

Appendix to the Paper:

X. Zhang and G. Ge, Existence of resolvable H-designs with group sizes 2, 3, 4 and 6, Designs, Codes and Cryptography, to appear.

Lemma 0.1. *There exists an RH(4^{19}).*

Proof. In [1, Lemma 5.4], Cao, Ji and Zhu constructed an H(2^{19}) on Z_{38} with group set $\{\{j, j + 19\}, j = 0, 1, \dots, 18\}$ and the following shortened list of base blocks.

$\{0, 2, 16, 25\}$	$\{0, 2, 15, 24\}$	$\{0, 4, 10, 11\}$	$\{0, 4, 31, 32\}$	$\{0, 8, 20, 22\}$	$\{0, 8, 24, 26\}$
$\{1, 3, 17, 26\}$	$\{1, 3, 16, 25\}$	$\{1, 5, 11, 12\}$	$\{1, 5, 32, 33\}$	$\{1, 9, 21, 23\}$	$\{1, 9, 25, 27\}$
$\{0, 1, 2, 3\}$	$\{1, 2, 4, 6\}$	$\{0, 1, 4, 5\}$	$\{1, 2, 5, 9\}$	$\{0, 1, 6, 12\}$	$\{1, 2, 7, 23\}$
$\{0, 1, 8, 16\}$	$\{0, 1, 9, 30\}$	$\{1, 2, 10, 13\}$	$\{0, 1, 10, 22\}$	$\{1, 2, 11, 31\}$	$\{0, 1, 13, 23\}$
$\{0, 1, 14, 15\}$	$\{1, 2, 15, 18\}$	$\{1, 2, 16, 37\}$	$\{1, 2, 17, 27\}$	$\{0, 1, 17, 21\}$	$\{1, 2, 19, 25\}$
$\{1, 2, 22, 35\}$	$\{1, 2, 24, 34\}$	$\{1, 2, 26, 30\}$	$\{0, 1, 26, 29\}$	$\{1, 3, 7, 31\}$	$\{0, 2, 8, 23\}$
$\{0, 2, 10, 12\}$	$\{1, 3, 21, 24\}$	$\{0, 3, 6, 15\}$	$\{0, 3, 8, 18\}$	$\{0, 3, 9, 21\}$	$\{1, 4, 14, 22\}$

Here, the blocks of the last five rows are developed by a multiplier 7 of order 3. These 90 blocks and the blocks in the first two rows form the set \mathcal{B}' of all base blocks, which are developed under the automorphism group $\langle (0 \ 2 \dots 34 \ 36)(1 \ 3 \dots 35 \ 37) \rangle$.

For each block $B = \{a, b, c, d\} \in \mathcal{B}'$, construct an H(2^4) with group set $\{\{x, x + 38\} : x \in B\}$ and block set $\mathcal{A}_B = \{\{a + 38i, b + 38(i + k), c + 38j, d + 38(j + k)\} : i, j, k \in Z_2\}$. Let $\mathcal{B} = \cup_{B \in \mathcal{B}'} \mathcal{A}_B$. It is clear that \mathcal{B} is the set of base blocks of an H(4^{19}) on $I_{76} = \{0, 1, 2, \dots, 75\}$ with group set $\{\{j, j + 19, j + 38, j + 57\}, j = 0, 1, \dots, 18\}$ and an automorphism group $\langle \alpha \rangle$, where $\alpha = (0 \ 2 \dots 34 \ 36)(1 \ 3 \dots 35 \ 37)(38 \ 40 \dots 72 \ 74)(39 \ 41 \dots 73 \ 75)$. Now, we need to show the resolution.

Note that there are several blocks in \mathcal{B} , each of which contains exactly one element from each cycle of α . We first list below some of these blocks, each of which gives a parallel class when developed under the automorphism group $\langle \alpha \rangle$.

$\{1, 2, 54, 75\}$	$\{1, 2, 60, 73\}$	$\{0, 1, 64, 67\}$	$\{0, 3, 44, 53\}$	$\{0, 7, 52, 59\}$
$\{0, 7, 63, 58\}$	$\{0, 7, 60, 67\}$	$\{7, 14, 74, 69\}$	$\{7, 14, 40, 55\}$	$\{0, 11, 44, 55\}$
$\{0, 11, 61, 64\}$	$\{11, 22, 51, 46\}$	$\{11, 22, 62, 65\}$	$\{0, 33, 66, 51\}$	$\{38, 39, 2, 3\}$
$\{38, 39, 9, 30\}$	$\{39, 40, 15, 18\}$	$\{39, 40, 16, 37\}$	$\{38, 39, 26, 29\}$	$\{38, 45, 14, 21\}$
$\{38, 45, 25, 20\}$	$\{45, 52, 32, 15\}$	$\{38, 45, 22, 29\}$	$\{45, 52, 29, 12\}$	$\{45, 52, 36, 31\}$
$\{45, 52, 2, 17\}$	$\{38, 45, 30, 13\}$	$\{38, 49, 22, 33\}$	$\{38, 49, 6, 17\}$	$\{38, 49, 23, 26\}$
$\{49, 60, 34, 29\}$	$\{49, 60, 14, 5\}$	$\{38, 49, 20, 15\}$	$\{38, 71, 28, 13\}$	$\{38, 1, 2, 41\}$
$\{38, 1, 4, 43\}$	$\{38, 1, 14, 53\}$	$\{38, 1, 26, 67\}$	$\{38, 3, 6, 53\}$	$\{38, 7, 14, 59\}$
$\{38, 7, 28, 73\}$	$\{38, 7, 22, 67\}$	$\{45, 14, 29, 50\}$	$\{38, 7, 30, 51\}$	$\{38, 21, 4, 67\}$
$\{38, 11, 6, 55\}$	$\{38, 11, 2, 51\}$	$\{38, 33, 28, 51\}$	$\{0, 39, 40, 3\}$	$\{0, 39, 42, 5\}$
$\{0, 39, 52, 15\}$	$\{1, 40, 53, 18\}$	$\{0, 39, 64, 29\}$	$\{0, 45, 66, 35\}$	$\{0, 45, 68, 13\}$
$\{0, 59, 42, 29\}$	$\{0, 49, 60, 33\}$	$\{0, 71, 66, 13\}$	$\{38, 1, 47, 30\}$	$\{38, 7, 63, 20\}$
$\{45, 14, 70, 15\}$	$\{45, 14, 74, 31\}$	$\{38, 11, 61, 26\}$	$\{49, 22, 72, 29\}$	$\{49, 22, 62, 27\}$
$\{0, 39, 9, 68\}$	$\{1, 40, 16, 75\}$	$\{1, 40, 22, 73\}$	$\{0, 45, 25, 58\}$	$\{7, 52, 32, 53\}$
$\{7, 52, 36, 69\}$	$\{0, 49, 23, 64\}$	$\{11, 60, 34, 67\}$	$\{11, 60, 24, 65\}$	$\{11, 60, 14, 43\}$

Then we shift each of the remaining base blocks in \mathcal{B} by a suitable automorphism α^i for some integer i . The result is listed below, where the blocks in each of the four consecutive rows, namely the i th, $(i + 1)$ th, $(i + 2)$ th and $(i + 3)$ th rows for $i \in \{4k + 1 : k = 0, 1, \dots, 38\}$, form a parallel class.

{0, 1, 2, 3}	{5, 6, 8, 10}	{12, 13, 16, 17}	{19, 20, 23, 27}	{24, 25, 30, 36}
{31, 32, 37, 15}	{33, 34, 4, 7}	{26, 29, 35, 9}	{14, 21, 18, 22}	{38, 39, 40, 41}
{43, 44, 46, 48}	{50, 51, 54, 55}	{57, 58, 61, 65}	{62, 63, 68, 74}	{59, 60, 75, 47}
{66, 73, 42, 49}	{56, 67, 52, 70}	{45, 53, 69, 71}	{64, 28, 72, 11}	
{0, 1, 8, 16}	{2, 3, 11, 32}	{4, 5, 14, 26}	{17, 18, 27, 9}	{6, 7, 19, 29}
{20, 21, 34, 35}	{22, 23, 10, 13}	{12, 33, 30, 24}	{25, 36, 31, 37}	{39, 40, 45, 61}
{42, 43, 50, 58}	{46, 47, 55, 38}	{51, 52, 60, 63}	{65, 66, 75, 57}	{72, 73, 48, 49}
{53, 54, 67, 70}	{64, 71, 69, 59}	{41, 62, 56, 74}	{28, 68, 44, 15}	
{1, 2, 15, 18}	{5, 6, 20, 3}	{7, 8, 23, 33}	{10, 11, 27, 31}	{13, 14, 34, 9}
{37, 0, 22, 32}	{25, 26, 12, 16}	{17, 24, 21, 19}	{38, 39, 48, 60}	{40, 41, 53, 63}
{49, 50, 64, 47}	{44, 45, 61, 65}	{51, 52, 69, 75}	{73, 74, 58, 68}	{59, 66, 42, 56}
{55, 62, 67, 71}	{70, 43, 29, 35}	{46, 54, 28, 30}	{72, 36, 4, 57}	
{1, 2, 19, 25}	{3, 5, 9, 33}	{4, 6, 12, 27}	{8, 10, 18, 20}	{11, 13, 31, 34}
{14, 17, 22, 32}	{16, 23, 30, 37}	{0, 7, 28, 35}	{39, 40, 60, 73}	{41, 42, 66, 70}
{46, 47, 72, 75}	{51, 53, 57, 43}	{48, 50, 56, 71}	{61, 64, 74, 44}	{38, 49, 67, 63}
{58, 62, 68, 69}	{29, 36, 65, 59}	{52, 54, 24, 26}	{15, 55, 21, 45}	
{0, 3, 6, 15}	{1, 4, 14, 22}	{11, 18, 32, 8}	{9, 16, 37, 27}	{12, 19, 30, 10}
{26, 33, 20, 28}	{36, 5, 13, 7}	{34, 17, 21, 29}	{31, 35, 24, 25}	{38, 40, 48, 50}
{39, 41, 59, 62}	{46, 49, 52, 61}	{64, 67, 72, 44}	{54, 57, 63, 75}	{66, 73, 60, 68}
{74, 47, 42, 53}	{70, 43, 55, 58}	{45, 56, 69, 65}	{71, 2, 23, 51}	
{0, 7, 25, 20}	{9, 16, 34, 17}	{11, 18, 5, 31}	{6, 13, 28, 35}	{19, 26, 3, 24}
{23, 30, 21, 15}	{32, 1, 37, 27}	{33, 2, 4, 36}	{38, 45, 66, 73}	{47, 54, 75, 65}
{42, 49, 46, 50}	{51, 58, 55, 53}	{64, 71, 44, 62}	{60, 74, 40, 69}	{61, 63, 39, 48}
{59, 67, 41, 43}	{57, 52, 10, 22}	{56, 29, 14, 70}	{12, 72, 8, 68}	
{7, 14, 36, 31}	{9, 16, 21, 25}	{13, 20, 8, 23}	{11, 18, 34, 24}	{30, 37, 22, 5}
{15, 29, 19, 35}	{17, 0, 32, 12}	{33, 6, 4, 10}	{38, 45, 63, 58}	{47, 54, 72, 55}
{49, 56, 43, 69}	{44, 51, 59, 53}	{57, 64, 41, 62}	{73, 42, 71, 65}	{52, 66, 46, 60}
{67, 40, 70, 61}	{27, 68, 2, 48}	{26, 75, 28, 39}	{1, 50, 3, 74}	
{0, 14, 18, 9}	{2, 16, 34, 10}	{11, 25, 37, 20}	{22, 5, 26, 13}	{6, 17, 28, 1}
{24, 35, 30, 3}	{31, 4, 21, 7}	{12, 23, 32, 27}	{38, 45, 60, 67}	{49, 56, 40, 73}
{47, 54, 42, 57}	{55, 62, 64, 58}	{39, 53, 65, 48}	{61, 72, 69, 59}	{63, 74, 50, 46}
{71, 75, 43, 44}	{8, 15, 51, 41}	{19, 68, 52, 36}	{66, 29, 70, 33}	
{11, 22, 6, 28}	{2, 13, 30, 20}	{12, 23, 24, 36}	{8, 19, 31, 34}	{15, 26, 0, 33}
{18, 29, 14, 32}	{10, 21, 1, 35}	{4, 37, 27, 7}	{17, 25, 3, 5}	{45, 52, 68, 58}
{40, 47, 70, 53}	{51, 65, 55, 71}	{42, 63, 67, 75}	{43, 54, 66, 61}	{38, 60, 72, 56}
{57, 41, 49, 44}	{74, 69, 59, 39}	{48, 50, 64, 73}	{46, 9, 16, 62}	
{11, 22, 7, 37}	{2, 13, 4, 15}	{17, 28, 19, 14}	{23, 34, 36, 1}	{5, 16, 29, 25}
{24, 35, 21, 27}	{33, 6, 3, 31}	{10, 18, 30, 32}	{38, 59, 42, 67}	{40, 61, 58, 52}
{44, 55, 66, 39}	{51, 62, 46, 68}	{53, 64, 49, 41}	{60, 71, 57, 63}	{54, 65, 74, 69}
{70, 72, 47, 56}	{9, 20, 43, 73}	{45, 8, 48, 12}	{26, 75, 0, 50}	
{11, 22, 14, 5}	{13, 24, 0, 34}	{19, 3, 9, 7}	{4, 26, 16, 29}	{10, 32, 6, 28}
{31, 15, 23, 18}	{17, 12, 8, 20}	{49, 60, 55, 61}	{40, 51, 68, 58}	{53, 64, 43, 67}
{62, 73, 74, 48}	{52, 63, 54, 65}	{59, 70, 72, 75}	{57, 41, 47, 45}	{56, 39, 36, 30}
{38, 21, 25, 71}	{27, 66, 69, 35}	{37, 44, 46, 2}	{1, 50, 42, 33}	
{0, 33, 28, 13}	{2, 35, 14, 10}	{4, 6, 20, 29}	{22, 24, 37, 8}	{32, 36, 25, 26}
{5, 7, 21, 30}	{19, 27, 1, 3}	{49, 60, 51, 46}	{53, 64, 62, 68}	{44, 66, 56, 69}
{42, 75, 70, 55}	{40, 73, 52, 48}	{50, 58, 74, 38}	{39, 41, 54, 63}	{17, 18, 65, 47}
{12, 15, 59, 71}	{61, 34, 9, 43}	{23, 72, 57, 11}	{67, 31, 45, 16}	
{0, 4, 10, 11}	{6, 14, 30, 32}	{1, 3, 16, 25}	{9, 13, 19, 20}	{49, 44, 40, 52}
{38, 42, 69, 70}	{46, 54, 66, 68}	{41, 45, 72, 73}	{7, 8, 48, 50}	{17, 18, 59, 63}
{21, 22, 65, 43}	{34, 35, 47, 57}	{5, 12, 71, 61}	{33, 28, 62, 74}	{29, 37, 53, 55}
{51, 58, 36, 26}	{39, 23, 31, 64}	{67, 2, 75, 27}	{15, 60, 24, 56}	
{0, 1, 44, 50}	{2, 3, 48, 56}	{4, 5, 52, 64}	{7, 8, 61, 71}	{10, 11, 65, 69}

{17, 18, 73, 41}	{13, 14, 74, 46}	{29, 30, 54, 58}	{19, 21, 63, 49}	{20, 22, 66, 43}
{24, 27, 70, 42}	{9, 12, 60, 68}	{25, 32, 57, 45}	{53, 16, 33, 39}	{37, 38, 55, 23}
{6, 51, 59, 15}	{62, 26, 72, 36}	{47, 31, 75, 35}	{28, 67, 34, 40}	
{0, 2, 48, 50}	{1, 3, 59, 62}	{7, 14, 66, 42}	{10, 17, 52, 56}	{9, 16, 51, 49}
{4, 11, 60, 40}	{6, 13, 38, 46}	{8, 15, 61, 55}	{19, 26, 69, 73}	{23, 30, 70, 64}
{21, 28, 44, 72}	{18, 32, 74, 65}	{12, 33, 75, 45}	{57, 58, 37, 5}	{29, 68, 54, 20}
{27, 41, 53, 36}	{71, 35, 39, 25}	{22, 63, 31, 43}	{34, 67, 24, 47}	
{7, 21, 49, 65}	{0, 14, 70, 46}	{9, 23, 73, 56}	{4, 25, 60, 54}	{11, 32, 64, 44}
{17, 28, 50, 72}	{13, 24, 57, 63}	{20, 31, 48, 38}	{15, 26, 43, 67}	{16, 27, 66, 40}
{8, 19, 75, 71}	{18, 29, 53, 59}	{30, 34, 61, 62}	{45, 52, 5, 37}	{74, 47, 10, 22}
{68, 33, 1, 51}	{69, 35, 3, 42}	{39, 12, 41, 36}	{6, 55, 2, 58}	
{0, 11, 72, 52}	{13, 24, 75, 71}	{15, 26, 61, 51}	{17, 28, 42, 38}	{21, 32, 68, 74}
{19, 3, 47, 45}	{14, 36, 64, 39}	{22, 6, 56, 40}	{23, 7, 53, 48}	{4, 37, 54, 50}
{2, 35, 63, 43}	{16, 18, 70, 41}	{59, 60, 25, 29}	{65, 67, 9, 12}	{62, 69, 1, 33}
{58, 31, 5, 46}	{57, 30, 27, 55}	{20, 66, 44, 8}	{34, 73, 10, 49}	
{0, 2, 53, 62}	{6, 10, 54, 55}	{8, 16, 66, 68}	{12, 20, 74, 38}	{7, 9, 61, 70}
{3, 5, 56, 65}	{11, 15, 59, 60}	{13, 17, 44, 45}	{19, 27, 39, 41}	{57, 58, 22, 24}
{48, 49, 18, 26}	{46, 47, 21, 31}	{51, 52, 29, 1}	{67, 32, 4, 50}	{64, 33, 30, 72}
{40, 35, 25, 43}	{34, 75, 42, 14}	{28, 73, 71, 23}	{63, 36, 69, 37}	
{38, 39, 6, 12}	{41, 42, 9, 25}	{44, 45, 16, 28}	{47, 48, 19, 1}	{50, 51, 29, 33}
{53, 54, 0, 10}	{55, 56, 4, 8}	{59, 61, 27, 13}	{60, 62, 30, 7}	{40, 43, 11, 23}
{49, 52, 24, 32}	{63, 70, 15, 5}	{64, 71, 31, 21}	{73, 37, 3, 65}	{58, 34, 14, 66}
{69, 26, 22, 72}	{17, 57, 75, 2}	{18, 67, 46, 36}	{35, 74, 20, 68}	
{38, 41, 8, 18}	{45, 52, 28, 4}	{40, 47, 6, 10}	{49, 56, 15, 13}	{54, 61, 34, 14}
{60, 67, 16, 24}	{51, 58, 7, 33}	{57, 64, 31, 35}	{55, 62, 26, 20}	{59, 73, 25, 3}
{66, 42, 22, 36}	{70, 53, 19, 27}	{39, 50, 9, 37}	{29, 43, 71, 11}	{12, 72, 46, 30}
{32, 65, 44, 2}	{74, 5, 68, 0}	{17, 69, 21, 75}	{23, 63, 1, 48}	
{38, 52, 18, 9}	{45, 59, 33, 16}	{47, 68, 24, 4}	{49, 60, 6, 28}	{51, 62, 19, 25}
{42, 53, 32, 22}	{55, 66, 7, 31}	{50, 61, 8, 26}	{63, 74, 21, 13}	{67, 40, 15, 11}
{43, 54, 14, 20}	{56, 58, 34, 5}	{65, 69, 37, 0}	{41, 10, 1, 71}	{64, 30, 36, 75}
{17, 57, 70, 3}	{44, 27, 48, 35}	{46, 12, 39, 2}	{23, 72, 29, 73}	
{38, 49, 29, 25}	{51, 62, 0, 34}	{53, 75, 5, 3}	{44, 66, 18, 31}	{46, 68, 4, 26}
{57, 41, 11, 6}	{40, 73, 14, 10}	{48, 43, 33, 13}	{50, 52, 27, 36}	{60, 64, 15, 16}
{72, 42, 20, 22}	{45, 47, 23, 32}	{69, 71, 8, 17}	{67, 30, 1, 59}	{56, 19, 35, 39}
{24, 63, 70, 2}	{74, 12, 54, 7}	{55, 21, 65, 28}	{9, 58, 37, 61}	
{38, 42, 10, 11}	{39, 43, 32, 33}	{41, 49, 23, 25}	{51, 59, 37, 1}	{53, 16, 18, 58}
{61, 24, 27, 69}	{40, 3, 8, 52}	{63, 26, 31, 47}	{44, 7, 15, 74}	{71, 34, 4, 45}
{50, 13, 22, 72}	{75, 0, 14, 73}	{48, 17, 35, 68}	{20, 65, 62, 28}	{21, 66, 46, 29}
{6, 55, 56, 30}	{67, 36, 57, 9}	{70, 5, 60, 12}	{19, 64, 2, 54}	
{38, 1, 13, 61}	{39, 2, 17, 65}	{41, 4, 24, 75}	{45, 8, 30, 40}	{51, 14, 0, 42}
{46, 10, 18, 58}	{43, 7, 25, 66}	{50, 15, 20, 68}	{67, 36, 12, 64}	{53, 22, 19, 55}
{54, 23, 34, 52}	{63, 32, 16, 49}	{57, 26, 31, 73}	{3, 74, 70, 6}	{62, 28, 72, 35}
{21, 60, 37, 47}	{9, 48, 27, 71}	{11, 56, 5, 69}	{33, 44, 29, 59}	
{45, 14, 32, 53}	{40, 9, 34, 42}	{47, 16, 3, 67}	{46, 15, 23, 55}	{38, 7, 5, 71}
{49, 18, 6, 59}	{51, 20, 22, 54}	{57, 26, 4, 70}	{61, 37, 27, 43}	{60, 36, 2, 69}
{63, 1, 13, 72}	{65, 0, 33, 39}	{29, 68, 50, 25}	{11, 56, 44, 21}	{10, 62, 66, 19}
{28, 74, 48, 12}	{35, 75, 17, 58}	{31, 52, 8, 64}	{24, 73, 30, 41}	
{38, 21, 18, 50}	{45, 28, 22, 40}	{51, 24, 8, 68}	{42, 15, 32, 60}	{53, 26, 5, 67}
{46, 19, 20, 70}	{57, 30, 4, 75}	{71, 6, 29, 59}	{44, 17, 35, 69}	{61, 34, 36, 39}
{52, 25, 11, 55}	{43, 16, 14, 58}	{13, 73, 41, 1}	{49, 12, 65, 37}	{47, 10, 72, 0}
{54, 27, 74, 31}	{9, 48, 23, 64}	{7, 56, 2, 62}	{3, 63, 33, 66}	
{49, 22, 14, 43}	{51, 24, 0, 72}	{53, 37, 5, 41}	{42, 26, 16, 67}	{44, 28, 2, 62}
{38, 33, 12, 46}	{40, 4, 18, 65}	{68, 32, 7, 54}	{64, 30, 19, 58}	{66, 36, 10, 50}
{45, 9, 23, 70}	{21, 60, 69, 13}	{20, 59, 75, 3}	{29, 74, 71, 31}	{34, 55, 52, 8}
{1, 47, 63, 27}	{73, 11, 39, 17}	{35, 57, 25, 61}	{15, 48, 6, 56}	
{38, 8, 24, 64}	{39, 3, 16, 63}	{41, 7, 34, 73}	{43, 13, 25, 65}	{45, 15, 31, 71}
{1, 40, 42, 6}	{10, 49, 54, 22}	{11, 50, 55, 33}	{12, 51, 59, 4}	{5, 44, 52, 17}
{18, 57, 66, 2}	{21, 60, 74, 19}	{37, 48, 46, 14}	{58, 27, 62, 28}	{29, 68, 32, 72}

{30, 70, 0, 53}	{9, 61, 35, 56}	{36, 47, 20, 69}	{26, 75, 23, 67}	
{0, 39, 51, 23}	{1, 40, 55, 27}	{3, 42, 64, 36}	{5, 45, 49, 35}	{6, 46, 52, 29}
{8, 48, 56, 20}	{12, 53, 59, 33}	{9, 50, 60, 30}	{13, 58, 72, 10}	{18, 63, 74, 16}
{32, 43, 61, 19}	{66, 31, 75, 11}	{68, 37, 73, 25}	{26, 65, 28, 67}	{15, 54, 2, 44}
{17, 62, 21, 57}	{34, 41, 14, 70}	{24, 38, 4, 71}	{7, 47, 22, 69}	
{7, 52, 73, 25}	{0, 45, 63, 20}	{2, 47, 72, 4}	{9, 54, 41, 29}	{11, 56, 40, 35}
{13, 58, 49, 5}	{15, 60, 65, 31}	{19, 64, 42, 32}	{14, 66, 46, 22}	{17, 38, 70, 12}
{53, 16, 57, 23}	{74, 37, 62, 27}	{39, 8, 43, 3}	{50, 33, 68, 24}	{61, 34, 48, 6}
{51, 21, 75, 1}	{28, 67, 36, 44}	{30, 69, 18, 59}	{26, 71, 10, 55}	
{0, 59, 63, 33}	{11, 60, 55, 23}	{13, 62, 41, 27}	{4, 53, 65, 30}	{17, 66, 40, 35}
{2, 51, 74, 16}	{21, 70, 72, 37}	{19, 68, 43, 1}	{12, 61, 47, 15}	{31, 42, 56, 14}
{34, 38, 44, 7}	{46, 9, 52, 20}	{67, 36, 50, 26}	{39, 3, 54, 25}	{6, 45, 10, 49}
{8, 48, 18, 58}	{28, 73, 5, 75}	{24, 69, 29, 57}	{22, 64, 32, 71}	
{11, 60, 57, 9}	{2, 62, 52, 27}	{13, 73, 43, 0}	{4, 75, 65, 7}	{6, 46, 59, 30}
{14, 56, 45, 8}	{1, 41, 55, 26}	{5, 47, 53, 16}	{3, 49, 61, 25}	{66, 29, 68, 31}
{71, 34, 51, 19}	{42, 15, 54, 28}	{74, 20, 48, 23}	{18, 63, 22, 64}	{33, 40, 17, 38}
{37, 44, 35, 67}	{36, 69, 21, 39}	{12, 58, 32, 72}	{24, 70, 10, 50}	
{1, 43, 70, 33}	{39, 2, 45, 23}	{40, 3, 48, 18}	{42, 5, 52, 26}	{53, 16, 63, 7}
{46, 9, 59, 31}	{50, 13, 64, 27}	{71, 34, 47, 12}	{51, 14, 74, 8}	{73, 4, 58, 10}
{38, 11, 44, 17}	{55, 28, 41, 37}	{75, 21, 67, 24}	{65, 22, 56, 30}	{72, 36, 49, 20}
{69, 35, 62, 25}	{15, 60, 0, 66}	{32, 54, 6, 57}	{19, 61, 29, 68}	
{38, 1, 55, 21}	{39, 3, 59, 24}	{40, 5, 46, 17}	{42, 7, 50, 22}	{43, 8, 56, 26}
{44, 13, 58, 27}	{64, 33, 54, 23}	{68, 37, 48, 28}	{47, 16, 41, 29}	{45, 14, 67, 12}
{71, 2, 69, 25}	{57, 30, 52, 36}	{62, 35, 53, 11}	{61, 34, 70, 0}	{6, 65, 10, 73}
{31, 4, 72, 63}	{66, 49, 32, 19}	{74, 9, 18, 51}	{15, 60, 75, 20}	
{38, 7, 53, 9}	{42, 11, 64, 33}	{45, 14, 57, 23}	{47, 16, 56, 12}	{44, 13, 74, 19}
{52, 28, 46, 22}	{49, 25, 75, 20}	{48, 31, 73, 5}	{51, 34, 66, 8}	{65, 0, 55, 3}
{54, 27, 50, 30}	{2, 41, 15, 63}	{32, 39, 26, 72}	{10, 21, 70, 43}	{18, 29, 58, 69}
{59, 60, 4, 17}	{61, 24, 37, 40}	{71, 6, 35, 68}	{67, 36, 62, 1}	
{38, 11, 60, 33}	{49, 22, 45, 37}	{40, 13, 42, 15}	{44, 17, 41, 9}	{50, 34, 46, 30}
{64, 21, 54, 1}	{62, 19, 74, 32}	{66, 23, 51, 31}	{56, 20, 72, 5}	{8, 47, 18, 68}
{12, 57, 26, 71}	{10, 55, 2, 61}	{4, 63, 29, 75}	{16, 65, 36, 69}	{35, 39, 28, 67}
{7, 14, 70, 53}	{6, 27, 48, 73}	{58, 59, 24, 25}	{3, 52, 43, 0}	
{38, 8, 58, 22}	{40, 10, 64, 28}	{39, 9, 59, 23}	{3, 42, 7, 49}	{5, 44, 11, 65}
{15, 54, 25, 45}	{2, 41, 19, 61}	{14, 55, 20, 67}	{16, 57, 24, 72}	{17, 62, 29, 71}
{0, 52, 32, 46}	{26, 27, 68, 69}	{33, 34, 47, 50}	{30, 37, 60, 43}	{74, 75, 12, 13}
{18, 63, 70, 1}	{6, 51, 66, 35}	{4, 53, 48, 21}	{73, 36, 56, 31}	
{0, 45, 28, 73}	{7, 52, 35, 63}	{6, 65, 24, 56}	{2, 51, 30, 58}	{4, 53, 33, 67}
{13, 62, 37, 71}	{11, 60, 19, 47}	{21, 70, 8, 42}	{23, 72, 32, 38}	{26, 27, 40, 41}
{5, 16, 66, 61}	{43, 44, 14, 17}	{54, 57, 22, 31}	{39, 50, 3, 36}	{75, 48, 12, 15}
{74, 9, 20, 69}	{10, 59, 68, 25}	{55, 18, 64, 29}	{1, 46, 34, 49}	
{0, 71, 12, 46}	{2, 42, 18, 65}	{4, 44, 19, 66}	{6, 48, 37, 38}	{1, 47, 21, 61}
{3, 49, 27, 67}	{10, 11, 50, 51}	{22, 23, 69, 52}	{7, 8, 54, 57}	{28, 35, 56, 63}
{17, 24, 39, 60}	{20, 31, 40, 73}	{74, 43, 26, 33}	{72, 45, 36, 9}	{14, 55, 58, 29}
{30, 41, 70, 5}	{53, 16, 68, 13}	{59, 32, 62, 15}	{25, 64, 34, 75}	

□

Lemma 0.2. *There exists an $RH(4^{41})$.*

Proof. In [1, Lemma 5.2], Cao, Ji and Zhu constructed an $H(2^{41})$ on Z_{82} with group set $\{\{j, j + 41\}, j = 0, 1, \dots, 40\}$ and the following shortened list of base blocks, which are developed by the automorphism group $\langle \alpha', \beta' \rangle$, where $\alpha' = (0\ 1 \dots 80\ 81)$ and β' is a multiplier 37 of order 5 in Z_{82} .

$$\mathcal{B}' : \begin{array}{cccccc} \{0,1,2,4\} & \{0,1,5,6\} & \{0,1,7,8\} & \{0,1,9,10\} & \{0,1,11,12\} & \{0,1,13,14\} \\ \{0,1,15,16\} & \{0,1,17,18\} & \{0,1,19,20\} & \{0,1,21,22\} & \{0,1,23,24\} & \{0,1,27,28\} \\ \{0,1,29,30\} & \{0,1,31,32\} & \{0,1,33,34\} & \{0,1,35,36\} & \{0,1,39,43\} & \{0,1,40,80\} \\ \{0,1,44,79\} & \{0,2,5,7\} & \{0,2,6,8\} & \{0,2,9,12\} & \{0,2,10,13\} & \{0,2,11,15\} \end{array}$$

$$\begin{array}{cccccc}
\{0,2,14,16\} & \{0,2,17,19\} & \{0,2,18,24\} & \{0,2,20,48\} & \{0,2,21,26\} & \{0,2,28,46\} \\
\{0,2,29,50\} & \{0,2,30,72\} & \{0,2,32,58\} & \{0,2,35,49\} & \{0,2,55,66\} & \{0,2,56,69\} \\
\{0,2,63,75\} & \{0,3,7,22\} & \{0,3,9,56\} & \{0,3,12,64\} & \{0,3,15,70\} & \{0,3,17,61\} \\
\{0,3,19,29\} & \{0,3,24,52\} & \{0,3,27,76\} & \{0,3,33,55\} & \{0,3,43,58\} & \{0,4,9,72\} \\
\{0,4,10,58\} & \{0,4,14,38\} & \{0,4,39,77\} & \{0,4,48,65\} & &
\end{array}$$

For each block $B = \{a, b, c, d\} \in \mathcal{B}'$, construct an $H(2^4)$ with group set $\{\{x, x + 82\} : x \in B\}$ and block set $\mathcal{A}_B = \{\{a + 82i, b + 82(i + k), c + 82j, d + 82(j + k)\} : i, j, k \in \mathbb{Z}_2\}$. Let $\mathcal{B} = \cup_{B \in \mathcal{B}'} \mathcal{A}_B$. It is clear that \mathcal{B} is the set of base blocks of an $H(4^{41})$ on $I_{164} = \{0, 1, 2, \dots, 163\}$ with group set $\mathcal{G} = \{\{j, j + 41, j + 82, j + 123\}, j = 0, 1, \dots, 40\}$ and an automorphism group $\langle \alpha, \beta \rangle$, where

$$\alpha = (0 \ 1 \dots 80 \ 81)(82 \ 83 \dots 162 \ 163) \text{ and}$$

$$\beta = \begin{cases} \beta'(x), & \text{if } x < 82, \\ \beta'(x - 82) + 82, & \text{if } x \geq 82. \end{cases}$$

Now, we need to show the resolution.

Note that there are several blocks in \mathcal{B} , each of which contains exactly one even and one odd elements from each cycle of α . We first list below some of these blocks, each of which gives a parallel class when developed under the automorphism group $\langle \alpha^2, \beta \rangle$.

$$\begin{array}{cccccc}
\{0, 1, 87, 88\} & \{0, 1, 93, 94\} & \{0, 1, 101, 102\} & \{0, 1, 103, 104\} & \{0, 1, 105, 106\} \\
\{0, 1, 109, 110\} & \{0, 1, 111, 112\} & \{0, 1, 113, 114\} & \{0, 1, 115, 116\} & \{0, 1, 117, 118\} \\
\{0, 1, 126, 161\} & \{0, 3, 91, 138\} & \{0, 3, 97, 152\} & \{0, 3, 125, 140\} & \{82, 83, 5, 6\} \\
\{82, 83, 7, 8\} & \{82, 83, 9, 10\} & \{82, 83, 11, 12\} & \{82, 83, 13, 14\} & \{82, 83, 15, 16\} \\
\{82, 83, 17, 18\} & \{82, 83, 19, 20\} & \{82, 83, 21, 22\} & \{82, 83, 23, 24\} & \{82, 83, 27, 28\} \\
\{82, 83, 29, 30\} & \{82, 83, 31, 32\} & \{82, 83, 33, 34\} & \{82, 83, 35, 36\} & \{82, 85, 9, 56\} \\
\{82, 85, 15, 70\} & \{82, 85, 43, 58\} & \{82, 1, 44, 161\} & \{82, 2, 5, 89\} & \{82, 2, 17, 101\} \\
\{82, 2, 35, 131\} & \{82, 4, 39, 159\} & \{0, 83, 126, 79\} & \{0, 84, 99, 19\} & \{0, 84, 117, 49\} \\
\{0, 84, 145, 75\} & \{0, 85, 9, 138\} & \{0, 85, 27, 158\} & \{0, 85, 43, 140\} & \{82, 1, 87, 6\} \\
\{82, 1, 89, 8\} & \{82, 1, 91, 10\} & \{82, 1, 93, 12\} & \{82, 1, 97, 16\} & \{82, 1, 101, 20\} \\
\{82, 1, 105, 24\} & \{82, 1, 109, 28\} & \{82, 1, 111, 30\} & \{82, 1, 113, 32\} & \{82, 1, 115, 34\} \\
\{82, 1, 117, 36\} & \{82, 2, 87, 7\} & \{82, 2, 93, 15\} & \{82, 2, 99, 19\} & \{82, 2, 117, 49\} \\
\{82, 2, 145, 75\} & \{82, 3, 89, 22\} & \{82, 3, 91, 56\} & \{82, 3, 97, 70\} & \{82, 3, 125, 58\} \\
\{82, 4, 121, 77\} & \{0, 83, 5, 88\} & \{0, 83, 7, 90\} & \{0, 83, 9, 92\} & \{0, 83, 11, 94\} \\
\{0, 83, 13, 96\} & \{0, 83, 15, 98\} & \{0, 83, 17, 100\} & \{0, 83, 19, 102\} & \{0, 83, 23, 106\} \\
\{0, 83, 27, 110\} & \{0, 83, 29, 112\} & \{0, 83, 31, 114\} & \{0, 83, 33, 116\} & \{0, 83, 35, 118\} \\
\{0, 84, 5, 89\} & \{0, 84, 11, 97\} & \{0, 84, 17, 101\} & \{0, 84, 63, 157\} & \{0, 85, 7, 104\} \\
\{0, 1, 89, 90\} & \{0, 1, 95, 96\} & \{0, 1, 97, 98\} & &
\end{array}$$

Then we shift each of the remaining base blocks in \mathcal{B} by a suitable automorphism $\alpha^i \beta^j$ for some integers i and j . The result is listed below, where the blocks in each of the eleven consecutive rows, namely the i th, $(i + 1)$ th, \dots , and $(i + 10)$ th rows for $i \in \{11k + 1 : k = 0, 1, \dots, 7\}$, form a parallel class.

$$\begin{array}{cccc}
\{0, 1, 2, 4\} & \{5, 6, 10, 11\} & \{7, 8, 14, 15\} & \{12, 13, 21, 22\} \\
\{16, 17, 27, 28\} & \{18, 19, 31, 32\} & \{23, 24, 38, 39\} & \{25, 26, 42, 43\} \\
\{29, 30, 48, 49\} & \{33, 34, 54, 55\} & \{35, 36, 58, 59\} & \{40, 41, 67, 68\} \\
\{44, 45, 73, 74\} & \{46, 47, 77, 78\} & \{81, 56, 76, 51\} & \{60, 37, 75, 52\} \\
\{20, 57, 69, 53\} & \{64, 66, 70, 72\} & \{61, 63, 79, 3\} & \{82, 83, 84, 86\} \\
\{87, 88, 92, 93\} & \{89, 90, 96, 97\} & \{94, 95, 103, 104\} & \{98, 99, 109, 110\} \\
\{100, 101, 113, 114\} & \{105, 106, 120, 121\} & \{107, 108, 124, 125\} & \{111, 112, 130, 131\} \\
\{115, 116, 136, 137\} & \{117, 118, 140, 141\} & \{122, 123, 149, 150\} & \{126, 127, 155, 156\} \\
\{128, 129, 159, 160\} & \{162, 139, 85, 157\} & \{153, 145, 138, 135\} & \{144, 147, 163, 91\}
\end{array}$$

{158, 161, 119, 134}	{142, 146, 151, 132}	{65, 148, 152, 71}	{50, 133, 143, 62}
{80, 154, 102, 9}			
{0, 1, 40, 80}	{2, 3, 46, 81}	{4, 6, 9, 11}	{5, 7, 14, 17}
{8, 10, 18, 21}	{13, 15, 24, 28}	{20, 22, 34, 36}	{25, 27, 42, 44}
{29, 31, 49, 77}	{30, 32, 51, 56}	{33, 35, 61, 79}	{69, 71, 16, 37}
{48, 50, 78, 38}	{43, 45, 75, 19}	{26, 62, 41, 47}	{72, 64, 57, 54}
{65, 67, 39, 52}	{53, 60, 70, 66}	{82, 83, 115, 116}	{84, 85, 119, 120}
{88, 89, 128, 86}	{90, 91, 134, 87}	{92, 94, 97, 99}	{96, 98, 102, 104}
{101, 103, 110, 113}	{112, 114, 122, 125}	{106, 108, 117, 121}	{93, 95, 107, 109}
{124, 126, 141, 143}	{127, 129, 145, 151}	{146, 138, 148, 118}	{131, 133, 152, 157}
{162, 154, 132, 142}	{147, 139, 130, 156}	{161, 111, 155, 158}	{137, 140, 144, 159}
{73, 23, 100, 160}	{153, 63, 76, 105}	{136, 74, 59, 150}	{12, 123, 149, 55}
{163, 58, 135, 68}			
{0, 2, 63, 75}	{1, 4, 8, 23}	{3, 6, 12, 59}	{7, 10, 19, 71}
{11, 14, 26, 81}	{13, 16, 30, 74}	{15, 18, 39, 67}	{28, 31, 55, 22}
{17, 20, 50, 72}	{21, 24, 64, 79}	{37, 41, 46, 27}	{56, 60, 66, 32}
{38, 42, 52, 76}	{69, 53, 36, 48}	{43, 25, 73, 58}	{82, 84, 111, 132}
{83, 85, 113, 155}	{86, 88, 118, 144}	{87, 89, 150, 162}	{90, 93, 99, 146}
{92, 95, 104, 156}	{91, 94, 106, 161}	{97, 100, 114, 158}	{102, 105, 126, 154}
{107, 110, 134, 101}	{130, 133, 163, 103}	{139, 143, 149, 115}	{121, 125, 135, 159}
{141, 145, 98, 136}	{124, 108, 96, 151}	{33, 34, 117, 119}	{77, 78, 116, 120}
{57, 49, 160, 152}	{44, 80, 122, 148}	{131, 123, 70, 62}	{138, 140, 65, 68}
{153, 9, 5, 147}	{29, 112, 128, 47}	{40, 142, 127, 54}	{45, 129, 157, 35}
{51, 137, 61, 109}			
{0, 1, 122, 162}	{2, 4, 90, 92}	{3, 5, 94, 97}	{6, 8, 98, 101}
{7, 9, 100, 104}	{10, 12, 106, 108}	{11, 13, 110, 112}	{14, 16, 116, 144}
{15, 17, 118, 123}	{18, 20, 128, 146}	{19, 21, 130, 151}	{23, 25, 135, 95}
{24, 26, 138, 82}	{30, 32, 85, 96}	{33, 35, 89, 102}	{36, 38, 99, 111}
{39, 42, 133, 103}	{44, 47, 143, 105}	{28, 31, 129, 139}	{34, 37, 140, 86}
{54, 57, 87, 109}	{41, 45, 132, 113}	{49, 53, 141, 107}	{46, 50, 142, 84}
{75, 79, 114, 152}	{69, 73, 117, 134}	{136, 137, 56, 58}	{119, 120, 76, 80}
{126, 163, 48, 52}	{148, 150, 72, 74}	{121, 157, 55, 68}	{88, 124, 81, 71}
{149, 153, 77, 43}	{147, 66, 78, 161}	{91, 29, 63, 155}	{127, 65, 62, 131}
{115, 22, 60, 159}	{154, 61, 51, 160}	{27, 145, 156, 67}	{70, 83, 125, 59}
{40, 158, 64, 93}			
{82, 84, 14, 16}	{83, 85, 18, 20}	{86, 88, 22, 28}	{87, 89, 25, 53}
{90, 92, 29, 34}	{91, 93, 37, 55}	{94, 96, 41, 62}	{95, 97, 43, 3}
{98, 100, 48, 74}	{99, 101, 52, 66}	{102, 104, 75, 4}	{103, 105, 77, 8}
{106, 108, 5, 17}	{109, 112, 39, 9}	{110, 113, 45, 7}	{114, 117, 51, 61}
{118, 121, 60, 6}	{119, 122, 70, 10}	{111, 115, 38, 19}	{124, 128, 56, 80}
{127, 131, 2, 40}	{129, 133, 13, 30}	{116, 35, 36, 120}	{135, 54, 58, 141}
{138, 57, 63, 146}	{140, 59, 67, 150}	{149, 68, 78, 161}	{153, 46, 24, 163}
{107, 26, 42, 125}	{134, 27, 69, 126}	{148, 21, 23, 142}	{156, 49, 73, 130}
{154, 47, 81, 145}	{155, 65, 33, 144}	{159, 15, 31, 147}	{132, 32, 71, 136}
{72, 143, 137, 12}	{50, 139, 123, 76}	{160, 79, 162, 0}	{1, 157, 11, 151}
{44, 158, 64, 152}			
{82, 1, 27, 110}	{83, 2, 30, 113}	{84, 3, 33, 116}	{85, 4, 36, 119}
{86, 5, 39, 122}	{89, 8, 47, 87}	{90, 10, 14, 98}	{91, 11, 23, 107}
{92, 12, 38, 138}	{93, 13, 40, 143}	{95, 15, 45, 153}	{101, 21, 75, 88}
{99, 20, 24, 121}	{104, 25, 31, 160}	{114, 35, 44, 96}	{97, 18, 32, 158}
{105, 26, 42, 134}	{130, 51, 72, 100}	{108, 29, 53, 102}	{147, 68, 16, 120}
{127, 49, 55, 103}	{124, 46, 56, 162}	{135, 57, 19, 118}	{48, 131, 132, 52}
{50, 133, 139, 58}	{54, 137, 145, 64}	{59, 142, 154, 73}	{66, 149, 163, 0}
{43, 126, 144, 63}	{17, 156, 148, 41}	{60, 117, 141, 34}	{9, 128, 106, 61}
{74, 157, 109, 28}	{7, 140, 161, 69}	{150, 70, 159, 80}	{151, 76, 125, 81}
{115, 37, 129, 71}	{79, 136, 6, 152}	{111, 65, 22, 94}	{62, 146, 155, 77}
{123, 78, 112, 67}			
{0, 83, 111, 30}	{1, 84, 114, 33}	{2, 85, 117, 36}	{3, 86, 124, 46}

{4, 88, 92, 12}	{5, 89, 96, 17}	{6, 90, 98, 19}	{7, 91, 103, 23}
{10, 94, 110, 34}	{11, 95, 113, 59}	{9, 93, 119, 55}	{15, 99, 126, 65}
{13, 97, 127, 71}	{16, 101, 105, 38}	{21, 106, 112, 77}	{40, 125, 134, 22}
{43, 128, 140, 31}	{24, 109, 130, 76}	{35, 120, 150, 8}	{47, 133, 138, 37}
{49, 135, 141, 25}	{58, 144, 154, 14}	{56, 142, 104, 39}	{107, 26, 146, 68}
{129, 48, 87, 45}	{131, 51, 137, 57}	{122, 42, 132, 53}	{147, 67, 161, 81}
{121, 41, 139, 63}	{160, 70, 162, 50}	{158, 78, 108, 52}	{82, 74, 149, 64}
{153, 60, 118, 66}	{73, 157, 20, 123}	{32, 143, 18, 152}	{80, 69, 145, 102}
{155, 75, 54, 148}	{72, 156, 159, 79}	{28, 100, 115, 61}	{151, 44, 136, 29}
{27, 116, 62, 163}			
{1, 87, 49, 148}	{82, 2, 103, 26}	{83, 3, 111, 47}	{84, 4, 113, 52}
{85, 5, 115, 75}	{89, 9, 145, 76}	{90, 11, 102, 72}	{91, 12, 110, 38}
{92, 13, 125, 65}	{88, 10, 97, 78}	{94, 16, 104, 70}	{96, 18, 144, 79}
{15, 98, 17, 101}	{29, 112, 69, 109}	{44, 127, 6, 123}	{24, 108, 30, 114}
{22, 106, 31, 116}	{23, 107, 33, 118}	{21, 105, 35, 119}	{51, 135, 71, 99}
{42, 126, 63, 150}	{7, 163, 59, 151}	{50, 134, 80, 122}	{55, 139, 28, 121}
{8, 93, 20, 154}	{39, 124, 56, 100}	{45, 130, 64, 156}	{74, 159, 25, 129}
{77, 149, 34, 143}	{0, 86, 14, 120}	{81, 147, 48, 142}	{66, 67, 157, 158}
{53, 54, 152, 153}	{57, 60, 146, 161}	{162, 137, 46, 73}	{131, 160, 62, 43}
{133, 140, 32, 37}	{132, 27, 141, 36}	{136, 61, 117, 40}	{19, 138, 58, 95}
{41, 155, 68, 128}			

□

References

- [1] H. Cao, L. Ji and L. Zhu, *Construction for generalized Steiner systems*, Des. Codes Cryptogr. **45** (2007), 185–197.