



THE UNIVERSITY OF VERMONT
DEPARTMENT OF MATHEMATICS AND STATISTICS
SIXTY-SEVENTH ANNUAL HIGH SCHOOL PRIZE EXAMINATION
TUESDAY, MARCH 12, 2024

THIS EXAMINATION IS TO BE ADMINISTERED DURING SCHOOL HOURS ON MARCH 12, 2024. AN EXAMINATION GIVEN AT ANY OTHER TIME WILL BE DISQUALIFIED.

THE TIME LIMIT ON THIS EXAMINATION IS 2 HOURS.

INSTRUCTIONS TO THE CONTESTANTS:

Do not begin the examination until the examiner tells you to do so.

The answer sheet is on the reverse side of this page. Before beginning the examination, carefully print your full name, your address, the complete name of your school and the town/city in which your school is located on the appropriate lines of the answer sheet. Check the circle corresponding to your grade level in school.

Answers must be written on the answer sheet in pencil or ink. You may work on problems in any order, but be sure that each answer is entered in the proper space on the answer sheet. (For example, if you solve number 12 first, make sure the answer is placed beside the 12 on the answer sheet.) All questions are weighted equally. Answer as many questions as you can in the allotted time. No contestant is expected to solve all of the problems.

CALCULATORS, COMPUTERS AND/OR ANY OTHER ELECTRONIC DEVICES ARE NOT PERMITTED.

UNLESS OTHERWISE INDICATED, ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST FORM.

A radical expression of index n is in simplest form if the radicand is not a fraction, denominators are rationalized and integer radicands do not have any factors that are n th powers of a prime. For example, $\sqrt{\frac{5}{12}}$ simplifies to $\frac{\sqrt{15}}{6}$.

Do NOT approximate the number π .

Do NOT approximate radicals.

The notation **log** is logarithm to the base 10.

The notation **log_a** is logarithm to the base a . The notation **ln** is logarithm to the base e .

The symbol **!** is the factorial symbol. For example, $3! = 3 \cdot 2 \cdot 1 = 6$.

The symbol ***i*** is the complex unit $\sqrt{-1}$.

All numbers are in base 10 unless otherwise indicated (e.g., 1001_2 is the base 2 representation of the decimal number 9).

Any answer that is a non-integer rational number must be expressed in the form $\frac{a}{b}$, where a and b are integers that have no common divisor other than 1.

ANSWER SHEET (2024)

PLEASE PRINT CLEARLY**ANSWERS**

STUDENT'S FULL NAME _____

STUDENT'S ADDRESS _____

NAME OF SCHOOL _____

TOWN (OR CITY) OF SCHOOL _____

WHAT GRADE ARE YOU IN? 9th 10th 11th 12th Other _____

1. 1 / 16

2. 14 triangles

3. 5 two-dollar bills

4. 8 square units

5. 75 newspapers

6. 2019 / 2 or 1009.5

7. 39 / 2 or 19.5 square units

8. 15 full circles

9. 1125

10. 6 liters

11. $x = 3$ and 15

12. $b = 4 / 133$

13. $p = 8$ $q = -19$

14. 7623

15. $x = 11 / 3$

16. 9:36 AM

17. $5 + 2\sqrt{2}$ (miles)²

18. $c = 210$

19. 1013 / 1012

20. 16 integers

21. 204 square units

22. 160 positive integers

23. 36 / 5 square units

24. 3 / 2

25. 2077₈

26. 58 units

27. 75 seconds

28. $\pi / 4$

29. 232 ice cream orders

30. 12 / 13

31. $a + p + q = -2$

32. 4

33. 1 / 56

34. $100\sqrt{13} / 13$ units

35. 108 children

36. $3\sqrt{5} / 5$ units

37. 25 ways

38. -12 / 13

39. 98 paths

40. $15\sqrt{7} / 4$ square units

41. (2, -1)