular direct product.

Suppose $n = a+b+c = x+y+z$. Let A, B and C be disjoint sets of sizes a, b and c, respectively; and let X, Y and Z be disjoint sets of sizes x, y and z, respectively. A latin square with rows and columns indexed by $A \cup B \cup C$ and symbols in the set $X \cup Y \cup Z$ is called $(a, b, c; x, y, z)-$*forbidden* if in cell (r, g) we find symbol s satisfying:

r in A and g in A implies s not in X
r in A and g in B implies s not in Z
r in A and g in C implies s not in Y
r in B and g in A implies s not in Z
r in B and g in B implies s not in Y
r in B and g in C implies s not in X
r in C and g in A implies s not in Y
r in C and g in B implies s not in X
r in C and g in C implies s not in Z.

The following gives the general picture of an $(a, b, c; x, y, z)-$forbidden latin square. The notation $\sim X$ denotes that the symbols in this region of the latin square contain no elements from the set X. $\sim Y$ and $\sim Z$ are defined analogously. Each region is indexed by the elements in A, B and C and this is also indicated.

	A	B	C
A	$\sim X$	$\sim Z$	$\sim Y$
B	$\sim Z$	$\sim Y$	$\sim X$
C	$\sim Y$	$\sim X$	$\sim Z$

We note that forbidden latin squares were defined in [3]. The (a, b, c)-forbidden latin square from that paper are $(a, b, c; c, a, b)$-forbidden latin square in this more general definition.

Example 2.1. A $(5,5,2;6,1,5)$-FLS. In this example, $X = \{1,2,3,4,5,6\}$, $Y = \{x\}$ and $Z = \{a,b,c,d,e\}$.

a	c	e	b	d	2	x	6	3	1	4	5
e	b	d	a	c	1	3	x	2	4	5	6
d	a	c	e	b	5	1	4	x	3	6	2
c	e	b	d	a	4	6	1	5	x	2	3
b	d	a	c	e	x	5	2	1	6	3	4
2	x	6	3	1	a	4	e	b	5	c	d
1	3	x	2	4	6	b	5	a	c	d	e
5	1	4	x	3	d	2	c	6	b	e	a
4	6	1	5	x	c	e	3	d	2	a	b
x	5	2	1	6	3	d	a	4	e	b	c
3	4	5	6	2	b	c	d	e	a	x	1
6	2	3	4	5	e	a	b	c	d	1	x

□

In the next theorem we give some necessary conditions for the existence of forbidden latin squares.

Theorem 2.2. *The following are necessary conditions for existence of an* $(a,b,c;x,y,z)$*-FLS:*

$$\begin{aligned} \max\{x,y,z\} &\leq \min\{a+b,a+c,b+c\} && (1) \\ \max\{a,b,c\} &\leq \min\{a+b,a+c,b+c,x+y,x+z,y+z\} && (2) \\ ax+by+cz &= ab+bc+ca. && (3) \end{aligned}$$

Proof. (1) and (2): Let $r \in B$. Each symbol $s \in X$ occurs in a different cell in row r. Also, no symbol occurs in cell (r,g) for any $g \in C$. Hence, $x \leq a+b$. The other inequalities can be proven in a similar manner.

(3): Given an $(a,b,c;x,y,z)$-FLS, construct a set of $(a+b+c)^2$ (ordered) triples in the obvious way. This set of triples forms a transversal design, in which the groups are $G_1 = A \cup B \cup C$, $G_2 = A \cup B \cup C$ and $G_3 = X \cup Y \cup Z$. This transversal design is in fact bicolored using three colors, where the color classes are of size $2a+x$, $2b+y$ and $2c+z$. This means that each triple contains exactly one "pure pair". The number of triples, $(a+b+c)^2$, is therefore equal to the number of pure pairs, namely, $a^2+b^2+c^2+2(ax+by+cz)$. The result follows from algebraic simplification. □

These necessary conditions are, in general, quite restrictive, but we note that in the case of the $(a,b,c;c,a,b)$-forbidden latin squares from [3], that condition (1) alone was shown to be necessary and sufficient. (Note that, in this case, condition (3) is vacuously true and condition (2) is equivalent to condition (1).)

We next construct an infinite class of forbidden latin squares that will be needed in the singular direct product construction in Section 3.

Theorem 2.3. *Suppose there is a latin square of order $a-1$ having b disjoint transversals, then there exists an $(a-1, a-1, b; a, b-1, a-1)$-FLS.*

Proof. Let X be a set of size a where $\infty \in X$, let Y be a set of size $b-1$, and let Z be a set of size $a-1$ with X, Y and Z mutually disjoint. Let L be a latin square of order $a-1$ on symbol set $X \backslash \{\infty\}$, having b disjoint transversals, denoted $T_1, \ldots, T_b$. Let L_Z be an isomorphic copy of L on symbol set Z.

Next, define L' to be the square obtained from L by replacing the transversals $T_1, \ldots, T_b$ with symbols from $Y \cup \{\infty\}$, respectively. Similarly, define L'_Z to be the square obtained from L_Z by replacing the transversals $T_1, \ldots, T_b$ from L_Z with the corresponding transversals $T_1, \ldots, T_b$ from L.

Now define M to be the $a-1$ by b rectangle whose columns are the transversals $T_1, \ldots, T_b$ from L; and define M_Z to be the $a-1$ by b rectangle whose columns are the transversals $T_1, \ldots, T_b$ from L_Z. Similarly, define N to be the b by $a-1$ rectangle whose rows are the transversals $T_1, \ldots, T_b$ from L; and define N_Z to be the b by $a-1$ rectangle whose rows are the transversals $T_1, \ldots, T_b$ from L_Z. Finally, let L'' be a latin square of order b on symbol set $Y \cup \{\infty\}$.

We construct the desired FLS, as follows:

L_Z	L'	M
L'	L'_Z	M_Z
N	N_Z	L''

It is straightforward to check that the square constructed above is indeed latin and that it satisfies the forbidden properties. □

Corollary 2.4. *An $(a-1, a-1, b; a, b-1, a-1)$-FLS exists for all positive integers $a > b$.*

Proof. By Theorem 2.3 the required forbidden latin square exists if there exists a latin square of side $a-1$ having b disjoint transversals. Such a square can be constructed if there exists a pair of orthogonal latin squares of side $a-1$. Hence if $a-1 \neq 2$ or 6, the result follows. Since there exists a pair of incomplete latin squares of side 6 missing a hole of side 2, there exists a latin square of side 6 having 4 (or fewer) disjoint transversals. So for all pairs $(a, b) \notin \{(3,1), (3,2), (7,5), (7,6)\}$ with $a > b$ the result follows. For these remaining ordered pairs (a, b), $(a-1, a-1, b; a, b-1, a-1)$-FLS can be found on the web page at the following URL:

`http://www.emba.uvm.edu/~Dinitz/forbiddenLS.html`

□

It is not necessary that $a > b$ for an $(a-1, a-1, b; a, b-1, a-1)$-FLS to exist. In the next example we describe a construction of such a square when $a = 6$ and $b = 10$.

Example 2.5. A $(5, 5, 10; 6, 9, 5)$-FLS.

Define $X = \{x_1, \ldots, x_5, \infty\}$, $Y = \{y_1, \ldots, y_9\}$ and $Z = \{z_1, \ldots, z_5\}$. Let U be a latin square of order five on symbol set $X \backslash \{\infty\}$; let V_1 be a latin square of order five on symbol set $\{y_1, \ldots, y_5\}$; let V_2 be a latin square of order five on symbol set $\{y_6, \ldots, y_9\} \cup \{\infty\}$; and let W be a latin square of order five on symbol set $\{z_1, \ldots, z_5\}$. Then the following array is the desired FLS:

V_1	V_2	W	U
V_2	U	V_1	W
W	V_1	U	V_2
U	W	V_2	V_1

□

We can adapt Stinson's hill climbing algorithm for Steiner triple systems [9] to find latin squares. One merely lets the triples be of the form (r, c, s) where the symbol s occurs in row r and column c and requires that every pair of row-column, row-symbol and column-symbol occurs exactly once. The algorithm can then be further modified to search for forbidden latin squares. We have done this and found $(a-1, a-1, b; a, b-1, a-1)$-FLS for many pairs (a, b) with $a \le b \le 2a-2$. (Note that Theorem 2.2 implies that $b \le 2a-2$ if an $(a-1, a-1, b; a, b-1, a-1)$-FLS exists.)

Using a hill-climbing algorithm, we have found $(a-1, a-1, b; a, b-1, a-1)$-FLS for $a = 4, 5, 6$ and 7 for all $a \le b \le 2a-2$. These squares are available from the above-mentioned web page.

3. The Singular Direct Product

We are now in a position to prove our main result, a $v \to 3(v-1)+1$ singular direct product theorem.

Theorem 3.1. *(Singular Direct Product) Suppose there is a* 3*-bicolorable* $STS(v)$ *with split* $(a, a-1, b)$*, and an* $(a-1, a-1, b; a, b-1, a-1)$*-FLS. Then there exists a* 3*-bicolorable* $STS(3v-2)$ *with split* $(3a-2, 2b+a-1, 2a+b-2)$.

Proof. Define sets V_{ij} of elements with $i, j \in \{0, 1, 2\}$, so that V_{ij} has $a-1, a-1, b$ elements for $j = 0, 1, 2$ respectively, when $0 \le i \le 1$; and $a, b-1, a-1$ elements for $j = 0, 1, 2$ respectively, when $i = 2$. The union of V_{ij} for $0 \le i \le 2$ then has $3a-2$, $2a+b-3$ and $2b+a-1$ elements for $j = 0, 1, 2$, respectively. Let ∞ be a new point.

For $i = 0, \ldots, 2$, place on the union of V_{ij} for $j = 0, 1, 2$ with $\{\infty\}$ an $(a, a-1, b)-$bicolored STS(v) in which the color classes are V_{i0}, $V_{i1} \cup \{\infty\}$ and V_{i2}. Now form an $(a-1, a-1, b; a, b-1, a-1)$-FLS. Use the latin square to construct triples in the obvious way (i.e., form the transversal design from the latin square and align the row, column, and symbol classes on the corresponding V_{fj}'s, V_{gj}'s and V_{hj}'s).

The result is a bicolorable STS$(3v-2)$ whose color classes have the specified sizes. □

Corollary 3.2. *Suppose there is a* 3*-bicolorable STS*(v) *with split* $(a, a-1, b)$*, where* $a > b$*. Then there is a* 3*-bicolorable STS*$(3v-2)$ *with split* $(3a-2, 2b+a-1, 2a+b-2)$.

Proof. This follows immediately from the singular direct product theorem above and Corollary 2.4. □

Theorem 3.4 below will show that it is necessary that $a > b$ in order for there to exist a 3-bicolorable STS(v) with split $(a, a-1, b)$. Hence the condition that $a > b$ in the above corollary is not needed.

One may ask whether there exists a singular direct product theorem of the form $v \to 3(v-1)+1$ which does not require that the original 3-bicolorable STS(v) has color split $(a, a-1, b)$. The answer to this is no, as exhibited in the next proposition.

Proposition 3.3. *In any singular direct product theorem of the form* $v \to 3(v-1)+1$ *it is necessary that the original 3-bicolorable STS(*v*) has color split* $(a, a-1, b)$ *for some* a *and* b.

Proof. Consider a hypothetical $v \to 3(v-1)+1$ construction in which we use, WLOG, an $(a, b-1, c; b, c-1, a)$-FLS. Note that this implies that the point ∞ again ends up in the second color class and that the original STS(v) has color split (a, b, c). Theorem 2.2 (3) implies that

$$a(b-1) + c(b-1) + ac = ab + (b-1)(c-1) + ac.$$

This simplifies to yield $b - 1 = a$, which is isomorphic to the construction presented above. □

Since our main ingredient in the singular direct product theorem (other than the forbidden latin square) is a 3-bicolorable STS(v) with split $(a, a-1, b)$, it is reasonable to determine the values of a and b for which this can exist.

Theorem 3.4. *A 3-bicolorable STS(v) with split $(a, a-1, b)$ can exist only for the following parameters:*

$$a = 3t^2 + 2t + 1 \quad \textit{and} \quad b = 3t^2 - t$$

or

$$a = 3t^2 + 4t + 2 \quad \textit{and} \quad b = 3t^2 + t$$

where t is any integer.

Proof. From Proposition 1.2 (a) it follows that

$$3\left(\binom{a}{2} + \binom{a-1}{2} + \binom{b}{2}\right) = \binom{2a+b-1}{2}.$$

This simplifies to give

$$(a-b)^2 = 3a - 2.$$

Then $a - b = 3t + 1$ or $3t + 2$ for an integer t. Solving for a and b gives the result. □

4. The $v \to 3(v-u) + u$ Construction

The next theorem gives a general $v \to 3(v-w) + w$ construction. Let $\mathcal{D} = (X, \mathcal{A})$ be a 3-bicolorable Steiner triple system of order v with split (a, b, c) which contains a subsystem $(Y, \mathcal{B})$ of order w (so $Y \subset X$, $|Y| = w$ and $\mathcal{B} \subset \mathcal{A}$). Let A, B, and C be the color classes of $\mathcal{D}$ and assume that $|A \cap Y| = i$, $|B \cap Y| = j$ and $|C \cap Y| = k$. Then D is said to have *color split* $(a, b, c; i, j, k)$.

Theorem 4.1. *Suppose there is a 3-bicolorable STS(v) with a sub STS(w) which has color split $(a, b, c; i, j, k)$ and a 3-bicolorable STS(v) with a sub STS(w) which has color split $(a, b, c; j, k, i)$. Suppose further that there exists an $(a-i, b-j, c-k; c-i, a-j, b-k)$-FLS. Then there exists a 3-bicolorable STS$(3(v-w)+w)$ with split $(2a+c-2i, 2b+a-2j, 2c+b-2k)$.*

Proof. Define disjoint sets V_{pq} of elements with $p, q \in \{0, 1, 2\}$, so that V_{pq} has $a-i, b-j, c-k$ elements for $q = 0, 1, 2$ respectively, when $p = 0$ or $p = 1$; and $c-i, a-j, b-k$ elements for $q = 0, 1, 2$ respectively, when $p = 2$. The union of V_{pq} for $0 \leq p \leq 2$ then has $2a+c-3i$, $2b+a-3j$ and $2c+b-3k$ elements for $q = 0, 1, 2$, respectively. Further, define three more disjoint sets I, J and K with $|I| = i, |J| = j$ and $|K| = k$.

For $p = 0$ and $p = 1$, place on $V_{p0} \cup V_{p1} \cup V_{p2} \cup I \cup J \cup K$ an STS(v) containing a sub-STS(w) (where the subsystem is on the points $I \cup J \cup K$)

with color split $(a, b, c; i, j, k)$ in which the color classes are $V_{p0} \cup I$, $V_{p1} \cup J$ and $V_{p2} \cup K$. Next, delete all blocks in the subsystem from both designs. For $p = 3$, place on the union of $V_{p0} \cup V_{p1} \cup V_{p2} \cup I \cup J \cup K$ an STS(v) containing a sub-STS(w) (the subsystem is again on the points $I \cup J \cup K$) with color split $(c, a, b; i, j, k)$ in which the color classes are $V_{p0} \cup I$, $V_{p1} \cup J$ and $V_{p2} \cup K$. But this time do not delete any blocks in the subsystem.

Now form an $(a - i, b - j, c - k; c - i, a - j, b - k)$-FLS. Use the latin square to construct triples in the obvious way (i.e., form the transversal design from the latin square and align the row, column, and symbol classes on the corresponding V_{0q}'s, V_{1q}'s and V_{2q}'s).

The result is a bicolorable STS$(3(v - w) + w)$ whose color classes have the specified sizes. □

Obviously, there will be many conditions on the parameters necessary for this construction to be used. We will not go into those here. We will, however note that the next case for w, after the $w = 1$ case considered in the previous section, is of course $w = 3$. When $w = 3$, the only possible color split is $(0, 1, 2)$. We now restate the above theorem in the case $w = 3$.

Corollary 4.2. *Suppose there is a* 3*-bicolorable STS*(v) *with split* (a, b, c)*, and an* $(a, b - 1, c - 2; c, a - 1, b - 2)$*-FLS. Then there exists a* 3*-bicolorable STS*$(3v - 6)$ *with split* $(2a + c, 2b + a - 2, 2c + b - 4)$*.*

A necessary condition for the existence of a $(a, b-1, c-2; c, a-1, b-2)$-FLS required above is $2a + 3 = b + c$. (This follows from condition (3) of Theorem 2.2.) The first parameter situation where all the necessary conditions are satisfied is when $a = 12$, $b = 10$ and $c = 17$. This would require the existence of a 3-bicolorable STS(39) with split $(12, 10, 17)$, and a $(12, 9, 15; 17, 11, 8)$-FLS. The 3-bicolorable STS(39) with color split $(12, 10, 17)$ was found to exist in [3], and using the modified hill-climbing algorithm we found a $(12, 9, 15; 17, 11, 8)$-FLS. (This latin square can also be obtained from the previously mentioned web page). Hence, in this particular parameter situation, the singular direct product can be used.

We believe that there are many other cases where this construction can be used; however we have not searched for others at this time.

5. Conclusion and Open Problems

The problem of bicolorable Steiner triple systems remains open, of course. We have shown in that the direct product construction from [3] can be modified to a singular direct product construction. The singular direct product construction depends on the existence of a generalized form of forbidden latin square, which is an interesting open problem in its own right. In particular, we ask if the necessary conditions from Theorem 2.2 are sufficient for existence of an $(a, b, c; x, y, z)$-FLS.

An obvious further generalization of forbidden latin squares is to specify possibly different partitions associated with rows, columns and symbols, i.e., an $(a, b, c; u, v, w; x, y, z)$-FLS. Necessary conditions analogous to those of Theorem 2.2 could easily be written down, and one could perhaps formulate more general versions of the singular direct product construction that would use these more general forbidden squares.

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References

[1] M. de Brandes, K. T. Phelps and V. Rödl. Coloring Steiner triple systems, *SIAM J. Alg. Disc. Meth.* 3 (1982), 241-249.

[2] C. J. Colbourn and J. H. Dinitz, eds. *The CRC Handbook of Combinatorial Designs*, CRC Press, Inc., 1996.

[3] C. J. Colbourn, J. H. Dinitz and A. Rosa. Bicoloring Steiner Triple Systems, *Electron. J. Combin.* **6** (1999), #R25, 16 pages.

[4] L. Milazzo and Zs. Tuza. Upper chromatic number of Steiner triple and quadruple systems, *Discrete Math.* 174 (1997), 247-259.

[5] L. Milazzo and Zs. Tuza. Strict colourings for classes of Steiner triple systems, *Discrete Math.* 182 (1998), 233-243.

[6] S. Milici, A. Rosa and V. Voloshin. Colouring Steiner systems with specified block colour patterns, preprint.

[7] A. Rosa. Steiner triple systems and their chromatic number, *Acta Fac. Rer. Nat. Univer. Comen. Math.* 24 (1970), 159-174.

[8] A. Rosa and C. J. Colbourn. Colorings of block designs. In *Contemporary Design Theory: A Collection of Surveys*, J. H. Dinitz and D. R. Stinson, eds., John Wiley & Sons, 1992, pp. 401-430.

[9] D. R. Stinson. Hill-climbing algorithms for the construction of combinatorial designs, *Ann. Discrete Math.* 26 (1985), 321-334.

[10] V. I. Voloshin. On the upper chromatic number of a hypergraph, *Australas. J. Combin.* 11 (1995), 25-45.

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