The notion of a monotonic directed design $(v,k,1)$-MDD was introduced to construct difference triangle sets in W. Chu, C. J. Colbourn and S. W. Golomb, A recursive construction for regular difference triangle sets, SIAM J. Discrete Math., vol. 18, pp. 741-748, 2005. In G. Ge, D. Huang and Y. Miao, Monotonic directed designs, SIAM J. Discrete Math., to appear, they described various constructions for monotonic directed designs, including those recursive constructions via a monotonic directed group divisible design (K,λ)-MDGDD. The following are some of the small designs they constructed which are essential to the establishment of the necessary and sufficient conditions for the existence of a monotonic directed design with block size 3, and with block size 4 having two definite exceptions and six possible exceptions.

1 Small examples of $(v,4,1)$-MDDs

The required blocks of the examples are listed below, where the points $\{0,1,\dots,v-1\}$ are linearly ordered by $0 \prec 1 \prec \cdots \prec v-1$.

$v=22:$
$(0,1,2,16)\ +2 \pmod{22}$, $(0,3,12,20)\ +2 \pmod{22}$, $(0,4,10,19)\ +2 \pmod{22}$,
$(0,5,17,21)\ +2 \pmod{22}$, $(1,4,11,0)\ +2 \pmod{22}$, $(1,7,15,20)\ +2 \pmod{22}$,
$(1,8,19,21)\ +2 \pmod{22}$.
$v=28:$
$(0,1,2,14)\ +2 \pmod{28}$, $(0,3,18,24)\ +2 \pmod{28}$, $(0,4,20,27)\ +2 \pmod{28}$,
$(0,8,21,25)\ +2 \pmod{28}$, $(0,9,19,26)\ +2 \pmod{28}$, $(1,3,23,26)\ +2 \pmod{28}$,
$(1,6,17,0)\ +2 \pmod{28}$, $(1,9,15,27)\ +2 \pmod{28}$, $(1,10,20,25)\ +2 \pmod{28}$.
$v=34:$
$(0,1,20,24)\ +2 \pmod{34}$, $(0,2,5,29)\ +2 \pmod{34}$, $(0,6,13,31)\ +2 \pmod{34}$,
$(0,11,22,32)\ +2 \pmod{34}$, $(0,14,23,33)\ +2 \pmod{34}$, $(0,17,21,30)\ +2 \pmod{34}$,
$(1,3,29,30)\ +2 \pmod{34}$, $(1,4,16,32)\ +2 \pmod{34}$, $(1,8,26,0)\ +2 \pmod{34}$,
$(1,9,15,31)\ +2 \pmod{34}$, $(1,13,18,33)\ +2 \pmod{34}$.
$v=40:$
$(0,1,2,22)\ +2 \pmod{40}$, $(0,3,5,34)\ +2 \pmod{40}$, $(0,4,10,35)\ +2 \pmod{40}$,
$(0,7,24,39)\ +2 \pmod{40}$, $(0,8,26,38)\ +2 \pmod{40}$, $(0,9,29,37)\ +2 \pmod{40}$,
$(0,13,23,36)\ +2 \pmod{40}$, $(1,4,25,37)\ +2 \pmod{40}$, $(1,5,19,35)\ +2 \pmod{40}$,
$(1,6,23,38)\ +2 \pmod{40}$, $(1,7,26,0)\ +2 \pmod{40}$, $(1,8,27,36)\ +2 \pmod{40}$,
$(1,12,28,39)\ +2 \pmod{40}$.
$v=46:$
$(2,18,20,42)\ +2 \pmod{46}$, $(2,27,35,45)\ +2 \pmod{46}$, $(2,13,41,43)\ +2 \pmod{46}$,
$(1,14,34,40)\ +2 \pmod{46}$, $(1,8,37,42)\ +2 \pmod{46}$, $(1,25,39,45)\ +2 \pmod{46}$,
$(1,13,35,44)\ +2 \pmod{46}$, $(1,16,26,43)\ +2 \pmod{46}$, $(2,14,21,37)\ +2 \pmod{46}$,
$(2,23,40,44)\ +2 \pmod{46}$, $(1,2,30,33)\ +2 \pmod{46}$, $(1,27,38,0)\ +2 \pmod{46}$,
$(2,11,15,38)\ +2 \pmod{46}$, $(1,4,36,41)\ +2 \pmod{40}$, $(2,32,0,1)\ +2 \pmod{46}$.
$v=52:$
$(2,13,23,49)\ +2 \pmod{52}$, $(2,21,33,37)\ +2 \pmod{52}$, $(2,11,39,1)\ +2 \pmod{52}$,
$(1,7,34,42)\ +2 \pmod{52}$, $(1,22,44,45)\ +2 \pmod{52}$, $(2,5,30,45)\ +2 \pmod{52}$,
$(2,12,29,51)\ +2 \pmod{52}$, $(1,25,33,48)\ +2 \pmod{52}$, $(2,9,43,48)\ +2 \pmod{52}$,
$(1,31,49,51)\ +2 \pmod{52}$, $(2,7,38,44)\ +2 \pmod{52}$, $(2,22,35,0)\ +2 \pmod{52}$,
$(1,2,47,50)\ +2 \pmod{52}$, $(1,8,20,0)\ +2 \pmod{52}$, $(2,16,40,42)\ +2 \pmod{52}$,
$(1,10,14,39)\ +2 \pmod{52}$, $(1,12,30,46)\ +2 \pmod{52}$.
$v=58:$
$(1,15,46,53)\ +2 \pmod{58}$, $(2,12,39,48)\ +2 \pmod{58}$, $(2,8,51,56)\ +2 \pmod{58}$,
$(2,20,21,55)\ +2 \pmod{58}$, $(1,21,56,0)\ +2 \pmod{58}$, $(1,2,40,54)\ +2 \pmod{58}$,
$(1,18,26,57)\ +2 \pmod{58}$, $(1,3,33,52)\ +2 \pmod{58}$, $(1,19,22,42)\ +2 \pmod{58}$,
$(2,24,28,52)\ +2 \pmod{58}$, $(2,14,31,46)\ +2 \pmod{58}$, $(1,7,34,55)\ +2 \pmod{58}$,
$(2,11,18,0)\ +2 \pmod{58}$, $(1,14,44,47)\ +2 \pmod{58}$, $(1,13,37,41)\ +2 \pmod{58}$,
$(2,25,36,47)\ +2 \pmod{58}$, $(2,27,43,53)\ +2 \pmod{58}$, $(2,7,49,57)\ +2 \pmod{58}$,
$(2,15,44,1)\ +2 \pmod{58}$.
$v=64:$
$(2,4,29,1)\ +2 \pmod{64}$, $(2,20,21,55)\ +2 \pmod{64}$, $(2,18,50,57)\ +2 \pmod{64}$,
$(1,6,57,0)\ +2 \pmod{64}$, $(2,7,36,0)\ +2 \pmod{64}$, $(2,5,38,59)\ +2 \pmod{64}$,
$(1,39,50,61)\ +2 \pmod{64}$, $(1,15,31,52)\ +2 \pmod{64}$, $(2,25,49,51)\ +2 \pmod{64}$,
$(2,32,42,56)\ +2 \pmod{64}$, $(2,33,39,58)\ +2 \pmod{64}$, $(1,16,42,59)\ +2 \pmod{64}$,
$(1,10,54,60)\ +2 \pmod{64}$, $(2,11,31,54)\ +2 \pmod{64}$, $(1,36,48,56)\ +2 \pmod{64}$,
$(1,5,45,53)\ +2 \pmod{64}$, $(1,18,51,63)\ +2 \pmod{64}$, $(1,33,43,46)\ +2 \pmod{64}$,
$(1,2,40,62)\ +2 \pmod{64}$, $(2,15,43,61)\ +2 \pmod{64}$, $(2,17,44,48)\ +2 \pmod{64}$.

$v = 76:$

$(0,12,48,56)\ +1 \pmod{76}$, $(0,19,37,68)\ +1 \pmod{76}$, $(0,46,72,73)\ +1 \pmod{76}$,
$(0,9,43,66)\ +1 \pmod{76}$, $(0,2,64,71)\ +1 \pmod{76}$, $(0,14,61,67)\ +1 \pmod{76}$,
$(0,22,51,54)\ +1 \pmod{76}$, $(0,13,24,52)\ +1 \pmod{76}$, $(0,41,58,74)\ +1 \pmod{76}$,
$(0,4,42,63)\ +1 \pmod{76}$, $(0,5,25,70)\ +4 \pmod{76}$, $(0,10,35,60)\ +4 \pmod{76}$,
$(0,20,65,75)\ +4 \pmod{76}$, $(5,30,60,75)\ +4 \pmod{76}$, $(5,35,40,4)\ +4 \pmod{76}$,
$(10,15,65,4)\ +4 \pmod{76}$, $(10,20,50,70)\ +4 \pmod{76}$, $(10,25,75,9)\ +4 \pmod{76}$,
$(15,35,75,14)\ +4 \pmod{76}$, $(15,45,50,9)\ +4 \pmod{76}$.

$v = 88:$

$(0,8,39,85)\ +1 \pmod{88}$, $(0,12,83,84)\ +1 \pmod{88}$, $(0,9,66,82)\ +1 \pmod{88}$,
$(0,27,48,86)\ +1 \pmod{88}$, $(0,22,26,63)\ +1 \pmod{88}$, $(0,52,58,69)\ +1 \pmod{88}$,
$(0,44,68,87)\ +1 \pmod{88}$, $(0,18,51,80)\ +1 \pmod{88}$, $(0,13,67,74)\ +1 \pmod{88}$,
$(0,14,42,78)\ +1 \pmod{88}$, $(0,47,49,81)\ +1 \pmod{88}$, $(0,3,56,79)\ +1 \pmod{88}$,
$(0,5,25,70)\ +4 \pmod{88}$, $(0,10,35,60)\ +4 \pmod{88}$, $(0,20,65,75)\ +4 \pmod{88}$,
$(5,30,60,75)\ +4 \pmod{88}$, $(5,35,40,80)\ +4 \pmod{88}$, $(10,15,65,80)\ +4 \pmod{88}$,
$(10,20,50,70)\ +4 \pmod{88}$, $(10,25,75,85)\ +4 \pmod{88}$, $(15,35,75,2)\ +4 \pmod{88}$,
$(15,45,50,85)\ +4 \pmod{88}$.

$v = 100:$

$(0,24,72,98)\ +1 \pmod{100}$, $(0,31,67,83)\ +1 \pmod{100}$, $(0,17,85,86)\ +1 \pmod{100}$,
$(0,32,89,91)\ +1 \pmod{100}$, $(0,38,47,80)\ +1 \pmod{100}$, $(0,23,76,87)\ +1 \pmod{100}$,
$(0,4,66,94)\ +1 \pmod{100}$, $(0,34,63,77)\ +1 \pmod{100}$, $(0,13,84,92)\ +1 \pmod{100}$,
$(0,49,56,93)\ +1 \pmod{100}$, $(0,39,51,97)\ +1 \pmod{100}$, $(0,22,41,95)\ +1 \pmod{100}$,
$(0,3,81,99)\ +1 \pmod{100}$, $(0,6,27,88)\ +1 \pmod{100}$, $(0,5,25,70)\ +4 \pmod{100}$,
$(0,10,35,60)\ +4 \pmod{100}$, $(0,20,65,75)\ +4 \pmod{100}$, $(5,30,60,75)\ +4 \pmod{100}$,
$(5,35,40,80)\ +4 \pmod{100}$, $(10,15,65,80)\ +4 \pmod{100}$, $(10,20,50,70)\ +4 \pmod{100}$,
$(10,25,75,85)\ +4 \pmod{100}$, $(15,35,75,90)\ +4 \pmod{100}$, $(15,45,50,85)\ +4 \pmod{100}$.

$v = 112:$

$(0,22,80,109)\ +1 \pmod{112}$, $(0,71,98,107)\ +1 \pmod{112}$, $(0,41,102,108)\ +1 \pmod{112}$,
$(0,2,49,83)\ +1 \pmod{112}$, $(0,3,51,88)\ +1 \pmod{112}$, $(0,33,99,106)\ +1 \pmod{112}$,
$(0,28,74,97)\ +1 \pmod{112}$, $(0,43,57,111)\ +1 \pmod{112}$, $(0,8,19,101)\ +1 \pmod{112}$,
$(0,24,96,100)\ +1 \pmod{112}$, $(0,39,77,95)\ +1 \pmod{112}$, $(0,13,92,104)\ +1 \pmod{112}$,
$(0,44,86,103)\ +1 \pmod{112}$, $(0,1,64,90)\ +1 \pmod{112}$, $(0,32,94,110)\ +1 \pmod{112}$,
$(0,21,52,105)\ +1 \pmod{112}$, $(0,5,25,70)\ +4 \pmod{112}$, $(0,10,35,60)\ +4 \pmod{112}$,
$(0,20,65,75)\ +4 \pmod{112}$, $(5,30,60,75)\ +4 \pmod{112}$, $(5,35,40,80)\ +4 \pmod{112}$,
$(10,15,65,80)\ +4 \pmod{112}$, $(10,20,50,70)\ +4 \pmod{112}$, $(10,25,75,85)\ +4 \pmod{112}$,
$(15,35,75,90)\ +4 \pmod{112}$, $(15,45,50,85)\ +4 \pmod{112}$.

$v = 118:$

$(0,11,78,114)\ +1 \pmod{118}$, $(0,21,62,115)\ +1 \pmod{118}$, $(0,33,71,117)\ +1 \pmod{118}$,
$(0,8,59,106)\ +1 \pmod{118}$, $(0,6,99,102)\ +1 \pmod{118}$, $(0,23,27,104)\ +1 \pmod{118}$,
$(0,29,86,87)\ +1 \pmod{118}$, $(0,82,91,113)\ +1 \pmod{118}$, $(0,42,76,108)\ +1 \pmod{118}$,
$(0,43,97,111)\ +1 \pmod{118}$, $(0,73,89,101)\ +1 \pmod{118}$, $(0,49,110,112)\ +1 \pmod{118}$,
$(0,24,88,107)\ +1 \pmod{118}$, $(0,17,56,69)\ +1 \pmod{118}$, $(0,37,109,116)\ +1 \pmod{118}$,
$(0,18,44,92)\ +1 \pmod{118}$, $(0,5,10,80)\ +2 \pmod{118}$, $(0,15,60,100)\ +2 \pmod{118}$,
$(0,20,50,95)\ +2 \pmod{118}$, $(0,25,85,105)\ +2 \pmod{118}$, $(5,20,55,110)\ +2 \pmod{118}$,
$(5,35,75,100)\ +2 \pmod{118}$, $(5,40,95,105)\ +2 \pmod{118}$.

$v = 124:$

$(0,6,66,113)\ +1 \pmod{124}$, $(0,22,32,94)\ +1 \pmod{124}$, $(0,65,95,108)\ +1 \pmod{124}$,
$(0,37,88,106)\ +1 \pmod{124}$, $(0,8,81,110)\ +1 \pmod{124}$, $(0,40,67,122)\ +1 \pmod{124}$,
$(0,80,85,121)\ +1 \pmod{124}$, $(0,44,119,120)\ +1 \pmod{124}$, $(0,12,58,111)\ +1 \pmod{124}$,
$(0,48,93,109)\ +1 \pmod{124}$, $(0,9,20,123)\ +1 \pmod{124}$, $(0,4,83,100)\ +1 \pmod{124}$,
$(0,33,101,104)\ +1 \pmod{124}$, $(0,25,115,117)\ +1 \pmod{124}$, $(0,31,54,118)\ +1 \pmod{124}$,
$(0,50,74,89)\ +1 \pmod{124}$, $(0,19,57,116)\ +1 \pmod{124}$, $(0,26,78,112)\ +1 \pmod{124}$,
$(0,7,35,98)\ +4 \pmod{124}$, $(0,14,49,84)\ +4 \pmod{124}$, $(0,28,91,105)\ +4 \pmod{124}$,
$(7,42,84,105)\ +4 \pmod{124}$, $(7,49,56,112)\ +4 \pmod{124}$, $(14,21,91,112)\ +4 \pmod{124}$,
$(14,28,70,98)\ +4 \pmod{124}$, $(14,35,105,119)\ +4 \pmod{124}$, $(21,49,105,2)\ +4 \pmod{124}$,
$(21,63,70,119)\ +4 \pmod{124}$.

$v = 130:$

$(0,22,79,126)\ +1 \pmod{130}$, $(0,71,88,124)\ +1 \pmod{130}$, $(0,41,83,84)\ +1 \pmod{130}$,
$(0,38,94,110)\ +1 \pmod{130}$, $(0,32,98,119)\ +1 \pmod{130}$, $(0,3,112,125)\ +1 \pmod{130}$,
$(0,59,96,120)\ +1 \pmod{130}$, $(0,77,121,123)\ +1 \pmod{130}$, $(0,9,48,117)\ +1 \pmod{130}$,
$(0,11,102,129)\ +1 \pmod{130}$, $(0,4,82,115)\ +1 \pmod{130}$, $(0,29,97,103)\ +1 \pmod{130}$,
$(0,26,93,127)\ +1 \pmod{130}$, $(0,28,51,114)\ +1 \pmod{130}$, $(0,14,106,113)\ +1 \pmod{130}$,
$(0,58,89,107)\ +1 \pmod{130}$, $(0,8,62,81)\ +1 \pmod{130}$, $(0,12,76,128)\ +1 \pmod{130}$,
$(0,5,10,80)\ +2 \pmod{130}$, $(0,15,60,100)\ +2 \pmod{130}$, $(0,20,50,95)\ +2 \pmod{130}$,
$(0,25,85,105)\ +2 \pmod{130}$, $(5,20,55,110)\ +2 \pmod{130}$, $(5,35,75,100)\ +2 \pmod{130}$,
$(5,40,95,105)\ +2 \pmod{130}$.

$v=142:$
$(0,48,67,124)+1 \pmod{142}$, $(0,53,119,121)+1 \pmod{142}$, $(0,37,133,134)+1 \pmod{142}$,
$(0,52,103,126)+1 \pmod{142}$, $(0,31,112,139)+1 \pmod{142}$, $(0,38,101,137)+1 \pmod{142}$,
$(0,82,129,136)+1 \pmod{142}$, $(0,11,109,122)+1 \pmod{142}$, $(0,3,86,92)+1 \pmod{142}$,
$(0,44,123,131)+1 \pmod{142}$, $(0,59,73,77)+1 \pmod{142}$, $(0,39,72,141)+1 \pmod{142}$,
$(0,16,104,132)+1 \pmod{142}$, $(0,24,117,138)+1 \pmod{142}$, $(0,34,128,140)+1 \pmod{142}$,
$(0,49,91,113)+1 \pmod{142}$, $(0,9,71,127)+1 \pmod{142}$, $(0,29,61,107)+1 \pmod{142}$,
$(0,26,43,84)+1 \pmod{142}$, $(0,5,10,70)+2 \pmod{142}$, $(0,15,90,120)+2 \pmod{142}$,
$(0,20,100,135)+2 \pmod{142}$, $(0,40,105,125)+2 \pmod{142}$, $(0,45,95,130)+2 \pmod{142}$,
$(5,15,115,130)+2 \pmod{142}$, $(5,30,85,140)+2 \pmod{142}$, $(5,45,75,135)+2 \pmod{142}$,
$(5,50,100,125)+2 \pmod{142}$.

$v=172:$
$(0,33,101,114)+1 \pmod{172}$, $(0,29,56,152)+1 \pmod{172}$, $(0,9,67,153)+1 \pmod{172}$,
$(0,11,42,159)+1 \pmod{172}$, $(0,47,104,163)+1 \pmod{172}$, $(0,22,24,143)+1 \pmod{172}$,
$(0,36,108,169)+1 \pmod{172}$, $(0,8,91,162)+1 \pmod{172}$, $(0,43,109,171)+1 \pmod{172}$,
$(0,14,77,156,)+1 \pmod{172}$, $(0,51,124,158)+1 \pmod{172}$, $(0,12,111,139)+1 \pmod{172}$,
$(0,76,122,129)+1 \pmod{172}$, $(0,52,146,149)+1 \pmod{172}$, $(0,26,132,164)+1 \pmod{172}$,
$(0,21,134,157)+1 \pmod{172}$, $(0,19,131,137)+1 \pmod{172}$, $(0,54,98,147)+1 \pmod{172}$,
$(0,38,102,141)+1 \pmod{172}$, $(0,151,167,168)+1 \pmod{172}$, $(0,37,78,126)+1 \pmod{172}$,
$(0,4,88,170)+1 \pmod{172}$, $(0,69,87,161)+1 \pmod{172}$, $(0,5,100,120)+2 \pmod{172}$,
$(0,10,25,145)+2 \pmod{172}$, $(0,30,65,155)+2 \pmod{172}$, $(0,55,110,160)+2 \pmod{172}$,
$(0,70,115,165)+2 \pmod{172}$, $(0,85,105,150)+2 \pmod{172}$, $(5,15,145,150)+2 \pmod{172}$,
$(5,20,80,160)+2 \pmod{172}$, $(5,40,130,170)+2 \pmod{172}$, $(5,45,75,155)+2 \pmod{172}$,
$(5,65,90,165)+2 \pmod{172}$.

$v=178:$
$(0,49,143,167)+1 \pmod{178}$, $(0,98,144,176)+1 \pmod{178}$, $(0,112,156,174)+1 \pmod{178}$,
$(0,39,103,107)+1 \pmod{178}$, $(0,2,121,124)+1 \pmod{178}$, $(0,12,153,161)+1 \pmod{178}$,
$(0,54,87,170)+1 \pmod{178}$, $(0,48,117,139)+1 \pmod{178}$, $(0,63,76,177)+1 \pmod{178}$,
$(0,81,152,163)+1 \pmod{178}$, $(0,6,142,168)+1 \pmod{178}$, $(0,99,106,173)+1 \pmod{178}$,
$(0,29,157,158)+1 \pmod{178}$, $(0,77,108,169)+1 \pmod{178}$, $(0,37,133,171)+1 \pmod{178}$,
$(0,16,113,164)+1 \pmod{178}$, $(0,21,147,175)+1 \pmod{178}$, $(0,56,73,109)+1 \pmod{178}$,
$(0,52,111,138)+1 \pmod{178}$, $(0,57,66,159)+1 \pmod{178}$, $(0,104,127,146)+1 \pmod{178}$,
$(0,84,131,172)+1 \pmod{178}$, $(0,14,72,151)+1 \pmod{178}$, $(0,34,123,166)+1 \pmod{178}$,
$(0,5,100,120)+2 \pmod{178}$, $(0,10,25,145)+2 \pmod{178}$, $(0,30,65,155)+2 \pmod{178}$,
$(0,55,110,160)+2 \pmod{178}$, $(0,70,115,165)+2 \pmod{178}$, $(0,85,105,150)+2 \pmod{178}$,
$(5,15,145,150)+2 \pmod{178}$, $(5,20,80,160)+2 \pmod{178}$, $(5,40,130,170)+2 \pmod{178}$,
$(5,45,75,155)+2 \pmod{178}$, $(5,65,90,165)+2 \pmod{178}$.

$v=184:$
$(0,62,111,183)+1 \pmod{184}$, $(0,17,88,149)+1 \pmod{184}$, $(0,31,128,179)+1 \pmod{184}$,
$(0,37,84,138)+1 \pmod{184}$, $(0,21,154,177)+1 \pmod{184}$, $(0,39,108,146)+1 \pmod{184}$,
$(0,18,144,152)+1 \pmod{184}$, $(0,57,123,181)+1 \pmod{184}$, $(0,73,102,176)+1 \pmod{184}$,
$(0,7,106,173)+1 \pmod{184}$, $(0,36,64,178)+1 \pmod{184}$, $(0,53,136,182)+1 \pmod{184}$,
$(0,14,93,112)+1 \pmod{184}$, $(0,11,158,174)+1 \pmod{184}$, $(0,41,68,159)+1 \pmod{184}$,
$(0,63,167,180)+1 \pmod{184}$, $(0,3,122,164)+1 \pmod{184}$, $(0,94,171,172)+1 \pmod{184}$,
$(0,43,52,139)+1 \pmod{184}$, $(0,2,34,143)+1 \pmod{184}$, $(0,127,131,153)+1 \pmod{184}$,
$(0,113,157,169)+1 \pmod{184}$, $(0,24,116,175)+1 \pmod{184}$, $(0,6,82,168)+1 \pmod{184}$,
$(0,33,81,170)+1 \pmod{184}$, $(0,5,100,120)+2 \pmod{184}$, $(0,10,25,145)+2 \pmod{184}$,
$(0,30,65,155)+2 \pmod{184}$, $(0,55,110,160)+2 \pmod{184}$, $(0,70,115,165)+2 \pmod{184}$,
$(0,85,105,150)+2 \pmod{184}$, $(5,15,145,150)+2 \pmod{184}$, $(5,20,80,160)+2 \pmod{184}$,
$(5,40,130,170)+2 \pmod{184}$, $(5,45,75,155)+2 \pmod{184}$, $(5,65,90,165)+2 \pmod{184}$.

$v=202:$
$(0,29,102,151)+1 \pmod{202}$, $(0,52,126,184)+1 \pmod{202}$, $(0,24,181,192)+1 \pmod{202}$,
$(0,82,114,193)+1 \pmod{202}$, $(0,38,61,179)+1 \pmod{202}$, $(0,76,109,153)+1 \pmod{202}$,
$(0,13,177,186)+1 \pmod{202}$, $(0,37,103,166)+1 \pmod{202}$, $(0,97,119,188)+1 \pmod{202}$,
$(0,144,147,178)+1 \pmod{202}$, $(0,12,84,171)+1 \pmod{202}$, $(0,78,121,167)+1 \pmod{202}$,
$(0,54,196,197)+1 \pmod{202}$, $(0,14,162,183)+1 \pmod{202}$, $(0,62,158,199)+1 \pmod{202}$,
$(0,2,108,191)+1 \pmod{202}$, $(0,7,88,156)+1 \pmod{202}$, $(0,174,182,201)+1 \pmod{202}$,
$(0,48,104,161)+1 \pmod{202}$, $(0,94,136,187)+1 \pmod{202}$, $(0,4,131,198)+1 \pmod{202}$,
$(0,17,124,163)+1 \pmod{202}$, $(0,28,99,200)+1 \pmod{202}$, $(0,47,133,139)+1 \pmod{202}$,
$(0,16,128,154)+1 \pmod{202}$, $(0,18,116,152)+1 \pmod{202}$, $(0,53,117,176)+1 \pmod{202}$,
$(0,5,10,110)+2 \pmod{202}$, $(0,15,25,170)+2 \pmod{202}$, $(0,20,50,175)+2 \pmod{202}$,
$(0,35,120,195)+2 \pmod{202}$, $(0,40,130,190)+2 \pmod{202}$, $(0,45,145,185)+2 \pmod{202}$,
$(0,65,115,180)+2 \pmod{202}$, $(5,20,125,185)+2 \pmod{202}$, $(5,25,95,175)+2 \pmod{202}$,
$(5,30,115,190)+2 \pmod{202}$, $(5,35,130,200)+2 \pmod{202}$, $(5,40,135,180)+2 \pmod{202}$,
$(5,60,140,195)+2 \pmod{202}$.

2 Small examples of $(\{4\},1)$-MDGDDs of type 2^u

The required blocks of the examples are listed below, where the point set is $\{0,1,\ldots,2u-1\}$, and the groups $\{\{i,u+i\} \mid 0 \le i \le u-1\}$ are linearly ordered by $\{0,u\} \prec \{1,u+1\} \prec \cdots \prec \{u-1,2u-1\}$.

$u = 16:$
$(0,20,7,15)\ +1 \pmod{32}$, $(0,22,11,13)\ +1 \pmod{32}$, $(0,3,12,29)\ +1 \pmod{32}$,
$(0,1,6,31)\ +1 \pmod{32}$, $(0,4,28,14)\ +1 \pmod{32}$.
$u = 19:$
$(0,28,16,36)\ +1 \pmod{38}$, $(0,24,13,18)\ +1 \pmod{38}$, $(0,10,33,17)\ +1 \pmod{38}$,
$(0,2,31,37)\ +1 \pmod{38}$, $(0,1,12,15)\ +1 \pmod{38}$, $(0,4,25,34)\ +1 \pmod{38}$.
$u = 22:$
$(0,7,16,43)\ +1 \pmod{44}$, $(0,2,34,42)\ +1 \pmod{44}$, $(0,1,15,20)\ +1 \pmod{44}$,
$(0,23,12,41)\ +1 \pmod{44}$, $(0,10,13,38)\ +1 \pmod{44}$, $(0,26,30,21)\ +1 \pmod{44}$,
$(0,6,37,17)\ +1 \pmod{44}$.
$u = 25:$
$(0,35,21,47)\ +1 \pmod{50}$, $(0,28,38,17)\ +1 \pmod{50}$, $(0,34,40,49)\ +1 \pmod{50}$,
$(0,7,18,20)\ +1 \pmod{50}$, $(0,31,14,22)\ +1 \pmod{50}$, $(0,27,43,23)\ +1 \pmod{50}$,
$(0,3,4,48)\ +1 \pmod{50}$, $(0,5,42,24)\ +1 \pmod{50}$.
$u = 28:$
$(0,34,40,54)\ +1 \pmod{56}$, $(0,33,18,50)\ +1 \pmod{56}$, $(0,31,21,23)\ +1 \pmod{56}$,
$(0,9,24,53)\ +1 \pmod{56}$, $(0,8,11,27)\ +1 \pmod{56}$, $(0,13,51,52)\ +1 \pmod{56}$,
$(0,4,47,26)\ +1 \pmod{56}$, $(0,7,12,49)\ +1 \pmod{56}$, $(0,30,10,55)\ +1 \pmod{56}$.
$u = 31:$
$(0,13,18,56)\ +1 \pmod{62}$, $(0,34,36,29)\ +1 \pmod{62}$, $(0,39,47,30)\ +1 \pmod{62}$,
$(0,35,19,25)\ +1 \pmod{62}$, $(0,44,60,61)\ +1 \pmod{62}$, $(0,12,22,54)\ +1 \pmod{62}$,
$(0,3,23,27)\ +1 \pmod{62}$, $(0,40,49,28)\ +1 \pmod{62}$, $(0,7,21,58)\ +1 \pmod{62}$,
$(0,11,26,59)\ +1 \pmod{62}$.
$u = 34:$
$(0,49,23,32)\ +1 \pmod{68}$, $(0,13,19,66)\ +1 \pmod{68}$, $(0,35,50,28)\ +1 \pmod{68}$,
$(0,37,55,67)\ +1 \pmod{68}$, $(0,40,20,64)\ +1 \pmod{68}$, $(0,17,62,33)\ +1 \pmod{68}$,
$(0,2,56,31)\ +1 \pmod{68}$, $(0,1,59,27)\ +1 \pmod{68}$, $(0,5,57,65)\ +1 \pmod{68}$,
$(0,22,25,63)\ +1 \pmod{68}$, $(0,7,11,21)\ +1 \pmod{68}$.
$u = 37:$
$(0,11,20,71)\ +1 \pmod{74}$, $(0,42,66,72)\ +1 \pmod{74}$, $(0,3,13,32)\ +1 \pmod{74}$,
$(0,39,56,57)\ +1 \pmod{74}$, $(0,44,69,35)\ +1 \pmod{74}$, $(0,49,21,36)\ +1 \pmod{74}$,
$(0,45,68,73)\ +1 \pmod{74}$, $(0,41,67,31)\ +1 \pmod{74}$, $(0,52,54,33)\ +1 \pmod{74}$,
$(0,7,50,34)\ +1 \pmod{74}$, $(0,4,16,63)\ +1 \pmod{74}$, $(0,8,22,70)\ +1 \pmod{74}$.
$u = 43:$
$(0,14,82,83)\ +1 \pmod{86}$, $(0,7,59,78)\ +1 \pmod{86}$, $(0,4,53,34)\ +1 \pmod{86}$,
$(0,21,23,77)\ +1 \pmod{86}$, $(0,9,74,85)\ +1 \pmod{86}$, $(0,46,66,35)\ +1 \pmod{86}$,
$(0,47,31,41)\ +1 \pmod{86}$, $(0,44,18,81)\ +1 \pmod{86}$, $(0,26,29,42)\ +1 \pmod{86}$,
$(0,15,32,40)\ +1 \pmod{86}$, $(0,24,72,36)\ +1 \pmod{86}$, $(0,45,73,38)\ +1 \pmod{86}$,
$(0,6,64,39)\ +1 \pmod{86}$, $(0,22,27,84)\ +1 \pmod{86}$.
$u = 46:$
$(0,55,34,81)\ +1 \pmod{92}$, $(0,12,35,41)\ +1 \pmod{92}$, $(0,11,65,33)\ +1 \pmod{92}$,
$(0,9,88,45)\ +1 \pmod{92}$, $(0,17,32,83)\ +1 \pmod{92}$, $(0,8,38,86)\ +1 \pmod{92}$,
$(0,50,25,43)\ +1 \pmod{92}$, $(0,16,21,73)\ +1 \pmod{92}$, $(0,10,74,87)\ +1 \pmod{92}$,
$(0,7,89,91)\ +1 \pmod{92}$, $(0,3,27,31)\ +1 \pmod{92}$, $(0,53,20,90)\ +1 \pmod{92}$,
$(0,1,69,40)\ +1 \pmod{92}$, $(0,56,75,44)\ +1 \pmod{92}$, $(0,58,72,42)\ +1 \pmod{92}$.
$u = 49:$
$(0,54,33,37)\ +1 \pmod{98}$, $(0,62,68,34)\ +1 \pmod{98}$, $(0,14,80,89)\ +1 \pmod{98}$,
$(0,7,78,90)\ +1 \pmod{98}$, $(0,63,24,41)\ +1 \pmod{98}$, $(0,69,91,92)\ +1 \pmod{98}$,
$(0,2,30,40)\ +1 \pmod{98}$, $(0,8,43,96)\ +1 \pmod{98}$, $(0,56,32,87)\ +1 \pmod{98}$,
$(0,11,36,97)\ +1 \pmod{98}$, $(0,13,18,39)\ +1 \pmod{98}$, $(0,3,85,45)\ +1 \pmod{98}$,
$(0,27,84,46)\ +1 \pmod{98}$, $(0,52,72,47)\ +1 \pmod{98}$, $(0,29,44,94)\ +1 \pmod{98}$,
$(0,51,67,48)\ +1 \pmod{98}$.
$u = 58:$
$(0,60,71,39)\ +1 \pmod{116}$, $(0,64,101,57)\ +1 \pmod{116}$, $(0,23,44,111)\ +1 \pmod{116}$,
$(0,8,91,115)\ +1 \pmod{116}$, $(0,7,99,42)\ +1 \pmod{116}$, $(0,97,100,110)\ +1 \pmod{116}$,
$(0,81,41,114)\ +1 \pmod{116}$, $(0,70,89,48)\ +1 \pmod{116}$, $(0,69,96,46)\ +1 \pmod{116}$,
$(0,68,80,50)\ +1 \pmod{116}$, $(0,79,28,113)\ +1 \pmod{116}$, $(0,14,32,54)\ +1 \pmod{116}$,
$(0,6,108,55)\ +1 \pmod{116}$, $(0,26,103,104)\ +1 \pmod{116}$, $(0,62,20,51)\ +1 \pmod{116}$,
$(0,61,90,106)\ +1 \pmod{116}$, $(0,36,38,53)\ +1 \pmod{116}$, $(0,4,47,56)\ +1 \pmod{116}$,
$(0,25,30,112)\ +1 \pmod{116}$.

$u = 61:$
$(0,4,111,116)\ +1 \pmod{122}$, $(0,83,91,105)\ +1 \pmod{122}$, $(0,7,42,57)\ +1 \pmod{122}$,
$(0,90,99,117)\ +1 \pmod{122}$, $(0,6,80,58)\ +1 \pmod{122}$, $(0,71,84,59)\ +1 \pmod{122}$,
$(0,10,88,36)\ +1 \pmod{122}$, $(0,94,51,113)\ +1 \pmod{122}$, $(0,72,46,109)\ +1 \pmod{122}$,
$(0,68,32,60)\ +1 \pmod{122}$, $(0,73,33,53)\ +1 \pmod{122}$, $(0,16,103,56)\ +1 \pmod{122}$,
$(0,3,47,48)\ +1 \pmod{122}$, $(0,11,49,115)\ +1 \pmod{122}$, $(0,81,24,54)\ +1 \pmod{122}$,
$(0,2,41,108)\ +1 \pmod{122}$, $(0,23,92,121)\ +1 \pmod{122}$, $(0,34,55,119)\ +1 \pmod{122}$,
$(0,17,93,118)\ +1 \pmod{122}$, $(0,31,43,120)\ +1 \pmod{122}$.
$u = 67:$
$(0,7,39,55)\ +1 \pmod{134}$, $(0,8,43,119)\ +1 \pmod{134}$, $(0,94,46,66)\ +1 \pmod{134}$,
$(0,73,103,129)\ +1 \pmod{134}$, $(0,99,110,61)\ +1 \pmod{134}$, $(0,95,124,126)\ +1 \pmod{134}$,
$(0,4,104,122)\ +1 \pmod{134}$, $(0,74,83,37)\ +1 \pmod{134}$, $(0,21,40,62)\ +1 \pmod{134}$,
$(0,78,34,58)\ +1 \pmod{134}$, $(0,25,116,52)\ +1 \pmod{134}$, $(0,79,47,60)\ +1 \pmod{134}$,
$(0,89,117,127)\ +1 \pmod{134}$, $(0,81,45,59)\ +1 \pmod{134}$, $(0,109,57,132)\ +1 \pmod{134}$,
$(0,3,87,131)\ +1 \pmod{134}$, $(0,5,54,125)\ +1 \pmod{134}$, $(0,72,15,123)\ +1 \pmod{134}$,
$(0,6,107,42)\ +1 \pmod{134}$, $(0,68,121,133)\ +1 \pmod{134}$, $(0,1,93,64)\ +1 \pmod{134}$,
$(0,17,50,130)\ +1 \pmod{134}$.

3 Small examples of $(\{4\},1)$-MDGDDs of type 3^u

The required blocks of examples are listed below, where the point set is $\{0,1,\dots,3u-1\}$, and the groups $\{\{i,u+i,2u+i\} \mid 0 \le i \le u-1\}$ are linearly ordered by $\{0,u,2u\} \prec \{1,u+1,2u+1\} \prec \cdots \prec \{u-1,2u-1,3u-1\}$.

$u = 11:$
$(0,23,29,32)\ +1 \pmod{33}$, $(0,12,30,31)\ +1 \pmod{33}$, $(0,27,8,10)\ +1 \pmod{33}$,
$(0,15,7,20)\ +1 \pmod{33}$, $(0,4,28,21)\ +1 \pmod{33}$.
$u = 12:$
$(2,7,23,12)\ +2 \pmod{36}$, $(1,27,10,35)\ +2 \pmod{36}$, $(1,15,33,12)\ +2 \pmod{36}$,
$(1,30,21,24)\ +2 \pmod{36}$, $(2,18,10,13)\ +2 \pmod{36}$, $(1,2,8,22)\ +2 \pmod{36}$,
$(2,3,31,25)\ +2 \pmod{36}$, $(1,14,32,0)\ +2 \pmod{36}$, $(2,4,19,0)\ +2 \pmod{36}$,
$(1,28,5,11)\ +2 \pmod{36}$, $(2,28,35,1)\ +2 \pmod{36}$.
$u = 13:$
$(0,1,35,25)\ +1 \pmod{39}$, $(0,16,9,36)\ +1 \pmod{39}$, $(0,31,22,11)\ +1 \pmod{39}$,
$(0,33,37,12)\ +1 \pmod{39}$, $(0,15,21,38)\ +1 \pmod{39}$, $(0,2,7,10)\ +1 \pmod{39}$.
$u = 14:$
$(1,6,27,0)\ +2 \pmod{42}$, $(2,6,11,14)\ +2 \pmod{42}$, $(1,3,39,14)\ +2 \pmod{42}$,
$(2,5,26,28)\ +2 \pmod{42}$, $(2,18,37,13)\ +2 \pmod{42}$, $(2,31,22,29)\ +2 \pmod{42}$,
$(1,32,38,28)\ +2 \pmod{42}$, $(2,32,24,15)\ +2 \pmod{42}$, $(1,7,8,26)\ +2 \pmod{42}$,
$(1,35,23,13)\ +2 \pmod{42}$, $(1,30,5,40)\ +2 \pmod{42}$, $(2,33,0,1)\ +2 \pmod{42}$,
$(2,17,25,41)\ +2 \pmod{42}$.
$u = 15:$
$(0,5,7,44)\ +1 \pmod{45}$, $(0,35,26,12)\ +1 \pmod{45}$, $(0,3,41,14)\ +1 \pmod{45}$,
$(0,21,10,27)\ +1 \pmod{45}$, $(0,4,13,29)\ +1 \pmod{45}$, $(0,19,42,43)\ +1 \pmod{45}$,
$(0,8,40,28)\ +1 \pmod{45}$.
$u = 17:$
$(0,1,41,33)\ +1 \pmod{51}$, $(0,2,7,11)\ +1 \pmod{51}$, $(0,6,45,16)\ +1 \pmod{51}$,
$(0,36,13,48)\ +1 \pmod{51}$, $(0,21,46,14)\ +1 \pmod{51}$, $(0,37,24,15)\ +1 \pmod{51}$,
$(0,3,30,50)\ +1 \pmod{51}$, $(0,18,26,49)\ +1 \pmod{51}$.
$u = 19:$
$(0,46,53,16)\ +1 \pmod{57}$, $(0,22,54,55)\ +1 \pmod{57}$, $(0,40,13,34)\ +1 \pmod{57}$,
$(0,24,11,17)\ +1 \pmod{57}$, $(0,39,8,18)\ +1 \pmod{57}$, $(0,23,48,52)\ +1 \pmod{57}$,
$(0,2,45,37)\ +1 \pmod{57}$, $(0,3,31,15)\ +1 \pmod{57}$, $(0,9,14,56)\ +1 \pmod{57}$.

4 Small examples of $(\{4\},1)$-MDGDDs of type 6^u

The required blocks of examples are listed below, where the point set is $\{0,1,\dots,6u-1\}$, and the groups $\{\{i,u+i,2u+i,3u+i,4u+i,5u+i\} \mid 0 \le i \le u-1\}$ are linearly ordered by $\{0,u,2u,3u,4u,5u\} \prec \{1,u+1,2u+1,3u+1,4u+1,5u+1\} \prec \cdots \prec \{u-1,2u-1,3u-1,4u-1,5u-1,6u-1\}$.

$u = 9:$
$(0, 39, 33, 26)\ +1 \pmod{54}$, $(0, 2, 14, 52)\ +1 \pmod{54}$, $(0, 20, 6, 17)\ +1 \pmod{54}$,
$(0, 19, 24, 53)\ +1 \pmod{54}$, $(0, 13, 16, 8)\ +1 \pmod{54}$, $(0, 10, 42, 25)\ +1 \pmod{54}$,
$(0, 1, 23, 44)\ +1 \pmod{54}$, $(0, 28, 4, 35)\ +1 \pmod{54}$.

$u = 11:$
$(0, 17, 29, 54)\ +1 \pmod{66}$, $(0, 34, 7, 21)\ +1 \pmod{66}$, $(0, 36, 27, 42)\ +1 \pmod{66}$,
$(0, 46, 50, 32)\ +1 \pmod{66}$, $(0, 2, 18, 10)\ +1 \pmod{66}$, $(0, 47, 30, 43)\ +1 \pmod{66}$,
$(0, 24, 19, 9)\ +1 \pmod{66}$, $(0, 60, 20, 65)\ +1 \pmod{66}$, $(0, 1, 41, 64)\ +1 \pmod{66}$,
$(0, 3, 38, 31)\ +1 \pmod{66}$.

$u = 12:$
$(0, 25, 6, 71)\ +1 \pmod{72}$, $(0, 1, 5, 21)\ +1 \pmod{72}$, $(0, 49, 31, 33)\ +1 \pmod{72}$,
$(0, 41, 19, 58)\ +1 \pmod{72}$, $(0, 15, 42, 70)\ +1 \pmod{72}$, $(0, 26, 22, 11)\ +1 \pmod{72}$,
$(0, 13, 43, 23)\ +1 \pmod{72}$, $(0, 51, 45, 59)\ +1 \pmod{72}$, $(0, 18, 9, 47)\ +1 \pmod{72}$,
$(0, 3, 67, 35)\ +1 \pmod{72}$, $(0, 37, 44, 34)\ +1 \pmod{72}$.

$u = 14:$
$(0, 76, 11, 13)\ +1 \pmod{84}$, $(0, 35, 23, 26)\ +1 \pmod{84}$, $(0, 1, 7, 40)\ +1 \pmod{84}$,
$(0, 16, 66, 83)\ +1 \pmod{84}$, $(0, 15, 64, 53)\ +1 \pmod{84}$, $(0, 45, 79, 69)\ +1 \pmod{84}$,
$(0, 46, 10, 68)\ +1 \pmod{84}$, $(0, 71, 65, 41)\ +1 \pmod{84}$, $(0, 59, 52, 12)\ +1 \pmod{84}$,
$(0, 57, 5, 9)\ +1 \pmod{84}$, $(0, 18, 61, 81)\ +1 \pmod{84}$, $(0, 29, 80, 27)\ +1 \pmod{84}$,
$(0, 30, 8, 55)\ +1 \pmod{84}$.

$u = 15:$
$(0, 61, 40, 73)\ +1 \pmod{90}$, $(0, 17, 53, 87)\ +1 \pmod{90}$, $(0, 4, 83, 43)\ +1 \pmod{90}$,
$(0, 6, 86, 44)\ +1 \pmod{90}$, $(0, 64, 9, 58)\ +1 \pmod{90}$, $(0, 67, 54, 29)\ +1 \pmod{90}$,
$(0, 46, 57, 59)\ +1 \pmod{90}$, $(0, 66, 23, 74)\ +1 \pmod{90}$, $(0, 16, 26, 88)\ +1 \pmod{90}$,
$(0, 37, 42, 28)\ +1 \pmod{90}$, $(0, 63, 82, 14)\ +1 \pmod{90}$, $(0, 31, 55, 56)\ +1 \pmod{90}$,
$(0, 7, 85, 27)\ +1 \pmod{90}$, $(0, 18, 21, 89)\ +1 \pmod{90}$.

$u = 17:$
$(0, 74, 100, 16)\ +1 \pmod{102}$, $(0, 42, 63, 30)\ +1 \pmod{102}$, $(0, 73, 98, 101)\ +1 \pmod{102}$,
$(0, 75, 27, 67)\ +1 \pmod{102}$, $(0, 1, 10, 33)\ +1 \pmod{102}$, $(0, 71, 62, 48)\ +1 \pmod{102}$,
$(0, 72, 46, 84)\ +1 \pmod{102}$, $(0, 89, 47, 83)\ +1 \pmod{102}$, $(0, 4, 57, 81)\ +1 \pmod{102}$,
$(0, 20, 78, 31)\ +1 \pmod{102}$, $(0, 56, 41, 49)\ +1 \pmod{102}$, $(0, 35, 13, 15)\ +1 \pmod{102}$,
$(0, 92, 29, 99)\ +1 \pmod{102}$, $(0, 52, 97, 14)\ +1 \pmod{102}$, $(0, 61, 66, 50)\ +1 \pmod{102}$,
$(0, 6, 43, 65)\ +1 \pmod{102}$.

$u = 18:$
$(0, 20, 82, 17)\ +1 \pmod{108}$, $(0, 77, 47, 50)\ +1 \pmod{108}$, $(0, 61, 84, 71)\ +1 \pmod{108}$,
$(0, 79, 87, 34)\ +1 \pmod{108}$, $(0, 92, 14, 51)\ +1 \pmod{108}$, $(0, 73, 31, 53)\ +1 \pmod{108}$,
$(0, 19, 44, 89)\ +1 \pmod{108}$, $(0, 40, 69, 52)\ +1 \pmod{108}$, $(0, 57, 7, 106)\ +1 \pmod{108}$,
$(0, 1, 65, 104)\ +1 \pmod{108}$, $(0, 4, 98, 32)\ +1 \pmod{108}$, $(0, 6, 33, 107)\ +1 \pmod{108}$,
$(0, 2, 85, 15)\ +1 \pmod{108}$, $(0, 41, 46, 35)\ +1 \pmod{108}$, $(0, 26, 11, 86)\ +1 \pmod{108}$,
$(0, 59, 80, 68)\ +1 \pmod{108}$, $(0, 76, 24, 16)\ +1 \pmod{108}$.