Block Disjoint Difference Families for Steiner Triple Systems: $v \equiv 3$ mod 6

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Abstract

A block disjoint (v,k,λ) difference family is a difference family with disjoint blocks. We show that disjoint $(v, 3, 1)$ difference families exist for all $v \equiv 3 \bmod 6$ with $v \geq 3$.

1 Introduction

Let G be a group of order v. A family of k-tuples of elements from G is a (v,k,λ) difference family if the collection of orbits of the k-tuples (disregarding repeated k-tuples) under the action of G form a balanced incomplete block design, BIBD(v, k, λ). If the k-tuples are pairwise disjoint, call the family a *block disjoint* (v,k,λ) *difference family.*

In this paper we show that there exists a block disjoint (v,3,1) difference family for all $v \equiv 3 \bmod 6$. This is a companion paper to [2] which considered the case of block disjoint (v,3,1) difference family for $v \equiv 1 \bmod 6$. The reader is refered to that paper for background information.

2 Constructions

In this section we give constructions for block disjoint (v,3,1) difference families when $v \equiv 3 \bmod 6$. We give a different construction for each congruence of v modulo 24.

Theorem 1 *There exists a block disjoint (24k+3,3,1) difference family for* $k \geq 0$.

Beginning with a (24k+3,3,1) difference family that does not have disjoint blocks, the idea is to translate the blocks so that no two intersect. The initial difference family is given below. It is obtained from the so called Rosa triple system of order $4k$ first found (by Rosa) in [3], but easier for the reader to find in [1], Theorem 8.3.8.

(1a)	0	$8k+1$	$16k+2$	
(1b)	0	1	$6k$	
(1c)	0	$2k+1$	$9k+1$	
(1d)	0	$2k$	$12k+1$	
(1e)	0	$4k$	$11k+1$	
(1f)	0	$2k+2r$	$11k+r+1$	$1 \le r \le k-1$
(1g)	0	$2k+2r+1$	$7k+r+1$	$1 \le r \le k-1$
(1h)	0	$2r+1$	$6k+r$	$1 \le r \le k-1$
(1i)	0	$2r$	$10k+r+1$	$1 \le r \le k-1$

From the difference family above we obtain the following block disjoint difference family by linear translation of the triples. This construction requires that $k \ge 3$ and $k \ne 4$. The smaller cases of k are treated separately.

(1j)	$2k+1$	$10k+2$	$18k+3$	
(1k)	$10k$	$10k+1$	$16k$	
(1ℓ)	$14k+1$	$16k+2$	$23k+2$	
(1m)	$18k+1$	$20k+1$	$6k-1$	
(1n)	$19k+3$	$23k+3$	$6k+1$	
(1o)	$2r$	$2k+4r$	$11k+3r+1$	$1 \le r \le k-1$
(1p)	$2r+1$	$2k+4r+2$	$7k+3r+2$	$1 \le r \le k-1$
(1q)	$14k+2r$	$14k+4r+1$	$20k+3r$	$1 \le r \le k-1$
(1r)	$6k+r+x$	$6k+3r+x$	$16k+2r+1+x$	$1 \le r \le k-1$
				$r \not\equiv 0 \bmod 3$
(1s)	$5k+2s+y$	$5k-4s+y$	$19k-s+2+y$	$1 \le s \le (k-1)/3$

where $x = 1$ if $k \equiv 0, 1 \bmod 3$ and $x = 3$ if $k \equiv 2 \bmod 3$, and $y = 0$ if k is odd and $y = 1$ if k is even. Note that the (1s) is the case where $r \equiv 0 \bmod 3$ from the case (1r) and that in this case $3s = r$.

To aid the reader, in the table below we will explicitly give the translation from the first difference family to the second one.

From	To	Add	Comments
(1a)	(1j)	$2k+1$	
(1b)	(1k)	$10k$	
(1c)	(1ℓ)	$14k+1$	
(1d)	(1m)	$18k+1$	
(1e)	(1n)	$19k+3$	
(1f)	(1o)	$2r$	for $1 \le r \le k-1$
(1g)	(1p)	$2r+1$	for $1 \le r \le k-1$
(1h)	(1q)	$14k+2r$	for $1 \le r \le k-1$
(1i)	(1r)	$6k+r+x$	for $1 \le r \le k-1$ if $r \not\equiv 0 \bmod 3$
(1i)	(1s)	$5k+2s+y$	to the triples $\{0, -6s, 14k-3s+2\}$ for $1 \le s \le (k-1)/3$

Since the first set given is a difference family and the second is just obtained from the first by translations (and one flip), then clearly the new set of triples is also a difference family. It is not difficult to check that the blocks are indeed disjoint.

The following small cases complete the proof.

$v = 3$ ($k = 0$): (0,1,2)

$v = 27$ ($k = 1$): (1,10,19), (2,3,8), (4,7,14), (11,13,24), (5,9,17)

$v = 51$ ($k = 2$): (1,18,35), (2,3,14), (4,9,23), (6,10,31), (5,13,28), (20,26,44), (8,15,24), (19,22,32), (25,27,47)

$v = 99$ ($k = 4$) : (29,35,90), (9,42,75), (40,41,64), (57,66,94), (73,81,23), (79,95,25), (2,12,48), (4,16,51), (6,20,54), (3,14,33), (5,18,36), (7,22,39), (58,61,83), (60,65,86), (62,69,89), (26,28,68) (27,31,70).

Theorem 2 *There exists a block disjoint (24k+9,3,1) difference family for* $k \ge 1$.

We again begin with a ($24k$+9,3,1) difference family that does not have disjoint blocks and then translate the blocks. The initial difference family is given below. It is obtained from the Rosa triple system of order $4k+1$ found in [1], Theorem 8.3.8.

(2a)	0	$8k+3$	$16k+6$	
(2b)	0	1	$11k+5$	
(2c)	0	$2k+3$	$12k+5$	
(2d)	0	$4k+1$	$10k+3$	
(2e)	0	$2r$	$6k+r+2$	$1 \leq r \leq 2k$
(2f)	0	$2k+2r+3$	$11k+r+5$	$1 \leq r \leq k-2$
(2g)	0	$2r+1$	$10k+r+3$	$1 \leq r \leq k$

From the difference family above we obtain the following disjoint difference family by linear translation of the triples. This construction requires that $k \geq 5,\ k \neq 10$.

$(2h)$	$3k+5$	$11k+8$	$19k+11$	
$(2i)$	$24k+6$	$24k+7$	$11k+2$	
$(2j)$	$12k-2+x$	$14k+1+x$	$24k+3+x$	
$(2k)$	$11k+3$	$15k+4$	$21k+6$	
(2ℓ)	r	$3r$	$6k+2r+2$	$1 \leq r \leq 2k$ and $r \not\equiv 0 \bmod 3$
$(2m)$	$17k-2s+2$	$17k+4s+2$	$23k+s+4$	$1 \leq s \leq (2k)/3$
$(2n)$	$10k+r+2$	$12k+3r+5$	$21k+2r+7$	$1 \leq r \leq k-2$
$(2o)$	$17k+2r+7$	$17k+4r+8$	$3k+3r+1$	$1 \leq r \leq k$

where $x=0$ if $k \equiv 0,1 \bmod 3$ and $x=1$ if $k \equiv 2 \bmod 3$. As an aid to verification we give the translations.

From	To	Add	Comments
(2a)	(2h)	$3k+5$	
(2b)	(2i)	-3	
(2c)	(2j)	$12k-2+x$	
(2d)	(2k)	$11k+3$	
(2e)	(2ℓ)	r	for $1 \leq r \leq 2k$, if $r \not\equiv 0 \bmod 3$
(2e)	(2m)	$17k-2s+2$	for $1 \leq s \leq (2k)/3$, if $r \equiv 0 \bmod 3$ where $3s=r$
(2f)	(2n)	$10k+r+2$	for $1 \leq r \leq k-2$
(2g)	(2o)	$2r-7k-2$	for $1 \leq r \leq k$

We complete the proof by giving the small cases.

$v=33$ $(k=1)$: (1,12,23), (2,3,10), (5,7,20), (6,9,15), (13,17,27), (14,19,31)

$v=57$ $(k=2)$: (19,26,48), (5,14,28), (11,30,49), (54,55,24), (1,3,16), (2,6,18), (4,12,22), (34,40,51), (43,46,10), (45,50,13)

$v = 81$ ($k = 3$): (9,18,50), (23,36,56), (14,41,68), (78,79,35), (1,3,22), (2,6,24), (4,12,28), (5,15,30), (51,57,74), (49,61,75), (33,44,72), (60,63,13), (62,67,16), (64,71,19)

$v = 105$ ($k = 4$): (9,10,58), (17,52,87), (46,57,99), (47,64,90), (1,3,28), (2,6,30), (4,12,34), (5,15,36), (7,21,40), (8,24,42), (68,74,97), (66,78,98), (43,56,93), (44,59,95), (77,80,16), (79,84,19), (81,88,22), (83,92,25)

$v = 249$ ($k = 10$): (47,130,213), (246,247,112), (118,141,243), (113,154,216), (1,3,64), (2,6,66), (4,12,70), (5,15,72), (7,21,76), (8,24,78), (10,30,82), (11,33,84), (13,39,88), (14,42,90), (16,48,94), (17,15,96), (19,57,100), (20,60,102), (170,176,235), (168,180,236), (166,184,237), (164,188,238), (162,192,239), (160,196,240), (103,128,219), (104,131,221), (105,134,223), (106,137,225), (107,140,227), (108,143,229), (109,146,231), (110,149,233), (179,182,34), (181,186,37), (183,190,40), (185,194,43),(187,198,46), (189,202,49), (191,206,52), (193,210,55), (195,214,58), (197,218,61).

Theorem 3 *There exists a block disjoint (24k+15,3,1) difference family for* $k \geq 0$.

Once again begin with a (24k+15,3,1) difference family that does not have disjoint blocks and then translate the blocks. The initial difference family is given below. It is obtained from the Rosa triple system of order $4k + 2$ found in [1], Theorem 8.3.8.

(3a)	0	$8k + 5$	$16k + 10$	
(3b)	0	1	$11k + 6$	
(3c)	0	2	$12k + 8$	
(3d)	0	$2k$	$10k + 6$	
(3e)	0	$4k$	$10k + 4$	
(3f)	0	$4k + 2$	$10k + 5$	
(3g)	0	$2r + 1$	$6k + r + 4$	$1 \leq r \leq 2k$
(3h)	0	$2r$	$10k + r + 5$	$2 \leq r \leq k - 1$
(3i)	0	$2k + 2r$	$11k + r + 6$	$1 \leq r \leq k - 1$

From the difference family above we obtain the following disjoint difference family by linear translation of the triples. This construction requires that $k \geq 5$ and $k \neq 10$.

$(3j)$	$k+x$	$9k+5+x$	$17k+10+x$	
$(3k)$	$24k+13$	$24k+14$	$11k+4$	
(3ℓ)	$3k-y$	$3k+2-y$	$15k+8-y$	
$(3m)$	$9k+6$	$11k+6$	$19k+12$	
$(3n)$	$9k+10$	$13k+10$	$19k+14$	
$(3o)$	$9k+12$	$13k+14$	$19k+17$	
$(3p)$	$2r$	$4r+1$	$6k+3r+4$	$1 \le r \le k$
$(3q)$	$4k+r'+1$	$6k+3r'+2$	$11k+2r'+5$	$1 \le r' \le k$
$(3r)$	$5k+r+z$	$5k+3r+z$	$15k+2r+5+z$	$2 \le r \le k-1$
$(3s)$	$13k-2s+4$	$13k+4s+4$	$13k+s+9$	$1 \le s \le \lfloor \frac{k-1}{3} \rfloor$
$(3t)$	$9k+2r+1+w$	$18k+r+7+w$	$20k+3r+7+w$	$1 \le r \le k-1$

where $x = 3$ if $k \equiv 0 \pmod 4$, $x = -2$ if $k \equiv 1 \pmod 4$, $x = 9$ if $k \equiv 3 \pmod 4$, and $x = 0$ if $k \equiv 3 \pmod 4$; $y = (k \bmod 2)$; $z = (k \bmod 3)$; and $w = 0$ if k is even and $w = 4$ if k is odd.

The translations from the first difference family to the second one now follow.

From	To	Add	Comments
(3a)	(3j)	$k+x$	
(3b)	(3k)	-2	
(3c)	(3ℓ)	$3k-y$	
(3d)	(3m)	$9k+6$	
(3e)	(3n)	$9k+10$	
(3f)	(3o)	$9k+12$	
(3g)	(3p)	$2r$	for $1 \le r \le k$
(3g)	(3q)	$4k+r'+1$	where $r' = r-k$ and $k+1 \le r \le 2k$
(3h)	(3r)	$5k+r+z$	for $2 \le r \le k-1$ if $r \not\equiv 0 \bmod 3$
(3h)	(3s)	$13k-2s+4$	$1 \le s \le (k-1)/3$, if $r \equiv 0 \bmod 3$ where $3s = r$
(3i)	(3t)	$9k+2r+1+w$	to the triples $\{0, -2k-2r, -11k-r-6\}$ for $1 \le r \le k-1$

The proof is completed with the small cases below.
$v = 15$ ($k = 0$): (2,7,12), (4,5,8), (1,3,9)

$v = 39$ ($k = 1$): (1,14,27), (2,3,19), (4,6,24), (5,8,16), (7,11,22), (10,15,22), (17,23,32)

$v = 63$ ($k = 2$): (1,22,43), (2,3,30), (4,6,36), (7,11,33), (8,16,32), (9,19,34), (10,13,27), (35,40,53), (5,12,24), (17,26,37), (15,21,44)

$v = 87$ ($k = 3$):(1,30,59), (2,3,41), (4,6,48), (7,13,43), (5,17,39), (9,23,44), (8,11,31), (27,32,51), (15,22,40), (19,28,45), (46,57,73), (24,37,52), (25,29,62), (34,42,74), (26,36,67)

$v = 111$ ($k = 4$): (1,38,75), (2,3,52), (4,6,60), (5,13,51), (9,25,53), (11,29,56), (12,15,41), (14,19,44), (16,23,47), (18,27,50), (22,33,55), (24,37,58), (28,43,63), (32,49,68), (26,30,73), (34,40,82), (35,45,86), (45,66,106), (57,71,110)

$v = 255$ ($k = 10$): (192,107,22),(105,106,221), (120,122,248), (235,215,129), (139,179,243), (141,183,246), (2,5,67), (4,9,70), (6,13,73), (8,17,76), (10,21,79), (12,25,82), (14,29,85), (16,33,88), (18,37,91), (20,41,94), (119,96,44), (118,93,42), (117,90,40), (116,87,38), (115,84,36), (114,81,34), (113,78,32), (112,75,30), (111,72,28), (110,69,26), (188,166,71), (186,162,68), (184,158,65), (182,154,62), (180,150,59), (178,146,56), (176,142,53), (174,138,50), (172,134,47), (244,240,137), (242,234,133), (241,231,131), (239,225,127),(238,222,125), (209,203,101), (210,198,99), (211,193,97)

Theorem 4 *There exists a block disjoint (24k+21,3,1) difference family for* $k \geq 0$.

Again begin with a (24k+21,3,1) difference family that does not have disjoint blocks and then translate the blocks. The initial difference family is given below. It is obtained from the Rosa triple system of order $4k + 3$ found in [1], Theorem 8.3.8. (with the addition of the base block $\{0, 8k + 7, 16k + 14\}$.

(4a)	0	$8k + 7$	$16k + 14$	
(4b)	0	1	$11k + 10$	
(4c)	0	$2k + 1$	$10k + 9$	
(4d)	0	$2k + 3$	$11k + 11$	
(4e)	0	$4k + 3$	$10k + 8$	
(4f)	0	$2r$	$6k + r + 5$	$1 \leq r \leq 2k + 1$
(4g)	0	$2r + 1$	$10k + r + 9$	$1 \leq r \leq k - 1$
(4h)	0	$2k + 2r + 3$	$11k + r + 11$	$1 \leq r \leq k - 1$

From the difference family above we obtain the following difference family by linear translation of the triples. This construction requires that $k \geq 8$,

$k \neq 12$.

$(4i)$	$4k-14+u$	$12k-7+u$	$20k+u$	
$(4j)$	$24k+18+v$	$24k+19+v$	$11k+7+v$	
$(4k)$	$2k-2+w$	$4k-1+w$	$12k+7+w$	
$(4o)$	$6+x$	$2k+9+x$	$11k+17+x$	
$(4p)$	2	$4k+5$	$10k+10$	
$(4q1)$	$14k+r+4$	$14k+3r+4$	$20k+2r+9$	$1 \leq r \leq 2k+1$ and $r \not\equiv 0 \bmod 3$
$(4q2)$	$7k-2s+y$	$7k+4s+y$	$13k+s+5+y$	$1 \leq s \leq (2k+1)/3$
$(4r)$	$2r-1$	$4r$	$10k+3r+8$	$1 \leq r \leq k-1$
$(4s1)$	$6k+2r+1$	$8k+4r+4$	$17k+3r+12$	$1 \leq r \leq (k/2)$ and $k \equiv 2,3 \bmod 4$
$(4s2)$	$6k+2r+3$	$8k+4r+6$	$17k+3r+14$	$1 \leq r \leq (k/2)$ and $k \equiv 0,1 \bmod 4$
$(4s3)$	$3k+2r+6\alpha+z$	$5k+4r+6\alpha+3+z$	$14k+3r+6\alpha+11+z$	for $(k/2)+1 \leq r \leq k-1$

where $u = 8$ if $k \equiv 0 \bmod 3$ and $u = 0$ if $k \equiv 1,2 \bmod 3$; where $v = 0$ if $k \equiv 0 \bmod 3$ and $v = 1$ if $k \equiv 1,2 \bmod 3$; where $w = 0$ if $k \equiv 0 \bmod 6$, $w = 1$ if $k \equiv 1,2 \bmod 3$ and $w = 2$ if $k \equiv 3 \bmod 6$; where $x = 4$ if $k \equiv 0 \bmod 3$ and $x = 0$ if $k \equiv 1,2 \bmod 3$; where $y = 0$ if $k \equiv 0 \bmod 2$ and $y = 1$ if $k \equiv 1 \bmod 2$; where $z = 4$ if $k \equiv 0 \bmod 4$, $z = 1$ if $k \equiv 1 \bmod 4$, $z = 0$ if $k \equiv 3 \bmod 4$, and $z = 3$ if $k \equiv 3 \bmod 4$; and where $\alpha = \lfloor k/12 \rfloor$.

We give the translations from the first difference family to the second one.

From	To	Add	Comments
(4a)	(4i)	$4k-14+u$	
(4b)	(4j)	$24k+18+v$	
(4c)	(4k)	$2k-2+w$	
(4d)	(4o)	6+x	
(4e)	(4p)	2	
(4f)	(4q1)	$14k+r+4$	if $1 \leq r \leq 2k+1$ and $r \not\equiv 0 \bmod 3$
(4f)	(4q2)	$7k-2s+y$	if $1 \leq s \leq (2k+1)/3$, $r \equiv 0 \bmod 3$ and $3s = r$
(4g)	(4r)	$2r-1$	$1 \leq r \leq k-1$
(4h)	(4s1)	$6k+2r+1$	$1 \leq r \leq (k/2)$ and $k \equiv 2,3 \bmod 4$
(4h)	(4s2)	$6k+2r+3$	$1 \leq r \leq (k/2)$ and $k \equiv 0,1 \bmod 4$
(4h)	(4s3)	$3k+2r+6\alpha+z$	$(k/2)+1 \leq r \leq k-1$, $\alpha = \lfloor k/12 \rfloor$

To complete the proof we now provide the small cases.

$v = 21$ ($k = 0$): (1,8,15), (2,3,12), (4,6,10), (11,14,19).

$v = 45$ ($k = 1$): (1,16,31), (2,3,12), (4,6,24), (8,11,22), (5,9,26), (14,19,27), (15,21,37), (23,30,42)

$v = 69$ ($k = 2$): (11,34,57), (28,29,60), (30,35,59), (31,38,64), (33,44,61), (13,16,43), (14,23,48), (1,3,19), (2,6,21), (4,12,25), (5,15,27), (20,26,40)

$v = 93$ ($k = 3$): (11,42,73), (43,44,86), (45,52,84), (46,55,90), (50,65,88), (32,35,72), (34,9,75), (13,24,58), (16,29,62), (1,3,25), (2,6,27), (4,12,31), (5,15,33), (7,21,37), (30,36,56), (47,59,76)

$v = 117$ ($k = 4$): (11,50,89), (57,58,111),(60,69,109), (59,70,114), (65,84,113), (13,16,63), (44,49,95), (41,48,93), (17,30,73), (20,35,77), (23,40,81), (1,3,31), (2,6,33), (4,12,37), (5,15,39), (7,21,43), (8,24,45), (62,80,100), (66,72,98), (67,76,99)

$v = 141$ ($k = 5$): (15,62,109), (52,65,118), (139,140,63), (9,20,68), (2,25,60), (75,77,111), (76,80,113), (78,86,117), (79,89,119), (81,95,123), (82,98,125), (84,104,129), (85,107,131), (34,40,72), (32,44,73), (30,48,74), (1,4,61), (3,8,64), (5,12,67), (7,16,70), (35,50,102), (37,54,105), (22,41,91) (24,45,94)

$v = 165$ ($k = 6$): (42,97,152), (57,72,134), (162,163,73), (10,23,79), (2,29,70), (89,91,131), (90,94,133), (92,100,137), (93,103,139), (95,109,143), (96,112,145), (98,118,149), (99,121,151), (101,127,155), (40,46,84), (38,50,85), (36,54,86), (34,58,87), (1,4,71), (3,8,74), (5,12,77), (7,16,80), (9,20,83), (39,56,117), (41,60,120), (43,64,123), (26,49,107), (28,53,110)

$v = 189$ ($k = 7$): (59,122,185), (187,188,85), (13,28,92), (6,23,94), (2,33,80), (103,105,151), (104,108,153), (106,114,157), (107,117,159), (109,123,163), (110,126,165), (112,132,169), (113,135,171), (115,141,175), (116,144,177), (48,54,98) (46,58,99), (44,62,100), (42,66,101), (40,70,102), (1,4,81), (3,8,84), (5,12,87), (7,16,90), (9,20,93), (11,24,96), (45,64,134), (47,68,137), (49,72,140), (32,57,124), (34,61,127), (36,65,130)

$v = 309$ ($k = 12$): (42,145,248), (306,307,139), (22,47,151), (10,37,153), (2,53,130), (173,175,251), (174,178,253), (176,184,257), (177,187,259), (179,193,263), (180,196,265), (182,202,269), (183,205,271), (185,211,275), (186,214,277), (188,220,281), (189,223,283) (191,229,287), (192,232,289), (194,238,293), (195,241,295), (197,247,299), (82,88,162), (80,92,163), (78,96,164), (76,100,165), (74,104,166), (72,108,167), (70,112,168), (68,116,169), (1,4,131), (3,8,134), (5,12,137), (7,16,140), (9,20,143), (11,24,146), (13,28,149), (15,23,152), (17,36,155), (19,40,158), (21,44,161), (77,106,221), (79,110,224), (81,114,227), (83,118,230), (85,122,233), (87,126,236), (60,101,210), (62,105,213), (64,109,216), (66,113,219), (86,135,240).

3 Conclusion

Noting that every difference family constructed thus far has been in the cyclic group, we combine the previous theorems to obtain our main result.

Theorem 5 *There exists a cyclic block disjoint* $(v, 3, 1)$ *difference family for all* $v \equiv 3$ *(mod 6), with* $v \geq 3$ *and* $v \neq 9$.

A cyclic $(9, 3, 1)$ difference family does not exist, however there is a $1-$rotational block disjoint $(9, 3, 1)$ difference family, namely $\{\{\infty, 0, 4\}, \{1, 2, 7\}\}$. Hence there exists a block disjoint $(v, 3, 1)$ difference family for all $v \equiv 3$ (mod 6) with $v \geq 3$ as claimed in the abstract.

The results from this paper combined with the earlier paper [2] prove that there exists a cyclic block disjoint $(v, 3, 1)$ difference family for all $v \equiv 1$ or 3 (mod 6) with $v \geq 3$ and $v \neq 9$. The techniques in this paper are very similar to those employed in [2] for the case of $v \equiv 1$ (mod 6).

In checking the correctness of these difference families, we have used a program which generates the prescribed triples and then tests them for disjointness of both their elements and their differences. This program can now be used to generate the triples in the $(v, 3, 1)$ disjoint difference families given in this paper (the general constructions – not the small special cases) for $v \equiv 3$ (mod 6). It is written in Pascal and is available at `http://www.emba.uvm.edu/~dinitz/disjoint.3mod6.pas`.

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